COMPLETE COURSE
IN
COMMERCIAL ARITHMETIC

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GENERAL INSTRUCTIONS

In the study of Arithmetic we have to deal with students in various stages of advancement. Some may want to carry the study right from the beginning. Others may feel that previous training will entitle them to start farther on in the study. Your first duty will be to scan the "Table of Contents," pages iii and iv of the course, and to notify us if any of the earlier parts of the work are to be omitted. Unless we hear to the contrary, we take it for granted that you wish to begin at the beginning.

Having settled where you wish to begin, always bear in mind that the instructor has arranged the matter in the course, page after page, in two divisions—

(1)	(2)
Instruction.	Exercises to test the student's grasp of the instruction.

This may seem obvious, but a long experience has shown us that most of the difficulties of the student who is taking a course by home study spring from the fact that the student is attempting the exercises without reading the instruction, or, if he has read the instruction, it has been done by a hit and miss plan, which skips over as much as it takes in. Just bear in mind that one of the great incidental benefits to be derived from any home study course is the formation of a habit of careful reading.

Read every line and be sure you understand every line. If you do this you will learn more than Arithmetic from this course.

We are not going to ask you to submit the work of the " Exercises " arranged in the course. These are for practice purposes only. The list of Answers given at the end of the text will enable you to check the correctness of your work on every question. We shall specify throughout the course the work that is to be submitted to the instructor for correction and criticism.

Should you have difficulty in solving any of the problems in the "Exercises" you may submit your working for these, pointing out, if possible, where your difficulty lies, and your instructor will explain to you the point that is not clear. Do not, however, submit several problems of the one type. An explanation with regard to one will, no doubt, help you to solve the others.

On the other hand, students may as well understand at the outset that requests such as the following: "Please send me solutions for all the questions in Exercise No. —," will be met by a polite but firm refusal. **The instructor is not taking a course in arithmetic.** It is the business of the student to send in solutions and the business of the instructor to go over those solutions, criticize, correct, offer help, and show the way, where the student is really in difficulty. The student who thinks he will learn Arithmetic by reading over solutions may as well learn at the outset that he is mistaken. Arithmetic is intended to develop reasoning power, and we do that by exercising **our own** brains, not by reading over the results of another's reasoning.

Method of Proceeding with Work.

These general instructions will not be repeated. Whether the student intends to start at the beginning or not, it will be well to follow through these instructions, as they relate to the earlier sections of the work, because they will serve as a model for succeeding sections.

1. Read carefully the explanatory matter given on pages 1 and 2.

2. Work the questions of Exercise 1, or as many of them as you feel are necessary to test your knowledge.

3. Check over your answers by the list of answers supplied to you.

4. Feel perfectly free to submit any difficulties to us, **but do not conclude that you have struck a difficulty until you have worked and re-worked the question** without success in getting the correct answer.

5. Read further explanation on page 3.

6. Exercise 2 may be dealt with according to instructions 2, 3 and 4 in this series.

7. With this start the student should now be able to work his way through the remainder of this section, always keeping in mind the plan of reading thoroughly the explanatory matter leading up to an exercise before attempting the questions in the exercise.

Test Questions.

As a test of your knowledge of the work we ask you to submit to us your solutions of certain questions (See "TEST PAPERS" Special Directions to Students). Work should be done neatly in pen and ink.

Make it your definite aim to do neat, rapid, accurate work throughout.

Form the habit, early in your course, and in the simplest questions, of giving an orderly and logical solution.

As you advance in the course the questions will naturally become more difficult and the solutions more involved, but the habit, once formed, of arranging your thoughts in order will help you over many a difficulty.

Question: If 5 yd. of lace cost \$7.50, what will 8 yd. cost ?

Solution:

5 yd. of lace cost	\$7.50
1 yd. of lace costs \$7.50 ÷ 5 =	\$1.50
8 yd. of lace cost \$1.50 × 8 =	\$12.00

Question: If 4 yd. of cloth cost \$8.00, how many yards can be bought for \$12.00 ?

Solution:

1st step:	4 yd. of cloth cost	\$8.00
	1 yd. of cloth cost \$8.00 ÷ 4 =	\$2.00
2nd step:	\$2.00 buys	1 yd.
	\$12.00 buys 12.00 ÷ 2.00 =	6 yd.

In sending in solutions to the "TEST PAPERS," prepare paper as follows—

Leave a margin of 1 in. at the left side of the paper. Rule a vertical line for this margin. Number the solutions consecutively in this margin. Leave one line blank after each solution. Separate one solution from another by a double line ruled across the paper. Underline each answer. Do all ruling in red ink. Follow this plan throughout the course.

COMMERCIAL ARITHMETIC

TEST PAPERS

SPECIAL DIRECTIONS TO THE STUDENT

Read the General Directions—pages v to viii immediately following the Table of Contents. Study same carefully until you know how to prepare and submit your work for checking.

The Test Papers are printed on coloured paper and follow the instructional text matter on which they are based.

Study the text and submit the Test Paper for checking in the order that follows—

1. Factors and Multiples—pages 1 to 7—Test Paper No. 1.

2. Common or Vulgar Fractions—pages 8 to 30—Test Paper No. 2.

3. Decimal Fractions—pages 31 to 53—Test Paper No. 3.

4. Denominate Numbers—pages 54 to 70—Test Paper No. 4.

5. Percentage—pages 71 to 98—Test Paper No. 5.

6. Simple Interest—pages 99 to 118—Test Paper No. 6.

7. Bank Discount—pages 119 to 126—Test Paper No. 7.

8. Profit and Loss, Commission, Brokerage—pages 127 to 143—Test Paper No. 8.

9. Stocks and Bonds—pages 144 to 161—Test Paper No. 9

10. Taxes, Insurance, Customs Duties—pages 162 to 173—Test Paper No. 10

11. Average and its Application—pages 174 to 183—Test Paper No. 11.

12. Compound Interest—pages 184 to 194—Test Paper No. 12.

13. Foreign Trade—pages 195 to 205—Test Paper No. 13.

14. Partnership, Partial Payments, Cash Balance—pages 206 to 241—Test Paper No. 14.

15 True Discount, Annuities—pages 242 to 258—Test Paper No. 15.

16. Involution and Evolution, Mensuration—pages 259 to 278—Test Paper No. 16.

17. Practical Measurements, Metric System—pages 279 to 291—Test Paper No. 17.

SECTION 1

FACTORS AND MULTIPLES

FACTORS

1. A Factor of a number is any exact divisor of that number, except itself and one.

Thus, 3 and 2 will each divide 6 exactly, so they are both said to be factors of 6.

Every factor of a number has its co-factor, which, when multiplied by it, produces the number.

Thus, 3 and 2 are co-factors in relation to the number 6.

2. A Prime Number is one that has no factors; as 2, 5, 7, 11, 13, etc.

3. A Composite Number is one that can be resolved into factors; as 8, 10, 24, etc.

4. A Prime Factor is a factor which is a prime number.

Thus, 3, 5, and 7 are prime factors of 105.

5. A Composite Factor is a factor which is a composite number.

Thus, 15, 21, and 35 are composite factors of 105.

6. Tests of Exact Divisibility. The following tests of exact divisibility are often useful in a search for the factors of a number.

(1) A number is exactly divisible by 2 if its right-hand figure is zero or a number exactly divisible by 2.

(2) A number is exactly divisible by 4 if its two right-hand figures are zeros or express a number exactly divisible by 4.

EXAMPLES.—173528 is exactly divisible by 4, for 28 is exactly divisible by 4; but 319378 is not a multiple of 4, for 78 is not exactly divisible by 4.

(3) A number is exactly divisible by 8 if its three right-hand figures are zeros or express a number exactly divisible by 8.

EXAMPLES.—536 is a multiple of 8, therefore 1397536 is exactly divisible by 8; but 356 is not a multiple of 8, consequently 4679356 is not exactly divisible by 8.

(4) A number is exactly divisible by 5, 25, 125, ... if the number expressed by the right-hand figure or the two, three, ... right-hand figures is exactly divisible by 5, 25, 125, ...

(5) A number is exactly divisible by 3 if the sum of its digits is exactly divisible by 3.

(6) A number is exactly divisible by 9 if the sum of its digits is exactly divisible by 9.

EXAMPLES.—Test whether 18637569 and 7385621 are divisible by 9.

$1 + 8 + 6 + 3 + 7 + 5 + 6 + 9 = 45 = 9 \times 5$, $\therefore$ 18637569 is exactly divisible by 9.

$7 + 3 + 8 + 5 + 6 + 2 + 1 = 32 = 9 \times 3 + 5$, $\therefore$ 7385621 is not exactly divisible by 9.

(7) A number is exactly divisible by 10 if its right-hand figure is zero.

EXERCISE 1

Make a list of the following numbers that are exactly divisible by (*a*) 2, (*b*) 3, (*c*) 4, (*d*) 5, (*e*) 8, (*f*) 9, (*g*) 10, without actually dividing—

1. 345672.
2. 2143560.
3. 3657247.
4. 2803652.
5. 5329170.
6. 4362416.
7. 3758362.
8. 43621545.
9. 213654829.
10. 3267037021.

7. Factoring is the process of separating a number into its prime factors.

Illustration.—Factor the number 1260.

2 | 1260	
2 | 630, first quotient.	
3 | 315, second quotient.	
3 | 105, third quotient.	
5 | 35, fourth quotient.	
7, last quotient.	

SOLUTION.—Since the number is even, divide it by 2. $1260 \div 2 = 630$. The first quotient being even, divide again by 2. $630 \div 2 = 315$. The sum of the digits in 315, $(3 + 1 + 5)$, being 9, which is divisible by 3, we divide by 3 and get the third quotient, 105. For the reason last given, divide by 3 again and we get 35. Now divide by 5, since 35 ends with 5. Dividing thus we get 7, the last quotient, which completes the work, the prime factors being 2, 2, 3, 3, 5, and 7.

8. Rule.

Divide the given number by its least prime factor; similarly divide the resulting quotient; and so continue until a prime quotient is obtained. The several divisors and the last quotient are the prime factors.

PROOF

The continued product of all the prime factors should equal the given number.

EXERCISE 2

Resolve the following numbers into their prime factors—

1. 64.	6. 1575.	11. 15625.
2. 144.	7. 2376.	12. 18216.
3. 512.	8. 2744.	13. 21659.
4. 1050.	9. 4367.	14. 25785
5. 1527.	10. 6435.	15. 38577.

9. A Common Factor, or Divisor, of two or more numbers is a factor which belongs to each of them.

Thus, since the prime factors of 6 are 3 and 2, and of 10 are 5 and 2, it will be seen that 2 is a common factor of 6 and 10.

10. The Highest Common Factor (or, as it is sometimes called, the Greatest Common Measure or Greatest Common Divisor) of two or more numbers is the largest factor common to the numbers.

Thus, all the exact divisors of 12 are 2, 3, 4, and 6; of 18, are 2, 3, 6, and 9; and of 30, are 2, 3, 5, 6, 10, and 15. The divisors common to the three numbers are 2, 3, and 6. The Greatest Common Divisor or Highest Common Factor of the three numbers is, therefore, 6.

11. Numbers are prime to each other when they have no common factor.

Thus, the prime factors of 9 are 3 and 3, and of 22 are 2 and 11. 9 and 22 are, therefore, prime to each other.

Illustration 1.—What is the G.C.D. or the H.C.F. of 30, 45, 60, and 75?

$30 = 2 \times 3 \times 5$
$45 = 3 \times 3 \times 5$
$60 = 2 \times 2 \times 3 \times 5$
$75 = 3 \times 5 \times 5$
H.C.F. $= 3 \times 5 = 15$

SOLUTION 1.—Factor each number and pick out all the common factors. The product of all the common factors will be the H.C.F.

SOLUTION 2.—The G.C.D. of these numbers cannot be greater than 30; if 30 is an exact divisor of the rest of the numbers, it is the G.C.D. By inspection it is found not to be a divisor of all the numbers. Dropping the smallest factor, 2, out of 30, and using the other one, 15, as a divisor, we find by inspection that 15 is an exact divisor of all the numbers; therefore, it is the G.C.D. of 30, 45, 60, and 75.

If the numbers are large and the factoring therefore difficult, the following method is commonly used—

Illustration 2.—Find the G.C.M. of 1645 and 1833.

SOLUTION.—First Method—

```
1645)1833(1
     1645
     ----
 188)1645(8
     1504
     ----
  141)188(1
      141
      ---
   47)141(3
      141
      ---
        0
```

G.C.M. = 47

12. Rule.

Divide the greater number by the less, and the last divisor by the last remainder, and so continue to divide until a quotient is obtained without a remainder. The last divisor obtained will be the greatest common divisor.

Where more than two numbers are given, first find the G.C.D. of any two of them; then of the G.C.D. thus found and the third number, etc.

The following arrangement of the work necessitates the placing of fewer figures on the paper—

SOLUTION.—Second Method—

		1833
1645	1	1645
1504	8	——
——		188
141	1	141
141	3	——
——		47
0		

G.C.M. = 47

EXERCISE 3

Find the G.C.D. of the following numbers—

Mentally.

1. 12 and 18.
2. 15 and 25.
3. 18 and 24.
4. 20 and 36.
5. 24 and 72.

By Factoring.

6. 48 and 84.
7. 60 and 90.
8. 75 and 125.
9. 84, 96, and 120.
10. 48, 72, and 360.

By Division.

11. 964 and 1272.
12. 729 and 1701.
13. 513 and 1368.
14. 4125 and 6750.
15. 8778 and 9702
16. 4853 and 5697.
17. 1177 and 1498.
18. 2827 and 10537.
19. 1027, 2133, and 2449.
20. 1157, 2047, and 2937.
21. 1268, 1472, and 1684.

EXERCISE 4

1. A man has four large lots, containing respectively 675,

1,125, 1,575, and 1,800 sq. ft. He wishes to divide them all into smaller lots of the largest possible size to contain an equal number of square feet. How many square feet will each of the smaller lots contain?

2. How many feet in circumference is the largest locomotive drive-wheel that will make an exact number of revolutions in going a distance of 513, 608, and 1,368 ft. respectively?

3. Find the largest number that will divide 49, 73, 97, and 121, and leave a remainder of 1 each time.

4. What is the largest number that will divide 179 and 301 and leave remainders of 17 and 13 respectively?

5. At the intersection of three avenues is a triangular piece of ground whose separate sides are 64, 80, and 112 ft. What is the length of the longest boards that can be used to build a fence around it, if the boards are of equal length and no allowance is made for waste?

6. A carpenter is directed to make the widest possible sidewalk, no wastage, out of the following planks each 1 foot in width and 2 in. in thickness: 9 planks 20 ft. long, 8 planks 16 ft. long, and 6 planks 12 ft. long. What will be the length and width of the walk? (Planks to run crosswise of the walk.)

MULTIPLES

13. A Multiple of a number is a number which will contain the given number exactly.

Thus, 12 is a multiple of 4, of 3, of 6, or of 2.

14. A Common Multiple of two or more numbers is a number which will contain each number exactly.

Thus, 12 is a multiple of 3, and 12 is also a multiple of 4. It is, therefore, a common multiple of 3 and 4.

15. The Lowest Common Multiple of two or more numbers is the lowest, or smallest, number that will contain each of them exactly.

16. Illustration.—What is the L.C.M. of 75, 120, 150, and 180?

SHORT METHOD SOLUTION.

2)~~75~~	120	150	180
2)	60	75	90
3)	30	75	45
5)	10	25	15
	2	5	3

$2 \times 2 \times 3 \times 5 \times 2 \times 5 \times 3 = 1800$

Set the numbers down in a row as for short division. Strike 75 out of the list, as it is contained exactly in 150, and we know that any number that will contain 150 will contain 75. Beginning with 2, the lowest prime number (except 1), we see that it will divide all three numbers, so we divide each number by it, setting the quotients immediately below the numbers. Again, we see that 2 will divide 60 and 90, so we divide them by it as before. 2 will not divide 75 exactly, so we simply set it down among the quotients. Now, we see that 2 will not divide *two* of the numbers, so we try 3, which will divide all of them. Then, since 3 will not divide two of the numbers again, we try 5, the next prime number, and see that it will divide all three of them. After dividing by 5, we see that there is no number that will divide any of the two quotients. The L.C.M. is the product of the final quotients and the various divisors, $2 \times 2 \times 3 \times 5 \times 2 \times 5 \times 3 = 1800$. It will be noted that these factors are the same as those in the previous solutions, with different arrangement. If at any stage of the work we had found any number, such as 75, which was exactly contained in any of the other numbers in the same line, we would have struck it out as we did the 75.

EXERCISE 5

Find the L.C.M. of—

1. 32 and 88.
2. 34 and 60.
3. 57 and 95.
4. 81 and 117.
5. 36 and 150.
6. 12, 44, 60.
7. 44, 88, 100.
8. 3, 5, 8, 9, 10, 15.
9. 5, 6, 10, 12, 15.
10. 4, 8, 12, 16, 50.
11. 144, 1728, 60.
12. 56, 512, 128.
13. 11, 33, 66, 165.
14. 9, 18, 45, 63.
15. 14, 49, 77, 22.

COMMERCIAL ARITHMETIC

TEST PAPER—No. 1

FACTORS AND MULTIPLES

Your work for this paper is to be submitted to the instructor for correction and criticism.

1. Define—Factor, Factoring, Multiple, a Prime Number, a Composite Number.

2. Write all the Prime Numbers less than one hundred.

3. Without actually dividing, make a list of the following numbers that are exactly divisible by: 2, 3, 4, 5, 6, 8, 9, 10, 11, 12. Give your reason for your answer in each case.

(*a*) 345,672.
(*b*) 2,143,560.
(*c*) 5,329,170.
(*d*) 4,362,416.
(*e*) 43,621,545

4 The product of four consecutive numbers is 73,440 Find the numbers.

5. Find the G.C.D. of 12,870,000 and 504,504 ; and the L.C.M. of 611, 141, 273, and 4,277.

6. Find the G.C.D. of 5,040, 7,770, 9,912, and 10,773.

7. In problem 6, divide the given numbers by their G.C.D. and resolve the quotients into their prime factors. Is any prime factor common to all four quotients ?

8. Find the L.C.M. of 7,920 yd. and 4,400 yd.

9. A, B, and C walk a distance of 103,950 in. together. How often will they step at the same moment, A taking 33 in. at a step, B 27, and C 30 ?

10. Which of the following numbers are prime, and which are composite: 2,491, 2,699, 14,787, and 1,477 ?

Business Advice from One of Canada's Foremost Manufacturers

Mr. J. H. Fortier

1. Work conscientiously.
2. Keep on studying.
3. Be enthusiastic. Pessimism gets you nowhere. But guard against extravagance.
4. Don't be afraid of responsibility.
5. Have definite hours of work.
6. Having made a decision, go to it. Correct as you go.
7. Undertake few things, but complete them. From small things train yourself to bigger ones.
8. Stick to your job. Hold what you have. Progress every day.
9. Worry is wearing, physically and mentally. Don't cross the river till you come to it.
10. Take recreation—but don't let play disinterest you in your business.
11. Meet successful people. Learn from them. Profit from every occasion.
12. Stand for something good in your community. Boost your village, town, province and country. It tends toward general progress—and it benefits yourself.

COMMON OR VULGAR FRACTIONS

17. A Fraction is simply a part.

Cut an apple in two equal parts, and each of the parts is known as one-half of the apple.

$\frac{1}{2}$ is a Proper Fraction, *one* being the numerator and *two* being the denominator.

$\frac{2}{2}$ is an Improper Fraction.

18. A Proper Fraction represents a part less than a whole.

19. An Improper Fraction represents either a whole amount, or more than a whole amount, as $\frac{9}{8}$; $\frac{7}{5}$; $\frac{4}{4}$.

If we cut several apples of the same size each into halves, we know that every time we take up two of these parts we have one whole apple. This we could represent as 1, or as $\frac{2}{2}$. Suppose we pick up 3 pieces. We would then have three-halves, or we could speak of them as one apple and one-half apple.

Expressing this in figures, $\frac{3}{2} = 1\frac{1}{2}$.

In the last expression we have a combination of a fraction and a whole number, or we speak of it as a Mixed Number.

EXERCISE 6

Exercise 1.—Divide a sheet of paper by ruling into three columns, heading the first column "Proper Fractions," the second "Improper Fractions," and the third "Mixed Numbers." Sort out the following expressions by placing each one in the column to which it belongs. Put the expressions one beneath the other in straight lines down the columns—

$\frac{1}{2}$, $\frac{3}{2}$, $4\frac{1}{2}$, $\frac{3}{8}$, $\frac{15}{4}$, $2\frac{3}{4}$, $\frac{5}{8}$, $\frac{12}{18}$, $6\frac{1}{8}$, $4\frac{1}{3}$, $\frac{2}{3}$, $\frac{12}{8}$, $6\frac{1}{5}$, $\frac{17}{5}$, $\frac{3}{5}$, $5\frac{1}{6}$, $\frac{16}{6}$, $\frac{4}{6}$, $2\frac{1}{6}$.

$14\frac{2}{7}$, $\frac{5}{7}$, $\frac{15}{7}$, $11\frac{1}{9}$, $\frac{5}{9}$, $\frac{14}{9}$, $\frac{21}{9}$, $\frac{2}{9}$, $\frac{12}{10}$, $\frac{1}{10}$, $5\frac{1}{10}$, $\frac{23}{10}$, $7\frac{1}{11}$, $\frac{2}{11}$, $\frac{15}{11}$, $8\frac{1}{15}$,

$\frac{17}{15}$, $23\frac{1}{3}$, $7\frac{2}{6}$, $4\frac{8}{11}$, $\frac{15}{27}$, $9\frac{7}{30}$, $2\frac{8}{17}$, $\frac{15}{22}$, $9\frac{8}{25}$, $\frac{47}{125}$, $29\frac{7}{250}$, $10\frac{12}{225}$, $\frac{17}{864}$,

$29\frac{1}{8}$, $\frac{74}{11}$, $\frac{86}{115}$.

Exercise 2.—Go down your list of Improper Fractions, and set opposite each one the Mixed Number or the Whole Number to which it is equivalent.

Exercise 3.—Go down your columns of Mixed Numbers and set opposite each one the Improper Fraction to which it is equivalent.

20. REDUCTION ASCENDING

If we cut an apple in two equal parts, we have in each part one-half of the apple. Should we take one-half of the apple and cut it in two equal parts, we have in each half two-quarters. Now the two-quarters are just exactly equal to the one-half. There is a difference in form, but not in value. If we continue cutting, and give each of the two-quarters a cut into two equal parts, we should have, out of our one-half apple, $\frac{4}{8}$. From this simple example we gather that $\frac{1}{2}$, $\frac{2}{4}$, and $\frac{4}{8}$ are the same in value. This shows us that there is another way in which we may change or reduce fractions. We may change the form without changing the value. This is of great use to us in such operations as addition and subtraction. In the same way we add $\frac{1}{2}$ and $\frac{1}{4}$ by figuring that $\frac{1}{2}$ is equal to $\frac{2}{4}$. The sum of $\frac{2}{4}$ and $\frac{1}{4}$ is $\frac{3}{4}$.

Illustration.—Reduce $\frac{2}{3}$ to sixths.

$\frac{2}{3} \times \frac{2}{2} = \frac{4}{6}$

SOLUTION.—Dividing 3 into 6, we get 2, which tells us that there are always twice as many sixths in any amount as there are thirds. If we have $\frac{2}{3}$, we must have twice 2 or four-sixths.

21. Rule.

Divide the required denominator by the denominator of the given fraction.

Multiply the numerator of the given fraction by the quotient thus obtained and write the product over the required denominator.

The result is the fraction in higher terms.

EXERCISE 7

Change—

1. $\frac{2}{3}$ to 15ths.
2. $\frac{1}{5}$ to 15ths.
3. $\frac{2}{7}$ to 21sts.
4. $\frac{2}{3}$ to 21sts.
5. $\frac{5}{12}$ to 36ths.
6. $\frac{1}{3}$ to 36ths.
7. $\frac{5}{9}$ to 36ths.
8. $\frac{3}{9}$ to 27ths.
9. $\frac{3}{40}$ to 120ths.
10. $\frac{5}{8}$ to 88ths.
11. $\frac{7}{13}$ to 52nds.
12. $\frac{2}{23}$ to 115ths.
13. $\frac{3}{16}$ to 128ths.
14. $\frac{5}{24}$ to 192nds.
15. $\frac{11}{21}$ to 147ths.

22. REDUCTION DESCENDING

We have seen that it is possible to change the form of a fraction without changing its value. This we did by multiplying both parts of the fraction by the same number. By reversing the process, we should be able to change a fraction of a higher denomination into an equivalent one of a lower denomination.

Illustration.—Reduce $\frac{8}{16}$ to its lowest terms.

$\frac{8}{16} \div \frac{8}{8} = \frac{1}{2}$

SOLUTION.—To get the fraction in its lowest terms, we must know the largest number that will divide both 8 and 16. In a simple case of this kind, we can tell by inspection that this number is 8. Dividing both 8 and 16 by 8, we produce the fraction $\frac{1}{2}$, which is an expression of $\frac{8}{16}$ in its lowest terms.

Illustration.—Reduce $\frac{192}{330}$ to its lowest terms.

$\frac{192}{330} \div \frac{6}{6} = \frac{32}{55}$

SOLUTION.—Where the fraction is large, we find the largest number that will divide both terms by finding the highest common factor. The H.C.F. of 192 and 330 is 6. Divide 6 into each of the numbers and we get the fraction $\frac{32}{55}$.

23. Rule.

Cancel all factors common to both numerator and denominator; or divide both terms by their greatest common divisor.

EXERCISE 8

Reduce to their lowest terms—

1. $\frac{16}{24}$
2. $\frac{25}{60}$
3. $\frac{78}{192}$
4. $\frac{50}{625}$
5. $\frac{112}{280}$
6. $\frac{39}{143}$
7. $\frac{504}{792}$
8. $\frac{396}{432}$
9. $\frac{385}{539}$
10. $\frac{498}{830}$
11. $\frac{549}{671}$
12. $\frac{936}{1368}$
13. $\frac{485}{679}$
14. $\frac{1680}{1920}$
15. $\frac{5184}{6912}$
16. $\frac{1623}{3787}$
17. $\frac{2695}{3080}$
18. $\frac{5608}{7711}$
19. $\frac{3178}{5221}$
20. $\frac{352211}{372248}$

24. REDUCTION TO A COMMON DENOMINATOR

To reduce a series of fractions to the same common denominator.

Illustration.—Reduce $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ to 12ths.

$\frac{1}{2} \times \frac{6}{6} = \frac{6}{12}$
$\frac{1}{3} \times \frac{4}{4} = \frac{4}{12}$
$\frac{1}{4} \times \frac{3}{3} = \frac{3}{12}$

SOLUTION.—The number expressing the denomination to which we may reduce halves, thirds, and quarters must be a number which will contain 2, 3, 4 exactly. This, in other words, must be the L.C.M. of 2, 3, 4. The L.C.M. of these three numbers is 12. $\frac{1}{2}$ reduced to twelfths gives us $\frac{6}{12}$; $\frac{1}{3}$ reduced to twelfths gives us $\frac{4}{12}$; and $\frac{1}{4}$ reduced to twelfths gives us $\frac{3}{12}$.

25. Rule.

1. *Find the L.C.M. of the denominators.*

2. *Divide this L.C.M. by the denominator of the fraction that is being reduced and multiply the terms of that fraction by the quotient thus obtained.*

3. *Continue with the fractions in regular order until all have been reduced.*

Before beginning the above operation, each fraction should be in its lowest terms, and mixed numbers should be reduced to improper fractions.

Whole numbers are written in the form of a fraction by writing 1 for the denominator.

EXERCISE 9

Reduce to equivalent fractions having the L.C.D.

1. $\frac{1}{2}$ and $\frac{2}{3}$
2. $\frac{1}{2}$ and $\frac{1}{4}$
3. $\frac{3}{5}$ and $\frac{7}{10}$
4. $\frac{5}{6}$, $\frac{7}{12}$, and $\frac{11}{18}$
5. $\frac{2}{9}$, $\frac{4}{27}$, and $\frac{19}{54}$
6. $\frac{7}{10}$, $\frac{9}{25}$, and $\frac{11}{75}$
7. $\frac{17}{36}$, $\frac{35}{48}$, $\frac{11}{12}$, and $\frac{43}{60}$
8. $\frac{13}{35}$, $\frac{31}{45}$, $\frac{15}{63}$, and $\frac{59}{84}$
9. $\frac{9}{10}$, $\frac{11}{15}$, $\frac{17}{20}$, and $\frac{21}{25}$
10. $\frac{6}{18}$, $\frac{8}{15}$, $\frac{9}{30}$, and $\frac{23}{60}$
11. $\frac{5}{9}$, $\frac{11}{30}$, $\frac{43}{60}$, and $\frac{14}{90}$
12. $\frac{7}{12}$, $\frac{14}{18}$, $\frac{13}{24}$, and $\frac{23}{36}$

26. ADDITION OF FRACTIONS

Illustration 1.—Find the sum of $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$.

SOLUTION. First Step—

$\frac{2}{3} \times \frac{4}{4} = \frac{8}{12}$
$\frac{3}{4} \times \frac{3}{3} = \frac{9}{12}$
$\frac{5}{6} \times \frac{2}{2} = \frac{10}{12}$

Change these fractions to equivalent fractions having the same common denominator by the process already illustrated. As a result we have $\frac{8}{12}$, $\frac{9}{12}$, $\frac{10}{12}$.

SOLUTION. Second Step—

$\frac{8}{12}$ may be expressed, 8 twelfths.
$\frac{9}{12}$ may be expressed, 9 twelfths.
$\frac{10}{12}$ may be expressed, 10 twelfths.
The sum of these is 27 twelfths.

Having once changed the fractions to the same common denominator, the addition may be just as easily carried out as the addition of eight pounds, nine pounds, and ten pounds.

$27 \text{ twelfths} = \frac{27}{12}$
$\frac{27}{12} = 2\frac{3}{12}$ or $2\frac{1}{4}$

Note that it is the numerator or the part which shows how many that enters into the addition. After we have found that there are 27 twelfths in the sum of these three fractions, we reduce this improper fraction to a mixed number, $2\frac{3}{12}$, which in turn we reduce to its lowest terms, $2\frac{1}{4}$.

Illustration 2.—Find the sum of $3\frac{1}{4}$, $1\frac{2}{3}$, $4\frac{1}{2}$.

$3 + 1 + 4 = 8$
$\frac{1}{4} = \frac{3}{12}$
$\frac{2}{3} = \frac{8}{12}$
$\frac{1}{2} = \frac{6}{12}$
$\frac{17}{12} = 1\frac{5}{12}$
$9\frac{5}{12}$

SOLUTION.—Find first the sum of the whole numbers, which is 8. Next find the sum of the fractions by the method already illustrated. They amount to $\frac{17}{12}$, or $1\frac{5}{12}$. Add 8 and $1\frac{5}{12}$, and we have the complete sum, $9\frac{5}{12}$.

27. Rule.

Reduce to equivalent fractions of a common denominator; add the resulting numerators; and place the sum over the common denominator.

NOTE 1.—To find the sum of two or more mixed numbers: *add the fractions and integers separately, and combine the two results.*

NOTE 2.—When necessary, all results, if proper fractions, should be reduced to lowest terms; and if improper fractions, to equivalent mixed numbers.

Add— **EXERCISE 10**

1. $\frac{3}{4}$, $\frac{2}{3}$, and $\frac{4}{5}$
2. $\frac{4}{5}$, $\frac{7}{8}$, $\frac{9}{10}$, and $2\frac{7}{20}$
3. $4\frac{1}{2}$, $2\frac{1}{3}$, $5\frac{3}{4}$ and $6\frac{7}{8}$
4. $\frac{7}{8}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{16}$, and $\frac{11}{24}$
5. $\frac{5}{9}$, $\frac{3}{5}$, $\frac{7}{15}$, $\frac{3}{10}$, and $\frac{3}{20}$
6. $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, and $\frac{5}{6}$
7. $1\frac{1}{2}$, $2\frac{2}{3}$, $3\frac{3}{4}$, and $4\frac{4}{5}$
8. $4\frac{1}{2}$, $\frac{3}{16}$, $5\frac{3}{8}$, and $\frac{1}{4}$
9. 7, $8\frac{3}{4}$, $9\frac{1}{7}$, $6\frac{5}{28}$, and 8
10. $3\frac{1}{2}$, $9\frac{5}{112}$, 8, $7\frac{5}{56}$, and $6\frac{4}{7}$
11. $\frac{3}{4}$, $\frac{7}{8}$, $\frac{9}{16}$, $\frac{17}{28}$, and $\frac{9}{56}$
12. $5\frac{11}{14}$, $9\frac{111}{112}$, $\frac{85}{56}$, $1\frac{7}{16}$, and $1\frac{5}{14}$
13. $2\frac{5}{11}$, $3\frac{2}{27}$, $4\frac{7}{55}$, and $8\frac{4}{135}$
14. $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{7}$, $\frac{1}{9}$, and $\frac{1}{11}$
15. $364\frac{7}{8}$, $243\frac{5}{6}$, $327\frac{7}{24}$, and $162\frac{9}{16}$
16. $123\frac{5}{16}$, $247\frac{2}{3}$, $842\frac{1}{48}$, and $375\frac{7}{24}$
17. $325\frac{5}{8}$, $426\frac{5}{12}$, $324\frac{5}{24}$, and $136\frac{5}{48}$
18. $243\frac{7}{16}$, $327\frac{7}{24}$, $436\frac{7}{48}$, and $244\frac{7}{96}$

28. SUBTRACTION OF FRACTIONS

Illustration 1.—Find the difference between $\frac{5}{8}$ and $\frac{2}{3}$.

$\frac{2}{3} = \frac{16}{24}$
$\frac{5}{8} = \frac{15}{24}$

SOLUTION.—First Step. As in the case of addition, we first change our fractions to equivalent fractions having the same common denominator. By this we find that $\frac{2}{3}$ is equal to $\frac{16}{24}$, while $\frac{5}{8}$ is equal to $\frac{15}{24}$.

$\frac{16}{24} - \frac{15}{24} = \frac{1}{24}$

SOLUTION.—Second Step. Subtracting $\frac{15}{24}$ from $\frac{16}{24}$, we have a remainder of $\frac{1}{24}$.

Illustration 2.—Find the difference between $8\frac{2}{3}$ and $3\frac{5}{8}$.

$8\frac{2}{3} = 8\frac{16}{24}$
$3\frac{5}{8} = 3\frac{15}{24}$
$5\frac{1}{24}$

SOLUTION.—As in the previous case, we change the fractions to equivalent fractions having the same common denominator. Having done this, we compare fractions and find the difference to be $\frac{1}{24}$, while a comparison of the whole numbers shows a difference of 5. The complete result is therefore $5\frac{1}{24}$.

Illustration 3.—Find the difference between $7\frac{5}{8}$ and $3\frac{2}{3}$.

$7\frac{5}{8} = 7\frac{15}{24}$
$3\frac{2}{3} = 3\frac{16}{24}$
$3\frac{23}{24}$

SOLUTION.—As in the previous cases, we change our fractions to equivalent fractions having the same denominator. Next we proceed to compare the fractions with the idea of subtracting $\frac{16}{24}$ from $\frac{15}{24}$. This, we see, is impossible unless we borrow one unit which is equal to $\frac{24}{24}$. Adding this $\frac{24}{24}$ to $\frac{15}{24}$, we get $\frac{39}{24}$. Now we subtract $\frac{16}{24}$ from $\frac{39}{25}$, getting a difference of $\frac{23}{24}$.

When we start to compare the whole numbers with the idea of getting their difference, we must remember the one that we borrowed, so that we are really subtracting 3 from 6, which leaves us 3. The whole answer is, therefore, $3\frac{3}{4}$.

29. Rule.

(1) To Subtract Fractions.—*When necessary, reduce the fractions to their least common denominator. Subtract the numerator of the subtrahend from the numerator of the minuend, and place the difference over the common denominator.*

(2) To Subtract Mixed Numbers.—*Reduce the fractions, if necessary, to a common denominator, and if the fraction in the subtrahend is smaller than that in the minuend, subtract one fraction from the other, and the smaller whole number from the larger whole number. But if the fraction in the subtrahend is larger than that in the minuend, borrow* one *from the whole number. After changing it to the same denominator as the fraction, add it to the fraction in the minuend. Then subtract as before.*

EXERCISE 11

Find the difference between—

1. $\frac{3}{8}$ and $\frac{3}{4}$
2. $\frac{8}{9}$ and $\frac{5}{21}$
3. $\frac{3}{4}$ and $\frac{7}{12}$
4. $\frac{9}{16}$ and $\frac{1}{24}$
5. $\frac{5}{8}$ and $\frac{1}{6}$
6. $\frac{3}{5}$ and $\frac{1}{2}$
7. $\frac{2}{6}$ and $\frac{7}{15}$
8. $\frac{5}{6}$ and $\frac{7}{9}$
9. $\frac{3}{8}$ and $\frac{5}{7}$
10. $\frac{3}{4}$ and $\frac{4}{5}$
11. $3\frac{1}{2}$ and $2\frac{3}{4}$
12. $5\frac{3}{8}$ and $1\frac{4}{6}$
13. $8\frac{5}{6}$ and $9\frac{3}{4}$
14. $6\frac{4}{5}$ and $4\frac{3}{8}$
15. $5\frac{4}{5}$ and $7\frac{1}{4}$

From—

16. $25\frac{3}{4}$ take $16\frac{1}{2}$
17. $83\frac{1}{2}$ take $19\frac{7}{8}$
18. 150 take $13\frac{1}{3}$
19. $144\frac{1}{8}$ take $50\frac{3}{4}$
20. $25\frac{17}{20}$ take $15\frac{1}{3}$
21. $51\frac{1}{3}$ take $18\frac{3}{4}$
22. 75 take $11\frac{3}{5}$
23. $62\frac{11}{15}$ take $8\frac{1}{9}$
24. $72\frac{3}{11}$ take $16\frac{8}{11}$
25. 195 take $101\frac{4}{17}$

MULTIPLICATION OF FRACTIONS

30. Multiplication of Fractions is a matter of multiplying the two numerators for a numerator, and the two denominators for a new denominator.

If we have to deal with mixed numbers we can bring them under the same rule.

Illustration.—Multiply $2\frac{1}{2}$ by $3\frac{1}{3}$.

SOLUTION.— $2\frac{1}{2} = \frac{5}{2}$, $3\frac{1}{3} = \frac{10}{3}$

$\frac{5}{2} \times \frac{10}{3} = \frac{50}{6} = 8\frac{2}{6} = 8\frac{1}{3}$

31. Cancellation. In all work of this kind, we can often shorten the operation by cancelling the factors that are common to both numerator and denominator, before proceeding with the multiplication.

Illustration. Multiply $\frac{2}{4}$ by $\frac{3}{8}$.

$\frac{\overset{1}{\cancel{2}}}{\underset{2}{\cancel{4}}} \times \frac{3}{\underset{4}{\cancel{8}}} = \frac{3}{16}$

SOLUTION.—It will be noted that 2 is a common factor of 2 and 8. We, therefore, divide 2 into 2, placing the result above the original figure after putting a cancellation stroke through it. In the same way we divide 2 into 8, placing the result, 4, underneath, after cancelling the 8. We next multiply the two figures of the numerator, 3 and 1, producing the new numerator, 3. We also multiply the two denominators, 4 and 4, thus producing the new denominator, 16.

32. A Compound Fraction is a fractional part of a whole number or mixed number, or another fraction; as $\frac{1}{8} \times \frac{1}{9}$, $\frac{1}{6} \times 2\frac{1}{2}$, $\frac{2}{5}$ of $\frac{9}{10}$.

Compound fractions may be reduced to simple fractions by the process of multiplication.

Thus— (1) $\frac{1}{8} \times \frac{1}{9} = \frac{1}{72}$

(2) $\frac{1}{6} \times 2\frac{1}{2} = \frac{1}{6} \times \frac{5}{2} = \frac{5}{12}$

(3) $\frac{2}{5}$ of $\frac{9}{10} = \frac{2}{5} \times \frac{9}{10} = \frac{9}{25}$

33. Rule.

1. *Change all whole or mixed numbers to fractions.*

2. *Eject all factors common to both the numerator and denominator.*

3. *Multiply the numerators together for a new numerator; multiply the denominators together for a new denominator.*

4. *If the new numerator is equal to, or greater than, the new denominator, reduce the fraction to a whole or mixed number.*

NOTE 1.—If one of the factors is an integer, change it to the form of a fraction by writing 1 under it for a denominator, or by imagining 1 to be thus written, and then apply the rule.

NOTE 2.—The word *of* written between fractions, or between fractions and integers, indicates that they are to be multiplied. Fractions thus connected are called *compound fractions.*

EXERCISE 12

Find the value of—

1. $18 \times \frac{5}{6}$
2. $23 \times \frac{2}{3}$
3. $125 \times \frac{3}{4}$
4. $240 \times \frac{9}{16}$
5. $84 \times \frac{3}{25}$
6. $25 \times \frac{7}{8}$
7. $36 \times \frac{11}{5}$
8. $93 \times 3\frac{3}{4}$
9. $125 \times 4\frac{2}{3}$
10. $625 \times 3\frac{7}{10}$
11. $1825 \times 2\frac{2}{3}$
12. $124 \times 7\frac{5}{16}$
13. $\frac{7}{8} \times 21$
14. $\frac{3}{16} \times 125$
15. $\frac{4}{15} \times 128$
16. $\frac{5}{16} \times 35$
17. $\frac{7}{32} \times 36$
18. $\frac{19}{64} \times 84$
19. $\frac{18}{5} \times 36$
20. $2\frac{1}{2} \times 47$
21. $3\frac{1}{3} \times 64$
22. $\frac{3}{4} \times \frac{7}{8} \times \frac{9}{16} \times \frac{8}{9} \times \frac{5}{6}$
23. $2\frac{1}{2} \times 4\frac{1}{2} \times 6\frac{3}{4} \times \frac{2}{3}$
24. $16\frac{2}{3} \times 9 \times 24\frac{3}{4} \times 12\frac{1}{2}$
25. $12\frac{1}{2} \times 12\frac{1}{2} \times 8 \times 4\frac{3}{4}$
26. $5\frac{1}{2} \times 5\frac{1}{2} \times 7\frac{1}{2} \times \frac{5}{6}$
27. $37\frac{7}{8} \times 4\frac{1}{2} \times 18\frac{3}{4}$
28. $88\frac{1}{2} \times 9\frac{3}{4} \times 7\frac{1}{2}$
29. $16\frac{2}{3} \times 30\frac{7}{8} \times 1\frac{15}{16}$

34. DIVISION OF FRACTIONS

When we know how to multiply fractions, there is little to learn in mastering the division of fractions. Suppose we wish to divide $\frac{8}{9}$ by $\frac{4}{7}$, or, as it is indicated, $\frac{8}{9} \div \frac{4}{7}$.

If this work is all put into one operation, it means that we must multiply $\frac{8}{9}$ by 7 and divide it by 4, or expressed as a multiplication question, it appears as follows—

$$\frac{\overset{2}{\cancel{8}}}{\underset{1}{\cancel{9}}} \times \frac{7}{\underset{1}{\cancel{4}}} = \frac{14}{9} = 1\frac{5}{9}.$$

Our rule for division may, therefore, be shortly stated as follows: *Invert the divisor and proceed as in multiplication.*

Illustration 1.—Divide $\frac{2}{8}$ by $\frac{3}{4}$.

$\frac{2}{\cancel{8}_{4}} \times \frac{\cancel{4}^{1}}{3} = \frac{1}{3}$

SOLUTION.—Following the rule, we set down the figure to be divided, $\frac{2}{8}$, followed by a multiplication sign, and after this the divisor, in an inverted position. The balance of the work is straight multiplication.

Illustration 2.—Divide $2\frac{1}{2}$ by $3\frac{1}{4}$.

SOLUTION.— $2\frac{1}{2} = \frac{5}{2}$, $3\frac{1}{4} = \frac{13}{4}$, $\frac{5}{2} \div \frac{13}{4} = \frac{5}{2} \times \frac{4}{13} = \frac{10}{13}$

35. A Complex Fraction is a fraction having one or both of its terms fractional; as, $\frac{\frac{3}{4}}{9}$, $\frac{7}{\frac{3}{4}}$, $\frac{\frac{3}{4}}{\frac{7}{8}}$, $\frac{2\frac{1}{2}}{\frac{9}{10}}$.

Complex fractions may be reduced to simple fractions by the process of division. Thus—

(1) $\frac{\frac{3}{4}}{9} = \frac{3}{4} \div \frac{9}{1} = \frac{3}{4} \times \frac{1}{9} = \frac{1}{12}$

(2) $\frac{7}{\frac{3}{4}} = \frac{7}{1} \div \frac{3}{4} = \frac{7}{1} \times \frac{4}{3} = \frac{28}{3}$

(3) $\frac{\frac{3}{4}}{\frac{7}{8}} = \frac{3}{4} \div \frac{7}{8} = \frac{3}{4} \times \frac{8}{7} = \frac{6}{7}$

(4) $\frac{2\frac{1}{2}}{\frac{9}{10}} = \frac{5}{2} \div \frac{9}{10} = \frac{5}{2} \times \frac{10}{9} = \frac{25}{9}$

EXERCISE 13

Divide—

1. $\frac{6}{7}$ by 3
2. $\frac{10}{17}$ by 5
3. $\frac{32}{35}$ by 8
4. $\frac{7}{9}$ by 6
5. 6 by $\frac{5}{8}$
6. $\frac{5}{6}$ by $\frac{3}{4}$
7. $1\frac{1}{2}$ by $\frac{2}{3}$
8. $\frac{15}{16}$ by $\frac{5}{6}$
9. $16\frac{2}{3}$ by $\frac{5}{7}$
10. $21\frac{7}{8}$ by 6
11. $2\frac{1}{2}$ by $2\frac{1}{3}$
12. $8\frac{1}{3}$ by $2\frac{1}{2}$

EXERCISE 14

NOTE TO THE STUDENT.—The order of procedure in simplification problems is as follows—

(1) Items joined by the word "of" are to be simplified first.

(2) Items joined by the multiplication and division signs are next simplified.

(3) Items joined by the addition and subtraction signs are the last to be simplified.

Find the value of—

1. $(\frac{2}{3}$ of $\frac{6}{7} \times \frac{3}{8}) \div (\frac{4}{9}$ of $\frac{3}{8}$ of $\frac{9}{11})$
2. $(3\frac{4}{7} \times \frac{5}{6}$ of $71\frac{2}{3}) \div 4\frac{1}{2} \times \frac{7}{8}$ of 16
3. $7\frac{8}{11} \div \frac{1}{4}$ of $\frac{3}{5}$ of $\frac{8}{9}$
4. $11 \div (\frac{2}{3} \times 5\frac{1}{2} \times 7)$
5. $(2\frac{1}{2} \div \frac{3}{4}) \times \frac{2}{3}$ of $\frac{4}{5}$ of $\frac{11}{18}$
6. $\dfrac{3\frac{1}{4} + 27\frac{5}{6} - 6\frac{1}{2} \times 3\frac{1}{4}}{(\frac{3}{5}$ of $25\frac{1}{4}) + (\frac{3}{4}$ of $27)}$
7. $\dfrac{75\frac{1}{3} + 8\frac{1}{8} + 7\frac{3}{4} - 1\frac{7}{8}}{31\frac{1}{4} \div 6\frac{1}{4}}$

REVIEW OF FRACTIONS

EXERCISE 15

1. A grocer packed in a box for shipment $15\frac{5}{8}$ lbs. sugar, $19\frac{3}{4}$ lbs. coffee, $2\frac{5}{16}$ lbs. tea, $17\frac{3}{8}$ lbs. ham, and $25\frac{1}{2}$ lbs. bacon. What was the total weight of the contents of the box?

2. If $18\frac{4}{5}$ galls. were sold from a barrel of molasses containing $45\frac{5}{32}$ galls., how many gallons remained in the barrel?

3. How many pounds of cheese at $18\frac{2}{3}$c. per pound can be bought for $\$1.23\frac{1}{5}$?

4. If $\frac{3}{5}$ of a bin holds 165 bus. of wheat, how many bushels would $\frac{3}{11}$ of the bin hold?

5. A merchant sold 75 lbs. of butter at $24\frac{3}{5}$c. per pound, 93 doz. eggs at $16\frac{2}{3}$c. per dozen, and $48\frac{5}{16}$ galls. of milk at 25c. per gallon. What was the total amount of the sales?

6. If $3\frac{1}{8}$ yds. of cloth cost \$12.50, what will $23\frac{7}{16}$ yds. cost?

7. A and B can do a piece of work in 10 days, which A can do alone in 18 days. In what time can B do the work alone?

8. A pole was broken off 12 ft. from the top, leaving 28 ft. still standing. What part of the pole remained standing?

EXERCISE 16

1. The distance by rail from Hamilton to Toronto is $39\frac{5}{16}$ miles.

If a way-station on the road is $18\frac{5}{12}$ miles from Hamilton, how far is it from Toronto?

2. I sold $\frac{5}{8}$ of a piece of goods containing $39\frac{3}{4}$ yds. How many yards remained in the piece?

3. A person owns $\frac{3}{8}$ of a ship and sells $\frac{2}{3}$ of his share for \$1,260. What is the value of the ship?

4. A man invested $\frac{2}{5}$ of his capital in bank stock, $\frac{3}{4}$ of the remainder in real estate, and had still \$6,000 left. Find his capital.

5. After taking out of a purse $\frac{2}{5}$ of its contents, $\frac{3}{4}$ of the remainder was found to be \$18. What part of \$54 was the whole sum?

6. A receives $\frac{3\frac{1}{8}}{7\frac{1}{2}}$ of an estate and B $\frac{4}{9}$ of $\frac{1}{3}\frac{8}{5}$ of the remainder. C gets what is left and finds that his share is worth \$872 more than A's. What is the value of the estate?

7. The divisor is $3\frac{3}{8} + 3\frac{6}{7}$ and the quotient is $\frac{\frac{3}{4} - \frac{1}{6}}{\frac{4}{9} \text{ of } \frac{15}{16}}$. Find the dividend.

8. I bought $\frac{2}{9}$ of a ship and sold $\frac{2}{5}$ of my share for \$18,000. What was the value of the ship at that rate?

EXERCISE 17

1. A man earns \$280 in $2\frac{1}{3}$ months. If he spends in $4\frac{1}{6}$ months what he earns in $3\frac{1}{2}$ months, how much will he save in a year?

2. A produce merchant exchanged $48\frac{3}{5}$ bus. of oats at $39\frac{3}{4}$c. per bushel, and $13\frac{1}{2}$ bbls. of apples at \$3.85 per barrel, for butter at $37\frac{1}{2}$c. per pound. How many pounds of butter did he receive?

3. What quantity taken from $159\frac{1}{7}$ will make it exactly divisible by $12\frac{5}{6}$?

4. A, B, and C own a vessel, each having equal shares. They sell respectively $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$ of their shares to D, who dies and leaves his share equally among them. If B's and C's interests in the ship be now worth \$37,300, what is the value of A's share?

5. A merchant sold 20 bbls. of flour for $\$127\frac{1}{2}$, which was $\frac{5}{6}\frac{1}{0}$ of what he received for what he had left and which he sold at $\$6\frac{1}{4}$ a barrel. How many barrels in all did he sell?

6. After spending \$10 less than $\frac{3}{5}$ of my money, I had \$15 more than $\frac{3}{10}$ of it left. How much had I at first?

7. I had a sum of money of which I paid away $\frac{1}{5}$, then $\frac{1}{2}$ of the remainder, then $\frac{2}{3}$ of what was still left, and found that I had still left half a dollar less than $\frac{1}{6}$ of $\frac{3}{4}$ of the whole. What sum had I at first?

8. A, B, and C counted their money and found that A had $\frac{2}{5}$ of the total amount, B $\frac{1}{4}$, and C the remainder. How much had B and C if A had \$30?

EXERCISE 18

1. A pedestrian travelled $32\frac{3}{8}$ miles on Monday, $23\frac{5}{32}$ miles on Tuesday, $37\frac{3}{4}$ miles on Wednesday, $19\frac{2}{3}$ miles on Thursday, $24\frac{4}{5}$ miles on Friday, and $16\frac{1}{2}$ miles on Saturday. What was the average distance travelled per day?

2. If coffee loses $\frac{1}{15}$ of its weight in roasting, how much green coffee will be needed to make 252 lbs. of roasted coffee?

3. A certain number divided by $8\frac{4}{17}$, the quotient increased by $2\frac{3}{4}$, the sum multiplied by $\frac{2\frac{1}{2}}{3}$, and the result diminished by $\frac{1}{2}$ of $\frac{3}{7}$ of $14\frac{4}{9}$, gives $2\frac{2}{3}$. What is the number?

4. $\frac{4}{5}$ of A's stock was destroyed by fire, $\frac{7}{8}$ of the remainder was injured by water and smoke; he sold the uninjured goods at cost price, and the injured goods at $\frac{1}{3}$ of cost price. He realized \$1,155. What did he lose by the fire?

5. A can do a piece of work in 35 days; B can do it in 40 days; C can do it in 45 days. In what time will they do it, all working together?

6. A and B can reap a field of wheat in 3 days; A and C in $3\frac{1}{2}$ days; and B and C in 4 days. In what time would each working alone do the work?

EXERCISE 19

1. A can do a piece of work in 27 days, and B in 15 days; A works at it alone for 12 days, B then works alone for 5 days,

and then C finishes the work in 4 days. In what time could C have done the work by himself?

2. A can do a piece of work in $\frac{2}{3}$ of a day and B in $\frac{1}{2}$ of a day. In what time can both together do it? If $1.40 be paid for the work, how much should A receive?

3. I bought $48\frac{2}{3}$ lbs. of tea at $63\frac{3}{4}$c. per pound. How many pounds more would have been received, if the price had been $7\frac{1}{2}$c. per pound less, for the same money?

4. A dealer bought 46 bales of cotton, averaging 435 lbs. per bale, at $11\frac{1}{3}\frac{7}{2}$c. per pound, and sold the entire quantity at $12\frac{3}{4}$c. per pound. If his expenses for freight, insurance, commission, etc., amounted to $95.30, what was his total net profit?

5. If a fruit vendor buys lemons at the rate of 5 for 3 cents, how many must he sell at the rate of 6 for 5 cents to gain 14c.?

6. A and B being $135\frac{3}{4}$ miles apart, started toward each other and met in 4 days, A travelling at the rate of $3\frac{1}{2}$ miles per hour and $6\frac{3}{4}$ hrs. per day, and B at the average rate of $2\frac{3}{4}$ miles per hour. How many hours per day did B travel?

7. A barrel has two faucets of different sizes. If only the larger faucet is opened, the barrel will be emptied in 8 mins.; and if both are opened, it will be emptied in 5 mins. In what time will the barrel be emptied if only the smaller faucet be opened?

8. A owes B $30.28\frac{1}{8}$, and pays it in $32\frac{1}{4}$ yds. of dress goods at $82\frac{1}{2}$c. per yard, and the remainder in calico at $5\frac{1}{4}$c. per yard. How many yards of calico should B receive?

EXERCISE 20

1. A laborer withdrew $\frac{5}{1\frac{1}{2}}$ of his deposit from a savings bank, and with $\frac{2}{5}$ of the money thus withdrawn paid his rent, which was $12. How much money has he still on deposit?

2. A speculator invested $\frac{1}{4}$ of his money and $200 in bonds, $\frac{1}{3}$ of his money and $500 in railway stocks, $\frac{1}{8}$ of his money and $600 in real estate, and the balance, which was $1,200, he deposited in a bank. How much was he worth?

3. I sold 5 jars of butter weighing $35\frac{1}{2}$, $32\frac{3}{4}$, $31\frac{7}{8}$, $29\frac{5}{8}$, and $36\frac{1}{4}$ lbs. respectively, at $18\frac{3}{4}$c. per pound. The jars weighed $5\frac{1}{2}$,

$6\frac{1}{4}$, $5\frac{5}{16}$, $6\frac{1}{4}$, and $7\frac{7}{8}$ lbs. respectively. I received in payment 20 lbs. of coffee at $33\frac{1}{3}$c. per pound, 5 lbs. of tea at $62\frac{1}{2}$c. per pound, and the balance in sugar at 4c. per pound. How many pounds of sugar did I receive?

4. An estate was divided among two brothers and a sister; the elder brother received $\frac{4}{7}$ of the estate, the younger $\frac{1}{3}$, and the sister the remainder, which was $5,740 less than the elder brother received. What was the value of the estate?

5. A tank whose capacity is 126 galls. is $\frac{5}{6}$ full; if $11\frac{2}{3}$ galls. more be poured into it, what part of the tank is full?

6. How much ironstone ore must be raised from a mine, so that on losing $\frac{17}{40}$ in roasting, and $\frac{8}{19}$ of the remainder in smelting, there may result 506 tons of pure metal?

7. Two wine glasses are filled with a mixture of spirit and water, one containing 3 parts of water and 1 of spirit, and the other 4 parts of water and 3 parts of spirit. When the contents of the two glasses are mixed in a tumbler, find how many parts of the whole mixture are spirit and water.

8. Find the total number of bushels that can be put in five bins based upon the following—

$\frac{2}{3}$ of 1st bin equal 150 bus.
$\frac{17}{20}$ of 2nd bin equal 850 bus.
$\frac{5}{19}$ of 3rd bin equal 95 bus.
$\frac{3}{13}$ of 4th bin equal 120 bus.
$\frac{13}{65}$ of 5th bin equal 260 bus.

EXAMINATION No. 1
(FRACTIONS)
Time 1 *hour*—80% *required*

1. From the sum of $\frac{3}{7}$ and $5\frac{2}{5}$, take the difference between $17\frac{1}{2}$ and 21.

2. An estate is so divided among A, B, and C, that A gets $\frac{2}{5}$, B $\frac{8}{10}$, and C the remainder, which was $4,200. What was the amount of the estate?

3. Having paid $115 for a watch and chain, I discover that the cost of the chain was only $\frac{7}{16}$ of the cost of the watch. What was the cost of each?

4. Three-quarters of a stock of goods was destroyed by fire and $\frac{7}{10}$ of the remainder so damaged by water that it was sold at half its cost. If the uninjured goods when sold at cost brought $10,800, what must have been the loss of Anderson, who owned $\frac{9}{16}$ of the stock?

5. A farmer having 650 bus. of wheat, kept for his own use $52\frac{3}{4}$ bus. less than $\frac{1}{5}$, sold to his neighbors for seed $45\frac{1}{8}$ bus. more than $\frac{1}{3}$, and marketed the remainder at 80c. per bushel. How much money was received from the market sales?

6. A, B, and C rented a pasture for $37. A put in 3 cows for 4 months; B, 5 for 6 months; and C, 8 for 4 months. How much ought each to pay?

7. If 8 horses consume $4\frac{1}{2}$ bus. of oats in $3\frac{1}{2}$ days, how many bushels will 12 horses consume in the same time?

8. A farmer has $\frac{3}{5}$ of his sheep in one pasture, $\frac{3}{7}$ in another, and the remainder of his flock, 72 sheep, in the third pasture. How many sheep had he?

EXAMINATION No. 2

(FRACTIONS)

Time 1 *hour*—80% *required*

1. If the ingredients are $\frac{3}{20}$ sulphur, $\frac{1}{10}$ saltpetre, and $\frac{3}{4}$ charcoal, what is the number of pounds of each in $2{,}154\frac{3}{11}$ lbs. of gunpowder?

2. How many fields of $9\frac{5}{8}$ ac. each can be made from a farm containing $125\frac{1}{8}$ ac.?

3. A grocer bought a cask of molasses containing $65\frac{1}{2}$ galls., from which he sold at one time $\frac{1}{8}$ of it, at another 5 galls. less than $\frac{1}{4}$ of what remained, and the remainder was sold with the cask for $20\frac{1}{2}$ dols. If the value of the cask was 1 dol., at what price per gallon was the last sale made?

4. A mechanic worked $21\frac{2}{5}$ days, and after paying his board with $\frac{3}{8}$ of his earnings, had $66\frac{7}{8}$ dols. left. How much did he earn per day?

5. If $\frac{1}{2}$ the trees of an orchard are apple, $\frac{1}{4}$ peach, $\frac{1}{8}$ plum, $\frac{1}{16}$ pear, and the remaining 21 trees cherry, how many trees in all?

6. A and B do a piece of work in 10 days, which A alone can do in 18 days. In what time can B alone do the work?

7. A and B have joint work for 21 days, but B can in a day do only $\frac{3}{4}$ as much as A; after B has worked alone for 3 days and A for 5 days, they unite and complete the work. How many days will they require? If \$75 be paid for the work, what part of it should each receive?

8. If you buy 60 lemons at the rate of 6 for 10c., and twice as many more at the rate of 5 for 8c., and sell the entire lot at the rate of 3 for 4c., will you gain or lose, and how much?

SHARING

Illustration.—Divide \$400 between A and B, giving A \$50 more than B.

Total sum to be divided = \$400
A's portion more than B = 50

Amount left to divide = \$350

A's share = $\frac{1}{2}$ of \$350, \$175, plus \$50 = \$225
B's " = $\frac{1}{2}$ of \$350, \$175, = 175

\$400

Illustration.—A, B, and C own three farms with a total of 1,200 ac. A has 100 ac. more than B and 140 ac. more than C. How many acres has each?

Let 1 share = A's farm
" 1 " —100 ac. = B's "
" 1 " —140 " = C's "

3 shares—240 ac. = 1,200 ac.
∴ 3 " = 1,200 ac. plus 240 ac.
3 " = 1,440 "
1 " = $\frac{1440}{3}$ = 480 ac. = A's farm
1 " —100 ac. = 380 ac. = B's "
1 " —140 " = 340 " = C's "

Illustration.—The sum of \$621 is divided among 12 men,

15 women, and 30 children in such a manner that a man and a woman shall receive as much as 4 children and all the children shall receive $225. Find the amount received by each man, woman, and child respectively.

12 men, 15 women, and 30 children receive $621
30 " " $225

∴ 12 " and 15 women " $396
12 " " 12 " receive 12 × $30 = 360

3 " must receive $36
1 woman " $12
All receive $621
15 women receive $180
30 children receive . . . $225 $405

∴ 12 men receive $216
1 man receives $18

EXERCISE 21

1. Two farms together cost $12,250. If one cost $750 more than the other, find the cost of each.

2. A merchant's profits for two successive years were $3,765. If he gained $486 more the second year than he did the first, find his gain for each year.

3. Two farms together contain 537 ac. If one contains 43 ac. more than the other, find the area of each.

4. Divide $2,367 among A, B, and C, giving A $522 more than either B or C.

5. If $3,529 were divided among Smith, Brown, and Jones, so that Smith and Brown each received $398 more than Jones, how much did each receive?

6. Three farms cost $14,355. The first cost $320 more than the second, but $590 less than the third. Find the cost of each.

7. The total length of three railroads is 3,551 miles. The first is 148 miles shorter than the second, which is 294 miles shorter than the third. Find the length of each.

8. A, B, and C have together $19,505. A has $5,885 more than B, who has $2,745 less than C. How much has each?

EXERCISE 22

1. Divide $868 between A and B, giving A $4 for every $3 given B. How much does each receive?

2. The sum of two numbers is 4764 and one is three times the other. Find the numbers.

3. Two men together have $4,635. If one has $\frac{2}{7}$ of the amount the other has, how much has each?

4. A ship and her cargo are worth $23,750. If $\frac{3}{4}$ of the value of the ship equals $\frac{5}{6}$ of the value of the cargo, find the value of each.

5. The sum of two numbers is 7293. If $\frac{2}{5}$ of one equals $\frac{4}{7}$ of the other, find the numbers.

6. The sum of two numbers is 2079. If the numbers bear to each other the ratio of 3 to 8, find them.

7. A mixture of three grades of sugar is made by putting 3 lbs. of the first grade and 4 lbs. of the second to every 6 lbs. of the third. How many pounds of each grade are there in 2,470 lbs. of the mixture?

8. A mixture of three grades of sugar is made by putting 2 lbs. of the first grade with every 3 of the second and 4 of the second to every 5 of the third. How many pounds of each grade are there in 1,645 lbs. of the mixture?

EXERCISE 23

1. In a factory there are 264 men, women, and boys. Find the number of each if there are $\frac{1}{3}$ more women than boys and $\frac{1}{4}$ more men than women.

2. The weekly wages of 4 men and 5 boys are $58.50. If a man earns twice as much as a boy, what are the daily wages of each?

3. A farmer bought 17 sheep and 15 lambs for $137.50, paying $1.50 more for a sheep than for a lamb. How much did he pay for a sheep and a lamb?

4. Divide $4,117 among four boys and three men, giving each boy $2 when you give each man $5.

5. Divide $234.85 among three men, giving the first four times as much as the second and the third twice as much as the second.

6. A has 6 cows on pasture for 3 weeks, B 5 cows for 4 weeks, and C 13 cows for 2 weeks. The total cost of the pasture is $12.80. How much should each pay?

7. The cost of a quantity of silk at $3.25 per yard and tweed at $2.50 per yard was $409.75, the whole cost of the tweed being 25c. more than that of the silk. Find the number of yards of each.

8. Two loads of wheat at 65c. a bushel cost $81.25. One load weighed 13 bus. more than the other. Find the number of bushels in each load.

EXERCISE 24

1. A and B enter into partnership for a year, and A receives $\frac{3}{8}$ of the profits and B the remainder. A invests $2,000 additional for the second year's partnership and at the end of the year receives $\frac{4}{5}$ of the profits. How much did each invest the first year?

2. Three persons, A, B, and C, trade together, having a joint capital of $4,700. A's money is in the business 6 months, B's for 8 months, and C's for 10 months. Each receives $600 as his share of the profits. How much of the capital did each contribute?

3. A can do a piece of work in $\frac{2}{3}$ of a day and B in $\frac{1}{2}$ a day. If both work together and $1.40 be paid for the job, how should the money be divided?

4. A can do as much work in 8 days as B can in 11 days, but A works only $\frac{3}{4}$ of the time. How should $97.50 be divided between them for work?

5. A and B work respectively 7 and 8 hrs. a day, and receive equal wages. On the second day they work 9 and 10 hrs. respectively. They receive for their whole work $12.50. How much should each get?

6. A bankrupt's assets are only half of his liabilities, but one-third of the assets are only worth 25c. on the dollar. How many cents on the dollar can he pay?

7. A grocer bought sugar and coffee for $96.36, the cost of the sugar being $\frac{3}{4}$ of the cost of the coffee. What did he pay for each?

8. The loss on a property was $8,000. If the property was insured in three different companies for $3,000, $2,500, and $4,000 respectively, what amount should each company pay?

EXERCISE 25

1. A, B, and C formed a partnership. A invested $7,000 for 10 mths., B $6,000 for 8 mths., and C $10,000 for 12 mths. The gain for the year being $12,000, how should it be divided?

2. Johnson and Harris engage in business Jan. 1, Johnson investing $5,000 and Harris $8,000. One year later their net gain was $3,900. What was the share of each partner, gain being divided according to investment?

3. A, B, and C began business. On Jan. 1, A invested $7,500; on Jan. 1, B invested $12,000; on Jan. 1, C invested $10,000. The net profit for the year being $8,850, what was each man's share, the gain being divided according to investment?

4. The weekly wages at a mill amounted to $874.80. In the mill there were six times as many women and twice as many men as there were boys. A man's wages were $2.40 per day, a woman's $1.10 per day, and a boy's 75c. per day. How many women were there in the mill?

5. Divide $105 among A, B, and C, so that B's share may be half as much again as A's, and C's a third as much again as A's and B's together.

6. A, B, and C caught a certain number of fish. When A's fish and B's fish are put together they make 110; B's and C's, 130; A's and C's, 120. How many did each catch?

7. A farmer shared his farm among his three sons. To the youngest he gave 80 ac., to the eldest $\frac{4}{9}$ of the whole, and to the second $\frac{3}{4}$ as much as to both the others. How many acres did the farm contain?

8. A man divided $17,940 among his three sons, whose ages are 16, 18, and 26 yrs., in proportion to their ages. Three years afterwards he similarly divided an equal sum, and again after three years more an equal sum. How much did each son receive altogether?

EXERCISE 26

1. The sum of \$1,416 is to be divided among 15 men, 20 women, and 30 children, in such a manner that a man and a child shall together receive as much as two women, and all the women together shall receive \$480. Find the amount received by each man, woman, and child respectively.

2. If 13 men, 19 women, and 25 boys earn \$15,190.44 in a year (309 working days), and if a woman earns $\frac{3}{5}$ of what a man earns, and a boy $\frac{5}{7}$ of what a woman earns, what is the daily earning of each?

3. What does each man woman, and child get when \$178.92 is divided among 6 men, 8 women, and 10 children, so that 2 men may get as much as 3 women, and 2 women as much as 3 children?

4. Divide \$171.50 into parts proportional to $\frac{1}{2}$, $\frac{1}{4}$, $\frac{2}{5}$.

5. A and B entered into partnership, their capitals being in the ratio of 7 to 9. After 3 mths, A withdrew part of his capital, so that the ratio was 2 to 3. At the end of the year, A's share of the gain was \$1,500; what was B's gain?

6. A farm is divided into two parts, whose areas are as 9 to 13; the area of the larger part exceeds that of the smaller by $18\frac{2}{11}$ ac. Find the number of acres in the farm.

7. A sum of money was divided among A, B, and C. A received $\frac{2}{5}$ of the sum; B, \$20 less than $\frac{2}{3}$ of what was left; and the remainder, which was $\frac{3}{4}$ of A's share, was given to C. Find the sum divided.

8. A and B have together 210 ac. of land, and $\frac{3}{4}$ of A's share is equal to $\frac{6}{7}$ of B's. B paid \$1,470 for his land; for how much per acre must he sell it to gain \$20 per acre?

EXERCISE 27

1. A flagstaff 120 ft. high was broken off by the wind, and it was found that $\frac{3}{4}$ of the shorter part was $\frac{1}{2}$ the longer part. Find the length of each part.

2. A and B were candidates for election in a constituency of 2,700 voters. The votes polled by A were to those polled by B as 23 to 25, and B was elected by a majority of 100. How many persons did not vote?

3. Gunpowder is composed of nitre, charcoal, and sulphur in the proportion of 15, 3, and 2. A certain quantity of gunpowder is known to contain 20 cwt. of charcoal; find its weight, and also the weight of nitre and of sulphur it contains.

4. Divide \$1,200 among A, B, and C, so that A may have \$70 more than B and twice as much as C.

5. A house and lot are together worth \$2,100, and $\frac{1}{4}$ of the value of the house is equal to $\frac{1}{3}$ of the value of the lot. Find the value of each.

6. A legacy of \$9,500 is to be divided among A, B, and C, so that A will get $\frac{5}{19}$ of the whole and B will get $\frac{3}{4}$ as much as C. Find the shares of each.

7. A grocer has 630 lbs. of a mixture containing chicory and coffee in the proportion of 3 to 4. What amount of coffee must be added to the mixture to make the proportion 7 to 10?

COMMERCIAL ARITHMETIC

TEST PAPER—No. 2

COMMON OR VULGAR FRACTIONS

Your work for this paper is to be submitted to the instructor for correction and criticism.

1. On $\frac{2}{3}$ of a field was planted corn ; on $\frac{2}{3}$ of the remainder was sown wheat ; on $\frac{2}{3}$ of the remainder was planted potatoes ; the rest, consisting of $\frac{1}{3}$ of an acre, was planted in beans. How large was the field ?

2. A, B and C have to do a piece of work. A does $\frac{3}{11}$ and goes away. B does $\frac{2}{5}$ of the remainder, and then C finishes it. What fraction of the work is done by C ? If $16.50 be paid for the whole work, how much should each receive ?

3. If $\frac{5}{9}$ of 2 lb. of sugar cost as much as $2\frac{1}{2}$ lb. of rice, and if $3\frac{3}{4}$ lb. of rice cost 15c., what is the price of the sugar per pound ?

4. By selling a house for $3,990 I lost $\frac{1}{8}$ of its cost. For what amount should I have sold it to gain $\frac{1}{8}$ of its cost ?

5. A man's wages are 3\frac{4}{5}$ a day and his daily expenses 1\frac{7}{8}$. How many days must he labour to enable him to buy a suit of clothes worth 46\frac{1}{5}$?

6. A merchant bought 5 hams, weighing respectively $12\frac{1}{2}$, $18\frac{5}{8}$, $15\frac{3}{4}$, $14\frac{5}{16}$, and 16 lb., at $11\frac{1}{2}$c. per pound. What was the total cost ?

7. A boy agreed to work a year for $300 and an overcoat. At the end of ten months he left, and received $245 and the overcoat as full pay for his services. What was the overcoat worth ?

8. If the regular fare on a railway is 3c. a mile, but $\frac{1}{6}$ is allowed off full fare when return tickets are bought, find the distance between two places if a return ticket costs $1.80.

9. The weekly wages at a mill amounted to $583.20. In the mill there were seven times as many women and twice as many men as there were boys. A man's wages were $1.90 per day, a woman's 90c. per day, and a boy's 70c. per day. How many women were there in the mill?

10. Find the value of—

$$\tfrac{1}{2} + \tfrac{1}{3} + \tfrac{1}{4} \text{ of } \tfrac{1}{9} - \tfrac{1}{6} \text{ of } \tfrac{15}{16} \div \tfrac{3}{8} \text{ of } \tfrac{4}{9} \times \tfrac{5}{7}.$$

DECIMAL FRACTIONS

DEFINITIONS

36. Ours is a decimal system of notation. The unit values of the digits in every number increase in a tenfold ratio from right to left. For instance, the figure 1 standing alone denotes 1 unit. Put two 1's side by side, thus 11, and they denote 1 ten and 1 unit. Put three 1's, thus, 111, and we have 1 hundred, 1 ten, and 1 unit.

If this plan works in expressing whole numbers, why not use it in expressing fractions? If values go up in tenfold steps as we go to the left from the unit's place, we may consider them as going down in tenfold steps as we go to the right from the unit's place. All we need is a plan of marking the unit's figure. This we do by placing a dot or period after it. This is called the *Decimal Point.* Then—

1.= 1 unit.
11.1 = 1 ten, 1 unit, 1 tenth, or eleven and one-tenth.
111.11 = 1 hundred, 1 ten, 1 unit, 1 tenth, 1 hundredth, or one hundred, eleven and eleven hundredths.

Again—

1.= 1 unit.
10.1 = 1 ten, 0 units, 1 tenth, or ten and one-tenth.
100.01 = 1 hundred, 0 tens, 0 units, 0 tenths, 1 hundredths, or one hundred and one hundredth.

From these illustrations we gather that, while figures to the left of the decimal point represent whole numbers, figures to the right of the decimal point represent fractional numbers.

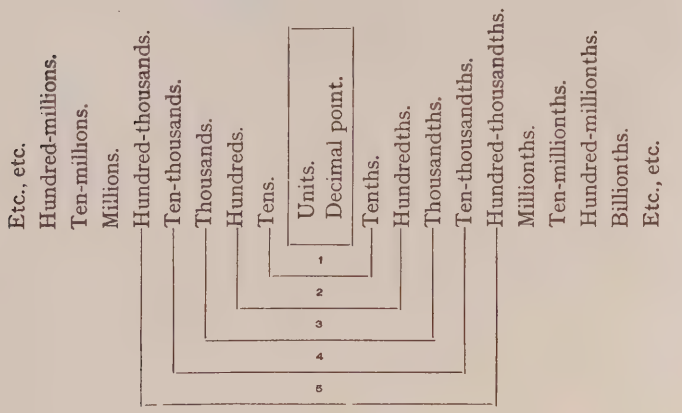
37. A Decimal Fraction is a fraction whose denominator is ten or some power of ten.

38. A Pure Decimal corresponds to a Proper Fraction, and has a value less than 1; as, .5, .26, .2794.

39. A Mixed Decimal corresponds to an Improper Fraction, and has a value more than 1; as, 7.4, 29.678.

40. A Complex Decimal corresponds to a Complex Fraction, and has a common fraction in its right-hand place; as, .66⅔.

41 DECIMAL NOTATION SCALE



Note that the units figure occupies the centre. One place to the left and we have tens. One place to the right and we have tenths. Two places to the left and we have hundreds. Two places to the right and we have hundredths, and so on.

42. NUMERATION OF DECIMALS

Question.—Express, orally, the decimal .0127.

METHOD.—Note that there are four decimal places occupied by figures. Starting at the decimal point, four places will carry us through tenths, hundredths, thousandths, and, lastly, ten-thousandths. Read the significant figures as in the case of the whole numbers (one hundred twenty-seven). As a name for this amount, use the name of the last decimal place (one hundred twenty-seven ten-thousandths).

Question.—Express, orally, the mixed decimal 475.175.

ANSWER.—Four hundred seventy-five, and one hundred seventy-five thousandths.

NOTE.—Never use the word "and" except to denote the break between the whole number and the decimal.

Hence the following rule—

Numerate from the decimal point to determine the denominator. Read the decimal as a whole number, and give to it the denomination of the right-hand figure.

EXERCISE 28

Express, orally, the following decimals—

1. .75	9. .00625	17. .0017834
2. .075	10. .937	18. 5.083291
3. .325	11. .0937	19. 7.0056832
4. .0325	12. .00937	20. 8.0010005
5. .215	13. .11625	21. 6.10070082
6. .0215	14. .83715	22. 1.00015008
7. .625	15. .517382	23. 6.005832¾
8. .0625	16. .8267513	24. 4.10075008⅞

43. NOTATION OF DECIMALS

Question.—Write, decimally, seventy-five hundred-thousandths.

METHOD.—Hundred-thousandths occupy the fifth place from the decimal point. To express seventy-five alone, we need only two figures (75), therefore we complete the five places needed by placing three ciphers after the decimal point, and follow with the two figures 75; thus, .00075.

44. Rule.

Write the decimal the same as a whole number, prefixing ciphers when necessary to give each figure its true local value. Place the decimal point before the left-hand figure of the decimal.

EXERCISE 29

Write the following as decimals—

1. Seven tenths. 2. Seven hundredths. 3. Twenty-seven thousandths. 4. Three-thousand-two-hundred-twenty-five ten-thousandths. 5. Seventy-five ten-thousandths. 6. Eighty-thousand-eight hundred-thousandths. 7. Nine-hundred-nine hundred-thousandths. 8. Eight hundred-thousandths. 9. 324 hundred-thousandths. 10. 34 ten-thousandths. 11. Eight, and 24

hundredths. 12. Twenty-five and 3125 ten-thousandths. 13. Two-hundred-twenty-six millionths. 14. Six-thousand-forty-five hundred-thousandths. 15. Five hundred-thousandths. 16. Twenty-five tenths.

17. $\frac{5}{10}$ 18. $\frac{204}{1000000}$ 19. $\frac{2400}{1000000}$ 20. $\frac{524}{10000}$

21. $\frac{25}{10000}$ 22. $\frac{4035}{1000}$ 23. $\frac{87}{1000000}$ 24. $\frac{120021}{100}$

45. REDUCTION OF DECIMALS

To reduce a decimal to a vulgar fraction.

Illustration 1.—Reduce .045 to a vulgar fraction.

$.045 = \frac{45}{1000} = \frac{9}{200}$

SOLUTION.—The decimal .045 is read forty-five thousandths. This in vulgar fraction form is $\frac{45}{1000}$ This fraction, brought to its lowest terms, is equal to $\frac{9}{200}$.

Illustration 2.—Reduce $.66\frac{2}{3}$ to a vulgar fraction.

SOLUTION.—

$$.66\frac{2}{3} = \frac{66\frac{2}{3}}{100} = \frac{\frac{200}{3}}{\frac{100}{1}} = \frac{200}{3} \times \frac{1}{100} = \frac{200}{300} = \frac{2}{3}$$

EXERCISE 30

Reduce to common fractions in their lowest terms—

1. .5
2. .8
3. .25
4. .68
5. .06
6. .375
7. .625
8. .075
9. .008
10. .5856
11. 16.75
12. 5.064
13. .00625
14. 5.00125
15. 18.03125
16. 7.31625
17. 3.00128
18. 5.34375
19. 50.0004
20. $.12\frac{1}{2}$
21. $.6\frac{1}{4}$
22. $.33\frac{1}{3}$
23. $.0\frac{7}{8}$
24. $25.16\frac{2}{3}$

46. To reduce a vulgar fraction to a decimal.

Illustration 1.—Reduce $\frac{7}{3125}$ to an equivalent decimal.

3125)7000(.00224
6250
——
7500
6250
——
12500
12500

SOLUTION.—Divide the denominator into the numerator, adding ciphers as required to complete the division. In the case given, 5 ciphers need to be added. Now, adding a cipher to a number multiplies it by 10, so that in adding 5 ciphers to the numerator we have really multiplied the numerator by 100000. Now, multiplying the dividend by 100000 would have the effect of multiplying the quotient by 100000. So that the quotient, to be made correct, must be divided by 100000. $224 \div 100000 = \frac{224}{100000}. = 224$ hundred-thousandths $= .00224$. It will be noted that in .00224 there are five figures to the right of the decimal point, which is the number of ciphers added in the division.

From this and similar cases we derive the following rule—

47. Rule.

Annex ciphers to the numerator and divide by the denominator.

Point off as many decimal places in the quotient as there are ciphers annexed.

If the quotient does not contain the required number of figures, add ciphers to the left of the quotient to complete the number.

NOTE 1.—If the division is not exact, when a sufficient number of decimal places has been obtained, the sign + may be annexed to show that the division is not complete, or it may be expressed as a complex decimal.

NOTE 2.—A fraction in its lowest terms can be reduced to a pure decimal only when its denominator contains no prime factors but 2 and 5. If the denominator contains any other prime factor, the division will not end. The decimals thus produced are called *Repeating* Decimals, and the figures repeated, *Repetends*.

Illustration 2.—Reduce $\frac{1}{3}$ to an equivalent decimal.

3)1000
333 +

$\frac{1}{3} = .333 +$
$= .\dot{3}$

SOLUTION.—The division in this case would never end. One-third, therefore, produces a repeating or circulating decimal. This is expressed, as illustrated, by placing a period above the three—thus, $\dot{3}$.

Illustration 3.—Reduce $\frac{5}{6}$ to an equivalent decimal.

$$\begin{array}{l} 6)50000 \\ \overline{8333}+ \\ =.8333+ \\ =\ .8\dot{3} \end{array}$$

SOLUTION.—In this case we have a mixed circulating decimal, which does not begin to repeat until after the first figure.

EXERCISE 31

Reduce to equivalent decimals. Do not extend beyond four significant figures.

1. $\frac{1}{2}$	6. $\frac{5}{9}$	11. $\frac{5}{12}$
2. $\frac{3}{4}$	7. $\frac{5}{16}$	12. $\frac{113}{500}$
3. $\frac{4}{5}$	8. $\frac{1}{3}$	13. $\frac{19}{25}$
4. $\frac{5}{8}$	9. $\frac{2}{3}$	14. $\frac{1}{64}$
5. $\frac{7}{8}$	10. $\frac{1}{16}$	15. $\frac{3}{12500}$

Reduce the following to decimals of—

2 *dec. places.*	3 *dec. places.*	4 *dec. places.*	5 *dec. places.*
16. $19\frac{2}{3}$	18. $75\frac{7}{12}$	20. $278\frac{3}{64}$	22. $8\frac{7}{128}$
17. $415\frac{5}{8}$	19. $91\frac{9}{32}$	21. $67\frac{5}{9}$	23. $15\frac{6}{11}$

5 *dec. places.*	6 *dec. places.*	7 *dec. places.*
24. $46.342\frac{7}{8}$	26. $812.9312\frac{13}{16}$	28. $2.91\frac{11}{64}$
25. $75.19\frac{17}{32}$	27. $73.41\frac{5}{6}$	29. $18.273\frac{2}{3}$

48. ADDITION OF DECIMALS

Since integers and decimals increase and decrease regularly by tenfold steps, it is clear that decimals may be added, subtracted, multiplied, or divided in the same manner as integers.

Illustration 1.—Add 7.41, 247.516, 17.2179, 6.003.

$$\begin{array}{r} 7.41 \\ 247.516 \\ 17.2179 \\ 6.003 \\ \hline 278.1469 \end{array}$$

SOLUTION.—Write the numbers so that the decimal points are in the same vertical line. This must bring tenths under tenths, hundredths under hundredths, and so on. Add as in the case of integers. Place the decimal point in the sum directly beneath the points in the addends.

Illustration 2.—Add 37.64, .48$\frac{3}{4}$, .12$\frac{1}{2}$, and .6$\dot{6}\dot{6}$.

37.64	SOLUTION.—Write the 37.64. The complex decimal, .48$\frac{3}{4}$, made pure, equals .4875. It is terminate. Write it as shown in the operation. .12$\frac{1}{2}$ equals .125. It is terminate also. Write it. .6$\dot{6}\dot{6}$ being interminate, it is necessary to extend it to four places, since one of the other decimals has four places. Place the sign + after .6666 to show that a small fraction must be added to it to make it exact. Add and point as directed.
.4875	
.125	
.6666 +	
38.9191 +	

EXERCISE 32

1. Add .125, 9.75, 31.5625, 3.6, and 75.0004.

2. Add .325, 300.025, .9375, 32.125462, and 7.5.

3. Add 49.327, .458, 8317.05, 341.875, and 32.4962.

4. Add 560.379, .45687, 350.0036, 7.074, and 52.257.

5. Add three, and seven-tenths; twenty-five, and one hundred twenty-five thousandths; ninety-five millionths; two hundred two ten-thousandths; two hundred, and two ten-thousandths.

6. What is the sum of 25, and 75 millionths; 4, and 24 hundredths; 700, and 25 thousandths; 925 thousandths; 7, and 8 hundredths; 125 hundred-millionths; 327 ten-thousandths; and 1000, and 625 thousandths?

7. Add 16$\frac{3}{4}$, 25$\frac{1}{2}$, 13$\frac{7}{8}$, 4$\frac{7}{16}$, 5$\frac{1}{32}$, and 3$\frac{5}{8}$, expressing the result decimally.

8. What is the sum of $\frac{1}{3}$, $\frac{7}{8}$, $\frac{5}{6}$, $\frac{1}{12}$, $\frac{3}{4}$, $\frac{2}{3}$, $\frac{2}{7}$, and $\frac{8}{9}$, correct to five decimal places?

9. What is the sum of .125, .75, .625, .5625, .9375, .0025, .075, .00875, and .00005, expressing the result as a mixed number?

10. Find the sum of 30.5, 28.25, 40.125, 52$\frac{7}{8}$, 174$\frac{31}{32}$, 5$\frac{1}{320}$, 49$\frac{13}{16}$, 245.01$\frac{3}{4}$, and .625.

11. Add 4.7$\frac{1}{2}$, 21.3$\frac{3}{4}$, 8.9$\frac{3}{8}$, 11.0$\frac{5}{16}$, and 9.

12. Find the sum, correct to six decimal places, of .48$\frac{2}{5}$, .006$\frac{5}{8}$, 46.38$\frac{1}{3}$, .0008$\frac{2}{5}$, 9.26, and 432.56$\frac{3}{7}$.

49. SUBTRACTION OF DECIMALS

Illustration.—What is the difference between 4.295 and 2.49167 ?

4.29500
2.49167
1.80333

SOLUTION.—Arrange, as in the case of subtracting whole numbers, so that the larger number is above and the smaller one is beneath it. Remember the size of the number is gauged by the whole number, not by the number of figures it may contain in the decimal part. If there are not as many figures in the decimal part of the minuend as there are in the subtrahend, supply 0's. Subtract, as in the case of whole numbers. Place the decimal point in the answer directly beneath the position it occupies in the minuend.

EXERCISE 33

	1.	2.	3.	4.
From	18.5	2.8706	.50376	.36
Take	2.3476	.49	.065	.12704

From—

5. 1.869 take .0374
6. .0061 take .00089
7. 6.723 take 2.7981
8. 9.305 take 7.9
9. 204.1 take 36.002
10. 1000 take 999.99
11. 2 take 1.3678
12. 17.36 take 9.0184

50. MULTIPLICATION OF DECIMALS

Illustration.—Find the product of .497 and .31.

.497
.31
497
1491
15407

.497 = $\frac{497}{1000}$ and .31 = $\frac{31}{100}$

SOLUTION.—Applying the rule for the multiplication of common fractions—
$\frac{497}{1000} \times \frac{31}{100} = \frac{15407}{100000} = .15407$.

Note that the denominator of the first fraction is 1, followed by three ciphers, and that the denominator of the second fraction is 1, followed by two ciphers. The denominator of the product is therefore 1, followed by three plus two, or five ciphers.

51. Rule.

Multiply as with whole numbers ; and from the right of the product point off as many decimal places as are contained in both factors.

NOTE 1.—If the product contains fewer figures than are to be pointed off, supply the deficiency by prefixing ciphers.

NOTE 2.—Terminate Complex Decimals are usually expanded to simple decimals before applying the rule ; and Interminate Complex Decimals are multiplied as convenience best dictates, either by reducing to equivalent common fractions or by expanding the decimal places sufficiently far for accuracy.

Thus, $31.87\frac{1}{2} \times .97\frac{1}{4}$ would be handled as $31.875 \times .97.5$.

Again, $.66\frac{2}{3} \times .14\frac{2}{7}$ would be handled as $\frac{66\frac{2}{3}}{100} \times \frac{4\frac{2}{7}}{100} = \frac{2}{3} \times \frac{1}{7} = \frac{2}{21}$.

NOTE 3.—Decimals are multiplied by 10, 100, 1000, etc., by moving the decimal point of the multiplicand as many places to the right as there are ciphers in the multiplier.

EXERCISE 34

Multiply—

1. .75 by .8
2. .17 by .9
3. .62 by .12
4. 6.2 by .39
5. 9.26 by .0025
6. .493 by .647
7. 35.24 by .005
8. 43.86 by 17.45
9. .0023 by .014
10. .4362 by 28
11. .562 by .00074
12. .0354 by 6.005
13. 6.354 by $.34\frac{3}{4}$
14. 7.59 by $.75\frac{2}{3}$
15. $30.03 \times .5 \times .0007$
16. $25 \times .125 \times .3$

DIVISION OF DECIMALS

52. Division of decimals is a process similar to division of whole numbers. The placing of the decimal point is the only thing that may occasion any difficulty, and even this point is made clear by the application of two principles that should already be in hand.

Illustration 1.—Divide .25 by 5.

5) .25
.05

SOLUTION.—Look at the decimal as a vulgar fraction $.25 = \frac{25}{100}$. Then $\frac{25}{100} \div 5 = \frac{25}{100} \times \frac{1}{5} = \frac{5}{100} = .05$.

Note that the quotient must be the same, in denomination, as the dividend. If we had to divide .025 by 5, it would be $\frac{25}{1000} \div 5$, or $\frac{25}{1000} \times \frac{1}{5} = \frac{5}{1000} = .005$. Note the application of the first principle. If the divisor is a whole number, the quotient will be of the same denomination as the dividend. 25 hundredths divided by 5 gives as a result 5 hundredths, just as 25 tens divided by 5 gives as a result 5 tens, and so on.

Illustration 2.—Divide 47.1346 by 7.35.

```
7.35)47.1346(
735.)4713.46(6.41
   4410
   ----
   3034
   2940
   ----
    946
    735
    ---
    211
```

SOLUTION.—Multiply both divisor and dividend by 100. Since the divisor is not a whole number, we make it a whole number by multiplying it by 100. To keep the equality, we also multiply the dividend by 100 (second principle). With the divisor a whole number, we proceed as in ordinary long division. In pointing off the places, we proceed, as in the first example, to give the quotient the same number of places as the dividend contains (two). If it were required to carry the division out to any further number of places, it may be done by affixing ciphers to the remainder until the required number of divisions can be performed.

53. Rule.

1. *Reduce the divisor to a whole number by moving the decimal point the required number of places to the right.*

2. *Move the decimal point in the dividend the same number of places to the right, annexing ciphers if necessary.*

3. *The quotient to the decimal point is a whole number, that to the right is a decimal.*

EXERCISE 35

Divide—

1. .5625 by 25
2. .1291136 by 262
3. .96351 by 346
4. .83094 by 75
5. .84375 by .25
6. .1291136 by .262
7. .763 by .125
8. .843 by 2.25
9. 375 by .25
10. 8.45 by .075
11. 94.3 by .0004
12. .635 by .000005
13. 29 by .06 (4 places)
14. 82.97 by 71 (3 places)
15. 2.3783 by 43 (4 places)
16. $75\frac{2}{3}$ by 27 (5 places)
17. $15\frac{7}{8}$ by $14\frac{1}{3}$ (4 places)
18. $\frac{5}{9}$ by 27 (3 places)
19. $\frac{5}{6}$ by 30 (4 places)
20. $1.008\frac{3}{4}$ by 9 (4 places)
21. $14\frac{2}{7}$ by $8\frac{1}{3}$ (2 places)
22. 35 by 21 (4 places)
23. 378.45 by 93.4 (6 places)
24. $63.71\frac{7}{16}$ by 25 (3 places)

54. SHORT METHODS IN DECIMALS

Approximations

Suppose that the exact result of an operation is 27.47186. For ordinary business purposes, three places may be sufficient. Reading our result to the nearest figure, and retaining but three places of decimals, it becomes 27.472, which is an approximate value of 27.47186 correct to three places of decimals. Short methods in decimals are, therefore, attempts at getting approximate values to a certain number of places.

55. Addition

(*a*)	72.142756	(*b*)	72.143
	15.2176		15.218
	42.71594		42.716
	130.076296		130.077

SOLUTION (*a*) shows the addition carried out complete.

SOLUTION (*b*) shows the solution correct to three decimal places. This is done by writing each addend, retaining only three places of decimals.

The addition is then performed in the usual way.

Subtraction may be handled in the same way where an approximation is sufficient.

56. Multiplication

Illustration.—Multiply 171.2478 by 8.4712, retaining only 4 decimal places.

(*a*)

171.2478		
8.4712		
342	4956	
1712	478	
119873	46	
684991	2	
13699824		
1450.6743	6336	

(*b*)

171.24780	
21748	
136998240	
6849912	
1198729	
17124	
3424	
1450.67429	

Ans.: 1450.6743

SOLUTION (*a*) shows the work carried out in full.

SOLUTION (*b*) shows the work contracted; so as to give an approximation to four decimal places.

57. Rule.

Reverse the multiplier, placing the units figure thereof directly under the decimal to which it is intended to extend the work (which should be one place further than an accurate answer is required). If the multiplier does not contain a whole number, place the tenths figures directly under the next figure to the left of the decimal to which it is intended to extend the work. Multiply as in ordinary multiplication, ignoring all figures in the multiplicand to the right of the figure which is used as multiplier. Arrange the several products so that the figures on the extreme right are in line vertically. Add and point off the number of places to which the work is extended.

EXERCISE 36

1. 36.275 × 4.3678 retaining 2 decimal places
2. 41.3075 × 467.32 retaining 3 decimal places
3. 17.0036 × .08245 retaining 4 decimal places
4. .43261 × .73158 retaining 3 decimal places
5. .003647 × .12739 retaining 4 decimal places
6. 700.375 × .02736 retaining 3 decimal places
7. .374825 × .693847 retaining 5 decimal places

58. Division

In dividing decimals, the quotient will take one of three different forms. It will be—

(*a*) A whole number, with or without a decimal part ;

(*b*) A decimal in which the first figure is a significant figure ; or

(*c*) A decimal in which there are one or more noughts between the decimal point and the first significant figure.

These three forms may be illustrated thus—

45.6 or 456 ; .456 ; and .00456.

In dividing by the contracted method, the first thing to be decided is, in the quotient, how many figures will there be in the whole number, or how many noughts will there be to the right of the decimal point.

59. Thus, in dividing 3754.26 by 13.254, it will be seen that there will be three figures in the whole number of the quotient. For, disregarding the decimals, 13 is contained in 37, and there will be one figure in the quotient for the 5 and one for the 4.

60. In dividing 32.4986 by 18376.25, there will be two noughts to the right of the decimal point in the quotient. For 18376 is not contained in 32 and, after reaching the decimal point, there must be a nought inserted after it in the quotient for every figure until the divisor is contained in the dividend, and 18376 is not contained in 324 nor 3249, but is contained in 32498.

61. In dividing 367.52 by 485.346, there will be no whole number in the quotient, and, at the same time, no noughts between the decimal point and first significant figure, since 485 is not contained in 367, but is contained in 3675.

62. In testing the quotient of .00375 by .2864, it will be necessary to shift the decimal point three figures to the right in both dividend and divisor, thus multiplying them both by the same number, 1000, which will not affect the quotient. The question then becomes $3.75 \div 286.4$, in which it is seen that the quotient takes the form in which there is one nought between the decimal point and the first significant figure.

63. Having decided how many figures there will be in the whole number in the quotient, or how many noughts there will be to the right of the decimal point, the next step is to decide how many figures there will be in the whole quotient. To decide this, if the quotient contains a whole number, we will add to the number of figures to which we want the answer to be correct the number of figures in the whole number. If the quotient contains no whole number, but takes the form in which there are noughts between the decimal point and the first significant figure, we will subtract from the number of figures to which we want the answer correct the number of noughts between the decimal point and the first significant figure. If there is no whole number in the quotient and there are no noughts between the decimal point and the first significant figure, the number of figures in the quotient will simply be the number of figures to which we want the answer correct.

64. Having decided how many figures there will be in the quotient, we will arrange our divisor to contain the same number of figures. Beginning at the left of the divisor, we will count off the number of figures desired in the quotient, and strike out the remainder. If there are not a sufficient number of figures in the divisor, we will add noughts to make up the number.

65. Having adjusted the divisor, the next step is to adjust the dividend. To do this, we will count off from the left side enough figures in the dividend to make it contain the divisor, rejecting the others, or supplying noughts to make up the deficiency.

66. Having adjusted both divisor and dividend, we will divide as in ordinary division, except that, instead of bringing down a figure from the dividend each time we divide, we will drop one figure from the right of the divisor, continuing to do so until the divisor is all eliminated.

Note 1.—It is not necessary to remove the decimal point from the divisor, as in ordinary division of decimals, since we know before commencing the division what form the quotient is to take, and can place the decimal point in the quotient in its proper place. So the decimal points may be disregarded.

Note 2.—It is always well to take the result one place farther than that really wanted, so as to be sure the last figure of the contracted quotient is correct; since, if the figure coming after the one to which we want the answer correct is 5 or more, 1 should be added to the last figure of the contracted quotient.

67. Illustration.—Divide 714.2965 by 14.367 correct to four places of decimals.

SOLUTION 1.—Ordinary Method

14.367)714.2965(

14367)714296.5(49.71786
7468

139616 Ans. = 49.7179
129303

103135
100569

25660
14367

112930
100569

123610
114936

86740
86202

SOLUTION 2.—Contracted Method

14367)7142965(4971793
5746800

1396165
1293030

103135
10056

566
1436

1130
001

29
126

3

Ans. = 49.7179

On examining the numbers, we see that there will be two figures in the whole number of the quotient. Since we wish the result correct to four places of decimals, we will take it out to five places, so we will have seven figures in our quotient. We will therefore need seven figures in the divisor. Since there are only five figures in the divisor, we will add two noughts to the right of it. On examining the dividend, we find that it will contain our divisor. So we proceed with our division, discarding one figure of the divisor each time. The quotient obtained is 4971793; and, since there are to be two figures in the whole number of the answer, we count off from the left

side two figures and place the decimal point. Then, since we were to have the answer correct to four decimal places, we drop the last figure of the quotient, since it is less than 5. Had it been 5 or more, we would have made our quotient 49.7180.

EXERCISE 37

Divide—

1.	27.3782	by	4.3267	correct to 3 decimal places
2.	487.24	by	1.003675	correct to 2 decimal places
3.	8.47326	by	75.43	correct to 5 decimal places
4.	.8487564	by	.075637	correct to 3 decimal places
5.	478.325	by	$1.43\frac{2}{3}$	correct to 3 decimal places
6.	8972.436	by	756.3452	correct to 4 decimal places
7.	1	by	1.007633	correct to 6 decimal places
8.	.953728	by	44.73654	correct to 3 decimal places

68. To Reduce British Currency to Canadian Currency

(*Note.*—See British Money Table on page 54)

METHOD NO. 1

Illustration.—Change £26 8s. 7d. to Canadian money when £1 is quoted at $4.75.

SOLUTION. 7d. $= \frac{7}{12}$s.

8s. $+ \frac{7}{12}$s. $= 8\frac{7}{12}$s.

$8\frac{7}{12}$s. $= \frac{8\frac{7}{12}}{20}$ or $\frac{103}{240}$ or ·429 of a £

£26.429 at $4.75 = $125.54.

METHOD NO. 2

The following is a short method of converting shillings and pence to a decimal of a pound:

(1) Divide the shillings by 2. This converts the shillings to a tenth part of a pound.

(2) Multiply the pence by 4 and add 1. This converts the pence to a thousandth part of a pound (approximately).

Illustration (*a*).—Where the amount of the shillings is an even number:

Change £26 8s. 7d. to Canadian money when £1 is quoted at $4.75.

SOLUTION. 8s. ÷ 2 = .4 represented as a tenth part of a pound.
7d. × 4 + 1 = .029 represented as a thousandth part of a pound.
.429 which is the decimal representing 8s. 7d.
and £26.429 at $4.75 = $125.54

Illustration (*b*).—Where the amount of the shillings is an odd number:

Change £26 7s. 4d. to Canadian money when £1 is quoted at $4.75.

SOLUTION. 7s. ÷ 2 = .3½ represented as a tenth part of a pound, which is equivalent to .35.
or .35
4d. × 4 + 1 = .017 represented as a thousandth part of a pound.
.367 which is the decimal representing 7s. 4d.
and £26.367 at $4.75 = $125.24.

Illustration (*c*).—Change £4 2s. 2d. to Canadian money when £1 is quoted at $4.80.

SOLUTION. 2s. ÷ 2 = .1 represented as a tenth part of a pound.
2d. × 4 + 1 = .009 represented as a thousandth part of a pound.
.109 which is the decimal representing 2s. 2d.
and £4.109 at $4.80 = $19.72.

EXERCISE 38

Change to Canadian currency—

	£	s.	d.			£	s.	d.	
1.	72	4	7	@ $4.85	6.	475	18	10	@ par
2.	47	15	4	@ $4.78	7.	547	8	5	@ $3.95
3.	57	12	6	@ $4.82½	8.	297	19	11	@ $4.28¼
4.	195	17	8	@ $4.72	9.	547	10	2	@ $4.90
5.	240	6	9	@ $4.81¼	10.	679	13	3	@ $4.82⅛

69. REPEATING OR CIRCULATING DECIMALS

In reducing common fractions to decimals, we find that sometimes the division does not terminate. Even in such fractions, if the division is carried far enough, a remainder would be obtained which had occurred before and a set of figures would be repeated in the same order in a never-ending succession. Such a decimal is called a *repeating*, or *circulating*, or *interminate decimal*, or simply a *repetend*.

Thus, $\frac{1}{3} = .3333 +$, or $\frac{1}{3} = .\dot{3}$. Again, $\frac{5}{7} = .714285714285 +$, or $\frac{5}{7} = .\dot{7}14285\dot{5}$.

Note that when the repetend consists of one figure, it is indicated by placing a point over it. If it consists of more than one figure, a point is placed over the first and last figures of the repeating part.

70. A Mixed Circulating Decimal is one in which all of the figures do not repeat.

Thus, $\frac{1}{12} = .08333 +$, or $\frac{1}{12} = .08\dot{3}$.

71. To reduce a pure repetend to an equivalent vulgar fraction.

Illustration.—Express $.\dot{2}\dot{5}$ as a common fraction.

$$.\dot{2}\dot{5} = \tfrac{25}{99}$$

SOLUTION.—

$.\dot{2}\dot{5} = .252525 +$

$\therefore$ 100 times $.\dot{2}\dot{5} = 25.252525 +$

And 1 times $.\dot{2}\dot{5} = .252525 +$

Subtracting, 99 times $.\dot{2}\dot{5} = 25$

$\therefore .\dot{2}\dot{5} = \frac{25}{99}$

72. Rule.

Put the period for numerator, and as many nines as there are places in the period for denominator.

73. To reduce a mixed repetend to an equivalent vulgar fraction.

Illustration.—Express $.24\dot{5}\dot{6}$ as a common fraction.

SOLUTION.—

$$\begin{array}{rl} & 2456 \\ & 24 \\ \hline & 2432 \quad \tfrac{2432}{9900} \text{ Ans.} \\ .24\dot{5}\dot{6} = & .2456565656\ldots + \\ \therefore \text{ 10000 times } .24\dot{5}\dot{6} = & 2456.565656\ldots + \\ \text{And 100 times } .24\dot{5}\dot{6} = & 24.565656\ldots + \\ \hline \text{Subtracting, } \quad \text{9900 times } .24\dot{5}\dot{6} = & 2456 - 24 \end{array}$$

$$\therefore .24\dot{5}\dot{6} = \frac{2456 - 24}{9900}$$

74. Rule.

Subtract the finite part from the whole and set down the difference for the numerator.

For denominator, put as many 9's *as there are places in the "infinite" part, followed by as many* 0's *as there are places in the "finite" part.*

EXERCISE 39

Express as circulating decimals—

1. $\frac{1}{3}$
2. $\frac{2}{3}$
3. $\frac{1}{6}$
4. $\frac{5}{6}$
5. $\frac{3}{7}$
6. $\frac{6}{7}$
7. $\frac{5}{12}$
8. $\frac{9}{11}$
9. $\frac{5}{14}$
10. $\frac{13}{28}$
11. $\frac{13}{30}$
12. $\frac{11}{21}$
13. $\frac{5}{18}$
14. $\frac{19}{60}$
15. $\frac{7}{13}$

Express as fractions in their lowest terms—

16. $.\dot{5}$
17. $.\dot{7}\dot{2}$
18. $.\dot{4}\dot{6}\dot{8}$
19. $\dot{1}5\dot{3}$
20. $.\dot{3}\dot{6}\dot{9}$
21. $.\dot{2}63\dot{5}$
22. $.\dot{3}12\dot{3}$
23. $.4\dot{2}\dot{5}$
24. $4.25\dot{3}$
25. $.04\dot{7}2\dot{6}$
26. $7.2\dot{0}1\dot{1}$
27. $2.53\dot{0}\dot{6}$
28. $.251\dot{6}$
29. $.03\dot{5}\dot{7}$
30. $.00245$

REVIEW OF DECIMALS

EXERCISE 40

1. If a pound of tea be worth $.62\frac{1}{2}$, what is .8 of a pound worth?

2. Bought a cord of wood for $4.625, a cheese for $7.56\frac{1}{4}$, and $14\frac{9}{16}$ lbs. of butter at 25c. per lb. What was the cost of the whole?

3. How many pairs of trousers can be made from 48.6 yds. of cloth, allowing 1.8 yds. per pair?

4. A certain number multiplied by 2.5 and the product divided by 5.2 produces 1. What is the number?

5. A man who owned .1875 of a mine sold $.1\dot{7}$ of his share. What fractional part had he left?

6. Divide the sum of .075 and .0075 by the difference between 7.5 and .75.

7. What decimal must be taken from the sum of $69\frac{1}{8}$, 8.2, 5.445, .065, and $20\frac{1}{12}$ so that it will contain 6.05 an exact number of times?

8. Multiply 350.4 by .0105 and divide the product by .0000219.

9. Express $.72\dot{5}6\dot{7}$ as a vulgar fraction in its lowest terms.

10. The average of seven numbers is 26.98. The average of the first two is 34.5 and of the next three is 19.3. Find the average of the remaining two.

EXERCISE 41

1. The average of four quantities is $18\frac{3.5}{2\,9\,7}$; the first is $26.2\dot{0}\dot{7}$, the second $3.\dot{5}9\dot{2}$, and the third $38.0\dot{6}$. Find the fourth.

2. At $1.75 per rod, what will it cost to fence a piece of land 63.5 rd. long and 27.75 rd. wide?

3. A drover lost .065 of his flock by wolves, .105 by disease, and .27 by theft; he then sold .75 of what remained, and has 280 sheep left. Find the number in his original flock.

4. A man left .375 of his estate to his wife, .4 of the remainder to his daughter, and the remainder to his son, who received $7,678.50. How much did his wife and daughter each receive?

5. A speculator has 640 ac. of land; he sells .25 of it at $43 per acre, $.1\dot{6}$ of the remainder at $30 per acre, as many acres as he sold the first and second times at $37\frac{1}{2}$ per acre, and the balance at $24 per acre. How much did he gain if he paid $25 per acre for the land?

6. If .375 of a barrel of sugar cost $4.77, what will .875 of a barrel cost?

7. A man has a flock of 720 sheep and sells .625 of his flock. What is the value of the remainder of the flock at $3\frac{1}{4}$ per head?

8. 427 is .875 of what number?

9. From a farm of $125\frac{3}{10}$ ac. there were sold at one time 27.63 ac. and at another $34\frac{3}{8}$ ac. How many acres remained?

10. By dividing 11.4575 by a certain divisor we obtain the quotient 2.5. What would be the quotient were we to divide .192486 by the same divisor?

EXERCISE 42

1. From an oil tank containing 375.087 galls. there leaked out each day $2\frac{5}{8}$ galls. How many gallons remained in the tank at the end of 27 days?

2. A certain decimal multiplied by 100 equals .65. What is .2 of the decimal?

3. I sold .355 of a cask of vinegar at $.35 a gallon and received $4.97 for it. How many gallons did the cask contain?

4. How many tons of coal in five carloads, weighing respectively 22.815 tons, 21.86 tons, 20.185 tons, 19.998 tons, and 18.125 tons?

5. I lack 15.035 bus. of having 24.08 bus. of wheat. How many bushels of wheat have I?

6. The sum of five addends is 375.045. If four of the addends are 72.05, 31.009, .4158, and 8.3, what is the fifth?

7. A man bequeathed .125 of his property to an orphan asylum, $\frac{5}{16}$ to his son, and the remaining $24,120 to his wife. What was the value of his property?

8. A and B at first had farms of equal size, but after A added 75.003 ac. to his farm and B sold 15.043 ac. from his, it was found that they had 360.12 ac. How many acres had each at first?

9. Multiply $\frac{3}{7}$ of .175 by $.\dot{2}85714\dot{2}$ and divide the result by .00425.

10. Assuming that £1 = $4.86\frac{2}{3}$, find the value of $2,300 in English currency.

EXERCISE 43

1. A farmer bought 48.125 tons of hay; for 20.25 tons of it he paid $16 per ton, and for the rest $18.2625 per ton; he sold the whole at an average price of $.945 per cwt.; how much did he gain or lose?

2. What decimal multiplied by 125 will give the sum of $\frac{5}{8}$, $\frac{7}{16}$, $\frac{3}{4}$, .09375, and 2.46?

3. The product of three vulgar fractions is $\frac{4}{7}$; two of them are expressed by the decimals $.\dot{6}\dot{3}$ and $.13\dot{6}$; by what fraction will the third one be expressed?

4. If a business produces an annual return of $6,000, and of three partners one has .475 and another $.3\dot{8}$ of the profits, how much money falls to the share of the third partner?

5. A man paid $120 for a horse; for a buggy, $36\frac{5}{16}$ more than $.\dot{3}$ of the cost of the horse; for harness, $.18\dot{5}$ of the cost of horse and buggy. Find his entire outlay.

6. If a pound of sugar cost .0093125 of $8, find the value of .0625 of 16 bbl. of 200 lbs. each.

7. A man who owned $\frac{3}{5}$ of a steamboat sold $.\dot{7}$ of his share for $1,400; what decimal part of the boat does he still own, and what was the boat worth?

EXAMINATION No. 3

(DECIMALS)

Time 1 *hour*—80% *required*

1. Multiply the sum of sixty-five hundred and sixty-five and one hundred and seven millionths by the product of nine hundred-millionths and one hundred twenty-seven and seventeen hundredths.

2. If a wheelman travels 10.3 hrs. per day, how many days will be required for him to travel 558.0025 miles at the rate of 7.88 per hour?

3. I sold a lumberman 381.25 lbs. of butter at $28\frac{3}{4}$c. per pound, 2468.375 lbs. of cheese at 11.4c. per pound, and 2356.5 lbs. of dressed beef at $7\frac{7}{8}$c. per pound, and received pay in lumber at $\$23.12\frac{1}{2}$ per thousand feet. How many thousand feet of lumber should I have received?

4. In a certain business school, .5 of the students study bookkeeping, .75 of the remainder study shorthand and typewriting, and the remainder, 125 pupils, study the English branches. How many students in each department, and in the entire school?

5. The sum of three numbers is 4.5. If the smaller is .95 and the larger 2.05, what is the product of the three numbers?

6. A and B are in partnership. A is to receive .75 of the profits and B the remainder. At the end of one year, B draws $1,250 as his share of the profits. If the total losses for the year were $950, what was the total gain for the year? The net gain?

7. January 1st, A and B join in the purchase of some real estate, A paying .4 of the purchase price, and B the remainder. They share the profits arising from the sale in proportion to their investments. The property is sold at a profit of $2,500 and B receives $6,000 as his share. How much did A and B pay for the real estate?

8. A merchant had on hand Jan. 1, 1927, a stock of merchandise aggregating $11750.90. During the year he bought goods amounting to $7315.90 and sold goods amounting to $15364.85. If on Dec. 31, 1927, he has stock on hand valued at $9215.75, has he gained or lost for the year and how much?

COMMERCIAL ARITHMETIC

TEST PAPER—No. 3

DECIMAL FRACTIONS

Your work for this paper is to be submitted to the instructor for correction and criticism.

1. Write in figures, add, and write the sum in words : five tenths ; two hundred, and fifteen thousandths ; five thousand, and four hundredths ; one, and two hundred-thousandths ; ten, and eleven millionths ; thirteen, and six thousand forty-five hundred-thousandths ; twenty-five tenths.

2. From $\frac{17}{125}$ take sixty-three ten thousandths, and write the difference in words, as a decimal, and as a fraction.

3. Find the product of—

(*a*) 1·732 and 1·4142.
(*b*) 2·236 and 2·4495.
(*c*) 147·4771 and ·30103.
(*d*) 57·29577 and 1·01.
(*e*) 20,403·1416 and ·000378.
(*f*) ·00004848 and 5,463.

4. Divide—

(*a*) 16·578 by 5·4.
(*b*) 48·591 by ·96.
(*c*) 2·56 by ·0032.
(*d*) 4·126 by 640.
(*e*) 3·1 by ·0025
(*f*) ·0012 by 1·6.

5. Add : $\cdot\dot{3}\dot{6}$; $\cdot 5\dot{3}\dot{9}$, and $\cdot 032\dot{4}3\dot{7}$ and express the result as a circulating decimal.

6. A piece of cloth was said to contain 84 yd., but it was found that the so-called yard measure with which it was measured was ·02083 of a yard too short. What was the correct length of the cloth ?

7. A man devotes ·12 of his income to charity, ·25 for educating his children, ·45 for household expenses, and saves the remainder, which is $284·76. What is his income ?

8. A ship whose cargo was worth $25,000, being disabled, ·45½ of the whole cargo was thrown overboard. What would a merchant lose who owned ·25 of the cargo ?

9. D. D. Wilson, of Seaforth, exported last year 8,360 barrels of eggs, each containing the same number. He received an average price of 14·85c. per dozen. Allowing the cost (including packing, etc.) to have been 13·5c. per dozen, and the entire profit to have been $7,900·20, find the number of eggs packed in each barrel.

10. Two lines are 41·06328 in. and ·0438 of an inch long respectively. How many lines as long as the latter can be cut off from the former, and what will be the length of the remaining line ?

It's What You Do That Counts

It isn't the job you intended to do,
  Nor the labor you've just begun
That puts you right on the ledger sheet,
  It's the work you have actually done.

Your credit is based on the things you do,
  Your debit on things you shirk.
The man that totals the biggest plus
  Is the man that completes his work.

Good intentions do not pay bills ;
  It's easy enough to plan.
To wish is the play of an office boy,
  TO DO is the work of a man.

DENOMINATE NUMBERS

TABLES OF VALUES, WEIGHTS AND MEASURES

75. Canadian Money

TABLE

10 Mills	. . .	= 1 Cent	. . .	*ct.* or *c.*
100 Cents	. . .	= 1 Dollar.	. .	*dol.* or $

The mill is defined by statute, but is not recognized in ordinary commercial transactions. Its use is practically confined to stating rates of local taxation, which are generally described as so many mills on the dollar of assessed value; thus, a rate of .015 is described as 15 mills on the dollar.

76. British Money

British or Sterling money is the currency of Great Britain.

The unit is the **Pound Sterling,** which is represented by a gold sovereign, and is equal in value to $4.86\frac{2}{3}$.

TABLE

4 Farthings (*qr.* or *far.*)	.	= 1 Penny	. .	*d.*
12 Pence	. . .	= 1 Shilling	. .	*s.*
20 Shillings	. . .	= 1 Pound or Sovereign		£
21 Shillings	. . .	= 1 Guinea		

77. Avoirdupois Weight

Avoirdupois Weight is used for all the ordinary purposes of weighing.

The **measuring unit** is the pound, which is equal to 7,000 grains.

TABLE

7,000 Grains	= 16 Ounces (*oz.*)	= 1 Pound . . *lb.*
100 Pounds . . .		= 1 Hundredweight . *cwt.*
2,000 Pounds, or 20 *cwt.* .		= 1 Ton . . . *T.*

The Dominion Weights and Measures Act declares that "all articles sold by weight shall be sold by Avoirdupois weight, except that gold, silver, platinum, and precious stones, and articles made thereof, may be sold by the ounce Troy or by any decimal part of such ounce."

In Great Britain and Ireland the grain, the ounce, and the pound are the same, but the hundredweight is equal to 112 lbs., and the ton to 2,240 lbs. (long ton).

British or Imperial Avoirdupois Weight

7,000 Grains (*gr.*)	= 16 Ounces (*oz.*)	= 1 Pound . . *lb.*
14 Pounds . . .		= 1 Stone . . *st.*
8 Stone . . .		= 1 Hundredweight *cwt.*
20 Hundredweight . .		= 1 Ton . . *T.*

78. Troy Weight

Troy Weight is used in weighing gold, silver, and jewels; in philosophical experiments.

The **measuring unit** is the **pound.**

TABLE

24 Grains (*gr.*) . .		= 1 Pennyweight . *dwt.*
20 Pennyweights . .		= 1 Ounce . . *oz.*
12 Ounces . . .		= 1 Pound . . *lb.*

The value of diamonds and other jewels is estimated by carats.
A carat is the weight of 4 grains.

79. Apothecaries Weight

Apothecaries Weight is used by druggists and physicians in compounding medicines, but drugs and medicines are bought and sold by Avoirdupois weight.

The **measuring unit** is the pound.

The pound, ounce, and grain are the same as in Troy weight.

rected by excluding from the leap years the centennial years, which are not divisible by 400. Hence, to find whether any year is a leap year or not—

Divide the number of centennial years by 400 *and all other years by* 4 *; if there is no remainder, the year is a leap year.*

The civil year includes both common and leap years, and is divided into 12 calendar months, viz.—

January (*Jan.*)	31 Days	July (*July*)	31 Days
February (*Feby.*)	28 Days	August (*Aug.*)	31 Days
In Leap Year	29 Days	September (*Sept.*)	30 Days
March (*Mar.*)	31 Days	October (*Oct.*)	31 Days
April (*Apr.*)	30 Days	November (*Nov.*)	30 Days
May (*May*)	31 Days	December (*Dec.*)	31 Days
June (*June*)	30 Days		

88. Standard Time

During the year 1883 the principal railroads of Canada and the United States adopted what is known as the "**Standard Time System.**" This system divides Canada and the United States into four sections or timebelts, each covering 15° of longitude, $7\frac{1}{2}$° of which are east and $7\frac{1}{2}$° are west of the governing or standard meridian, and the time throughout each belt is the same as the astronomical or local time of the governing meridian of that belt.

The governing meridians are the 75th, the 90th, the 105th, and the 120th, west of the Greenwich Observatory, London, England, and as these meridians are just 15° apart, there is a difference in time of **exactly** one hour between any one of them and the one next on the east, or the one next on the west; the standard meridian next on the east being one hour faster, and the one next on the west one hour slower. Hence, the 60° of longitude is four hours, the 75° five hours, the 90° six hours, the 105° seven hours, and the 120° eight hours slower than Greenwich time, making five different standards of time between the Atlantic and the Pacific Oceans, viz.: Intercolonial, Eastern, Central, Mountain, and Pacific.

89. Circular or Angular Measure

Circular Measure is used principally in surveying, navigation, astronomy, and geography, for reckoning latitude and longitude, determining locations of places and of vessels, and in computing difference of time.

Every circle, great or small, is divided into the same number of equal parts: as quarters, called quadrants; twelfths, called signs; three hundred and sixtieths, called degrees, etc. Consequently the parts of different circles, although having the same names, are of different lengths.

The unit is the degree, which is $\frac{1}{360}$ part of the circumference of any circle.

TABLE

60 Seconds (″)	= 1 Minute	′
60 Minutes	= 1 Degree	°
30 Degrees	= 1 Sign	*S.*
12 Signs, or 360°	= 1 Circle	*C.*

90. Miscellaneous Tables

COUNTING

12 Things	=	1 Dozen
12 Dozen	=	1 Gross
12 Gross	=	1 Great Gross
20 Things	=	1 Score

PAPER

24 Sheets	=	1 Quire
20 Quires	=	1 Ream
2 Reams	=	1 Bundle
5 Bundles	=	1 Bale

BOOKS

2 Leaves	=	1 Folio
4 Leaves	=	1 Quarto, or 4to
8 Leaves	=	1 Octavo, or 8vo
12 Leaves	=	1 Duodecimo, or 12mo

The terms **folio, quarto, octavo,** denote the number of leaves into which a sheet of paper is folded in making books.

91. REDUCTION OF DENOMINATE NUMBERS

In reduction, the *unit* or *denomination* of a number changes, but not the *value*. When the change is from a higher to a lower denomination, the process is called *reduction descending*; and when from a lower to a higher, *reduction ascending*.

To reduce denominate numbers from higher to lower denominations.

Illustration.—Change 1 bus., 1 pk., 1 pt. to pints.

1 bus. 1 pk. 0 qts. 1 pt.
4
4 pecks
1 peck
5 pecks
8
40 quarts
2
80 pints
1 pint
81 pints

SOLUTION.—Since 1 bus. = 4 pks., 1 bus. and 1 pk. = 5 pks. Since 1 pk. = 8 qts., 5 pks. = 5 × 8 qts. = 40 qts. Since 1 qt. = 2 pts. 40 qts. = 40 × 2 pts. = 80 pts., and 80 pts. + 1 pt. = 81 pts.; therefore, 1 bus., 1 pk., 1 pt. = 81 pts.

EXERCISE 44

Reduce to the lowest denomination named—

1. 4 mi. 17 rd. 3 yd. 1 in.
2. £52 10s. 4d.
3. 6 bus. 1 pk. 3 qt.
4. 43° 7′ 23″.
5. 15 gal. 3 qt. 1 gi.
6. 7 cu. yd. 4 cu. ft. 11 cu. in.
7. 190 sq. rd. 15 sq. in.
8. 2 mi. 10 ch. 43 lk.
9. 4 t. 5 cwt. 3 lb. 1 oz.
10. 20 rd. 5 in.
11. 15 sq. rd. 5 sq. yd. 3 sq. ft.
12. 10 a. 31 sq. rd. 5 sq. yd. 4 sq. ft.
13. 7 wk. 210 hr. 5 min. 31 sec.
14. 8 cu. yd. 2 cu. ft.
15. 1 bbl. 2 gal. 1 pt.
16. 2 mi. 15 rd. 11 ft. 10 in.

92. Reduction from a lower denomination to a higher.

Illustration.—Reduce 473 pt. to bushels.

2 | 473 pt.	SOLUTION.—Since 2 pt. equal 1 qt., 8 qt. 1 pk., and 4 pk. 1 bus., the successive divisors for reducing given pints to bushels are 2, 8, and 4 respectively.
8 | 236 qt. + 1 pt.	
4 | 29 pk. + 4 qt.	Divide 473 pt. by 2 and the result is 236 qt. with a remainder of 1 pt.; divide 236 qt. by 8 and the result is 29 pk., with a remainder 4 qt.; divide 29 by 4 and the result is 7 bus. with a remainder 1 pk.
7 bus. + 1 pk.	
473 pt. = 7 bus. 1 pk. 4 qt. 1 pt.	

Write the last quotient and the several remainders in order and the required result is 7 bus. 1 pk. 4 qt. 1 pt.

EXERCISE 45

Reduce—

1.	813551	ounces	to tons, etc.
2.	23769	inches	to miles, etc.
3.	17150	pounds	to long tons, etc.
4.	4276	pints	to gallons, etc.
5.	185760	seconds	to days, etc.
6.	278644	cubic inches	to cubic yards, etc.
7.	32359	farthings	to £, etc.
8.	477960	cubic feet	to cords, etc.
9.	213546	sheets of paper	to reams, etc.
10.	2368	pence	to half-crowns, etc.
11.	45630	mills	to dollars, etc.
12.	2562	pints	to bushels, etc.

93. To reduce a fractional denominate number to a lower denomination.

Illustration. — Reduce £$\frac{7}{16}$ (.4375) to integers of lower denominations.

SOLUTION

$12\text{d.} \times 20 \times \frac{7}{16} = 105\text{d.}$

$105\text{d.} = 8\text{s. } 9\text{d.}$

or

$20\text{s.} \times \frac{7}{16} = 8\frac{5}{4}\text{s.} = 8\frac{3}{4}\text{s.}$

$12\text{d.} \times \frac{3}{4} = 9\text{d.}$

$\therefore \pounds\frac{7}{16} = 8\text{s. } 9\text{d.}$

or

$$\begin{array}{r} 7 \\ 20 \\ \hline 16)140(8\text{s.} \\ 128 \\ \hline 12 \\ 12 \\ \hline 16)144(9\text{d.} \\ 144 \end{array}$$

$12\text{d.} \times 20 \times .4375 = 105\text{d.}$

$105\text{d.} = 8\text{s. } 9\text{d.}$

or

$$\begin{array}{r} \pounds.4375 \\ 20 \\ \hline \text{s.}8.7500 \\ 12 \\ \hline \text{d.}9.0000 \end{array}$$

$\therefore \pounds.4375 = 8\text{s. } 9\text{d.}$

EXERCISE 46

Reduce to lower denominations—

1. $\frac{3}{4}$ of a day
2. $\frac{7}{12}$ of a mile
3. $\frac{4}{7}$ of a month (30 days)
4. $\frac{7}{11}$ of an acre
5. $\frac{5}{12}$ of a cubic yard
6. $\frac{8}{9}$ of a gallon
7. $\frac{1}{320}$ of a bushel
8. $\frac{1}{160}$ of a rod
9. .625 of a mile
10. .727 of a ton
11. .625 of an acre
12. .4225 of a cord
13. .5375 of a gallon
14. $\frac{3}{7}$ of a pound, Apoth.

94. To reduce a fractional denominate number to a higher denomination.

Illustration 1.—Reduce 9s. 6d. 3 far. to the decimal of a pound sterling.

(*a*)

$$\begin{array}{r|l} 4 & 3 \text{ far.} \\ \hline 12 & 6.75\text{d.} \\ \hline 20 & 9.5625\text{s.} \\ \hline \pounds.478125 & \end{array}$$

SOLUTION (*a*).—The successive divisors to reduce farthings to pounds are 4, 12, and 20 respectively Dividing 3 far. by 4, the result is .75d. Putting with this the 6d., the result is 6.75. Dividing 6.75d. by 12, the result is .5625s. Putting with this the 9s., the result is 9.5625. Dividing by 20, the result is .478125 pounds sterling. Or,

(*b*)
9s. 6d. 3 far. = 459 far.
£1 = 960 far.
459 ÷ 960 = £.478125

SOLUTION (*b*).—In 9s. 6d. 3 far. there are 459 far., and in £1 there are 960 far. Hence, 9s. 6d. 3 far. is $\frac{459}{960}$ of a pound sterling. $\frac{459}{960}$ = £.478125.

Illustration 2.—Reduce £19 9s. 6d. 3 far. to dollars and cents.

£19 9s. 6d. 3 far.
= £19.478125
£1 = $4.86⅔
£19.478125 = 19.478125 times $4.86⅔
= $94.79354 +
= $94.79

SOLUTION.—By previous illustration, 9s. 6d. 3 far. are equal to £.478125. Then £19.478125, at the par value of £1 ($4.86⅔) are equivalent to $94.79.

Illustration 3.—Reduce 4 yd. 2 ft. 6 in. to the fraction of a rod.

(*a*)
6 ÷ 12 = ½ ft.
⅔(2½) ÷ 3 = ⅔ yd.
$\frac{20}{6}(4\frac{2}{3})$ ÷ 5½ = $\frac{20}{33}$ rd.
(*b*)
4 yd. 2 ft. 6 in. = 174 in.
1 rd. = 198 in.
174 ÷ 198 = $\frac{174}{198}$ = $\frac{29}{33}$.

SOLUTION (*a*).—The successive divisors to reduce inches to rods are 12, 3, and 5½ respectively. 6 in. divided by 12 equal ½ ft. Putting with this the 2 ft., the result is 2½ ft. 2½ ft. divided by 3 equal ⅔ yd. Putting with this the 4 yd., the result is 4⅔ yd. 4⅔ yd. divided by 5½ equal $\frac{20}{33}$ rd. Or,

SOLUTION (*b*).—4 yd. 2 ft. 6 in. equal 174 in. 1 rd. equals 198 in. 4 yd. 2 ft. 6 in. is, therefore, $\frac{174}{198}$ of 1 rd., or $\frac{29}{33}$ rd.

EXERCISE 47

1. What fractional part of a bushel is 3 pk. 4 qt. 2 pt.?
2. What decimal part of 2½ mi. is 10 rd. 4 ft.?
3. Reduce 3d. 3 far. to the fraction of a shilling.
4. Reduce 20 rd. 12 ft. 10 in. to the fraction of a mile.
5. Reduce 2 sq. yd. 6 sq. ft. 100 sq. in. to the fraction of a sq. rod.

Reduce to Canadian currency—£ at par, $4·86⅔.

6. £47 12s. 6d.
7. £250 15s. 9d.
8. £75 10s. 11d.
9. £124 12s. 9d.
10. £176 9s. 8½d.
11. £29 17s. 7¾d.
12. £212 9s. 5¼d.

95. Addition of denominate numbers.

Illustration.—Add 1 bus. 2 pk. 1 pt., 1 bus. 1 qt., 1 bus. 3 pk. 7 qt. 1 pt.

bs.	pk.	qt.	pt.	
1	2	0	1	
1	0	1	0	
1	3	7	1	
3	5	8	2	= first sum.
4		1	0	= equivalents.

SOLUTION.—Add each denomination separately, and the simple denominations are 3 bus., 5 pks., 8 qts., and 2 pts. Changing to equivalents, the 2 pts. = 1 qt., which added to the 8 qts. = 9 qts.; 9 qts. = 1 pk. and 1 qt. remaining. Write the 1 qt. remaining under the column of quarts and add the 1 pk. to the 5 pks. 6 pks. = 1 bus. and 2 pks. Write the 2 pks. under the column of pecks, and add the 1 bus. to the 3 bus. above. Therefore, the equivalent of 3 bus., 5 pks., 8 qts., and 2 pts. = 4 bus., 2 pks., 1 qt.

In practice, omit the first sum and write only the equivalents.

96. Rule.

1. *Write the numbers so that units of the same name shall stand in the same column.*

2. *Begin at the right, add each column, and reduce the sum to the next higher denomination; write the remainder, if any, and add the quotient with the next column.*

EXERCISE 48

1.

£	s.	d.	far.
75	5	8	3
13	16	5	1
96	8	11	2
52	13	9	1
7	2	4	2
33	18	8	3

2.

mi.	rd.	yd.	ft.	in.
5	175	3	1	8
17	248	4	2	6
25	315	1	1	11
41	214	3	1	7
50	16	2	2	9
48	296	4	1	5

3. An importer paid £375 8s. 6d. for an invoice of cutlery, £251 5s. 8d. for an invoice of files, and £73 17s. 6d. for an invoice of saws. The charges for transportation amounted to £19 2s. 4d. What was the total cost of the goods in sterling money?

4. Add £.832, .0125s., 5.275s., and .17d.

97. Subtraction of denominate numbers.

Illustration.—From 71 rd. 3 yd. 2 ft. 6 in. take 15 rd. 5 yd. 2 ft. 8 in.

rd.	yd.	ft.	in.
71	3	2	6
15	5	2	8
55	$2\frac{1}{2}$	2	10
	$\frac{1}{2}=1$		6
55	3	1	4

SOLUTION.—Write the numbers so that the units of the same denominations stand in the same vertical columns, and beginning at the right subtract as in simple numbers.

Since 8 in. cannot be subtracted from 6 in., take 1 ft. (12 in.) from 2 ft. and add it to 6 in., making 18 in. 18 in. minus 8 in. leaves 10 in., which write as inches in the remainder.

Inasmuch as 1 ft. was added to 6 in., there is but 1 ft. remaining in the minuend. Since 2 ft. cannot be subtracted from 1 ft., take 1 yd. (3 ft.) from the 3 yd. and add it to the 1 ft., making 4 ft. 4 ft. minus 2 ft. leaves 2 ft., which write as feet in the remainder.

Inasmuch as 1 yd. was added to 1 ft., there are but 2 yd. remaining in the minuend. Since 5 yd. cannot be subtracted from 2 yd., take 1 rd. ($5\frac{1}{2}$ yd.) from 71 rd. and add to the 2 yd., making $7\frac{1}{2}$ yd. $7\frac{1}{2}$ yd. minus 5 yd. leaves $2\frac{1}{2}$ yd., which write as yards in the remainder. 70 rd. minus 15 rd. leaves 55 rd., which write as rods in the remainder.

Reducing $\frac{1}{2}$ yd. to lower denominations and adding, the required result is found to be 55 rd. 3 yd. 1 ft. 4 in.

EXERCISE 49

1.

mi.	rd.	yd.	ft.
12	75	3	2
8	318	4	1

2.

bus.	pk.	qt.	pt.
18	1	3	1
16	3	2	1

3. A merchant sold 25 gal. 2 qt. 1 pt. of vinegar from a barrel containing 46 gal. 1 qt. 1 pt. What quantity remained?

4. A farmer engaged to deliver 15 ton 16 cwt. 75 lb. of hay, and subsequently delivered 9 ton 18 cwt. 34 lb. How much hay had he yet to deliver?

5. An English merchant's sales for a year amounted to £15,396 5s. 8d. 2 far., and his purchases £18,352 2s. 9d. 1 far. How much greater were his purchases than his sales?

98. Multiplication of denominate numbers.

Illustration.—Each of seven bars of silver weighs 15 lb. 5 oz. 13 dwt. 16 gr. Find the total weight.

lb.	oz.	dwt.	gr.
15	5	13	16
			7
108 lb.	3 oz.	15 dwt.	16 gr.

SOLUTION.—Write the multiplier under the lowest denomination of the multiplicand, and multiply as in simple numbers, thus—

16 gr. × 7 = 112 gr. = 4 dwt. 16 gr Put down 16 under gr. Carry 4 to dwt.

13 dwt. × 7 + (4 dwt. carried) = 95 dwt. = 4 oz. 15 dwt. Put down 15 under dwt. Carry 4 to oz.

5 oz. × 7 + (4 oz. carried) = 39 oz. = 3 lb. 3 oz. Put down 3 under oz. Carry 3 to lb.

15 lb. × 7 + (3 lb. carried) = 108 lb. Put down 108 under lb.

99. Rule.

Multiply each denomination separately and change the partial products to units of higher denominations.

EXERCISE 50

1. Multiply 7 bus. 3 pk. 5 qt. 1 pt. by 18.

2. How much wood is there in 36 piles, each measuring 3 cd. 40 cu. ft. 1,020 cu. in. ?

3. If 60 ac. 40 sq. rd. 9 sq. yd. 6 sq. ft. 100 sq. in. be multiplied by 72, what will be the product ?

4. 32 ac. of oats averaged 42 bus. 3 pk. 5 qt. 1 pt. Find the entire crop.

5. How far will an engine travel in making 20 round trips between two places 25 mi. 80 rd. 4 yd. 2 ft. 6 in. apart ?

100. Division of denominate numbers. (1) Where divisor is abstract number.

Illustration.—Divide 5 bus. 1 pk. 3 qt. by 4.

bus.	pk.	qt.	pt.
4)5	1	3	0
1	1	2	1½

SOLUTION.—Dividing 5 bus. by 4 gives a quotient of 1 bus. and an undivided remainder of 1 bus.; reduce this remainder to pecks (4) and add to the 1 pk. of the dividend, obtaining 5 pk., which divided by 4 gives 1 pk. and an undivided remainder of 1 pk. Reduce the 1 pk. remaining to quarts (8), and adding to the 3 qt. of the dividend gives 11 qt., which divided by 4 gives 2 qt. and an undivided remainder of 3 qt. 3 qt.= 6 pt., which divided by 4 gives 1½ pt.

101. Rule.

Divide as in simple numbers, and when necessary reduce remainders to lower denominations.

Should the highest dividend order not contain the divisor, reduce its units to the order next lower, and so proceed to the end.

EXERCISE 51

Find the quotient of—

1. 17 bus. 3 pk. ÷ 9.
2. 17 ton 9 cwt. 20 lb. ÷ 16.
3. £28 15s. 5d. 2 far. ÷ 6.
4. From the sum of £25 4s. 10d. and £10 5s. 2d. take their difference, divide the result by 5, and reduce the quotient to Canadian money.
5. Reduce $5,164.28 to equivalent British money.

$4.86⅔) $5164.28 (
3) 3 (
1460) 1549284 (£1061
1460 (
8928
8760
1684
1460
224
20
1460) 4480 (3s.
4380
100
12
1460) 1200 (1d.

SOLUTION.—Clear of fractions by multiplying through by 3. Reduce remainder of £'s to shillings by multiplying by 20, and divide. Reduce remainder of shillings to pence and again divide.

Hence $5164.28 = £1061 3s. 1d.

6. Reduce $185 to equivalents in British money; $2,500.

102. (2) **Where divisor is denominate number.**

Illustration.—Divide 39 bus. 1 pk. 1 qt. 1 pt. by 5 bus. 3 pk. 3 qt. 1 pt.

SOLUTION

39 bus. 1 pk 1 qt. 1 pt. = 1875 pts.
5 bus. 3 pk. 3 qt. 1 pt. = 375 pts.
1875 ÷ 375 = 5

NOTE.—In this solution both quantities have been reduced to the lowest denomination mentioned in either, viz., the pint, but the same result would have been obtained had both expressions been reduced to any of the other denominations of the table, as the bushel, peck, gallon, or quart.

103. Rule.

Reduce both quantities to the same denomination and divide in the ordinary way. The result will show how many times the one quantity is contained in the other.

EXAMINATION No. 4

Time 1½ *hours*—80% *required*

1. How often will a wheel 5 ft. 6 in. in circumference turn in going 11 miles?

2. How many bags, each holding 2 bus. 2 pk. 1 gal. 3 qt. will be required to hold 1,003 bus. 1 gal. 3 qt.?

3. How many parcels each containing 1 lb. 4 oz. can be made from 1 ton 35 lb. of tea?

4. 798 ton 3 cwt. 64 lb. of hay is divided among a certain number of people so that each receives 5 ton 3 cwt. 66 lb. How many people are there?

5. How many fields, each containing 45 ac. 100 sq. rd. 25 sq. yd. can be made out of a tract of land containing 684 ac. 72 sq. rd. 12. sq. yd.?

6. How many portions of time, each equal to 1 da. 14 hr. 57 min. 33 sec. are contained in 365 da. 5 hr. 48 min. 45 sec.?

7. Among how many persons must £641 14s. 11d. 1 far. be divided, so that the share of each may be £2 15s. 6d. 3 far.?

8. If a steamer, on a voyage of 3,087 mi. 6 rd. 3 yd., average 14 mi. 93 rd. 2 yd. per hour, how many days will it take her to make the trip?

COMMERCIAL ARITHMETIC

TEST PAPER—No. 4

DENOMINATE NUMBERS

Your work for this paper is to be submitted to the instructor for correction and criticism.

1. A farm consists of 27 ac. 4,466 sq. yd. of land under grain ; 4 ac. 2,917 sq. yd. in roots ; 7 ac. 2,087 sq. yd. under hay ; 12 ac. 2,352 sq. yd. under pasture ; 37 ac. 2,934 sq. yd. woodland, and 4 ac. 4,750 sq. yd. in orchard, garden, barnyard, and building sites. What is the total area of the farm ?

2. (*a*) Reduce 468,093 cub. in. to higher denomination.

(*b*) Reduce 132 cub. yd., 11 cub. ft., 981 cub. in. to cubic inches.

3. I wish to put 111 bus. 2 pk. 4 qt. of grain into bags that shall contain 2 bus. 1 pk 4 qt. each. How many bags will be required ?

4. I bought 20 lb. of opium by avoirdupois weight, at 55c. an ounce, and sold by troy weight at 60c. an ounce. Did I gain or lose, and how much ?

5. Find the total cost of the following—

2,745 lb. of wheat at $1.20 per bushel.
867 lb. of oats, at 36c. per bushel.
1,936 lb. of barley at 60c. per bushel.
1,650 lb. of hay at $8.00 per ton.
2,675 ft. of lumber at $10.00 per thousand feet.

6. Reduce £29 19s. 9d. to Canadian currency.

7. It costs $23.10 to fence a square field at 3½c. per yard. How many acres are in the field ?

8. If 4 horses consume 24 bus. of oats in 12 days, how many bushels will 20 horses eat in 16 days ?

9. A ship 40 miles from the shore springs a leak, which admits $3\frac{3}{4}$ tons of water in 12 min. Sixty tons would suffice to sink her, but the ship's pumps can throw out 12 tons of water in an hour. Find the average rate of sailing, that she may reach the shore just as she begins to sink.

10. A manufacturer buys a mixed car of packing lumber at $12 per M., f.o.b. cars at mill, and ships same into Toronto, paying freight at the rate of 19c. per cwt. The lumber, on being unloaded and measured, is found to be as follows: 2 M. feet Basswood, 3 M. feet Red Pine, and 4 M. feet Ash. Estimated weight of Basswood is 2,700 lb. per M. feet, Pine 3,000 lb. per M., and Ash 3,500 lb. per M. Find cost delivered per M. feet of each kind of lumber.

SECTION 2

PERCENTAGE

TERMS USED IN PERCENTAGE CALCULATIONS

To the Student: The terms used in connection with percentage should be thoroughly studied and understood.

104. The **Prime Cost** of an article is the first cost.

105. The **Gross Cost** is the prime cost plus charges of freight, drayage, etc.

106. The **Selling Price** is the price for which an article is sold.

107. The **Profit** or **Gain** is the amount by which the selling price exceeds the gross cost.

108. The **Loss** is the amount by which the gross cost exceeds the selling price.

109. A **Cash Discount** is a reduction made from the total invoice price of a bill for payment within a limited time, as : 2% 10 days means 2% off the bill if paid within 10 days.

110. A **Trade Discount** is an allowance or deduction made by a dealer or manufacturer from catalogue or list prices.

111. An **Agent** is one who acts under authority for another.

112. The **Principal** is the one for whom an agent acts.

113. A **Commission Agent** is one who buys and sells goods or property, or collects money for his principal.

114. A **Broker** is an agent who effects purchases or sales in the interests of a buyer or seller. He brings the buyer and seller together, as it were, and is chiefly engaged in the case of stock transactions.

115. **Commission** is the name applied to the commission agent's charge.

116. **Brokerage** is the name applied to the broker's charge.

117. A **Consignment** is the property received by a commission agent to be sold for a principal.

118. The **Consignor** is the person who ships the goods.

119. The **Consignee** is the commission agent who receives the goods to be sold.

120. A **Del Credere Agent** is one who guarantees to his principal the payment of sales made on time.

121. **Guaranty** is the charge made for guaranteeing sales made on time.

122. The **Gross Proceeds** of a sale or collection is the total amount received by the agent before deducting charges.

123. The **Net Proceeds** is what remains after all charges have been deducted.

124. An **Account Sales** is the statement rendered by the agent to his principal, showing sales and all charges and the net proceeds.

125. An **Account Purchase** is the statement rendered by an agent to his principal, showing cost of goods bought and the charges, together with the total amount of the purchase.

126. **Duties** or **Customs** are taxes levied by the Dominion Government on goods imported into Canada. Duties are levied for the purpose of raising revenue and protecting home industry.

127. An **Ad Valorem Duty** is a certain percentage of the actual cost of the goods imported.

128. A **Specific Duty** is a tax assessed at a certain sum per foot, yard, gallon, or other weight or measure. Upon certain goods both kinds of duties are levied.

129. A **Tariff** is a list of goods, with the rates of duties imposed on the same.

130. The **Free List** includes goods that are exempt from duty.

131. A **Preferential Duty** is a duty lower than the common, imposed on goods from certain countries. Thus, goods imported from Great Britain or British possessions are given a preferential duty in Canada.

132. A **Surtax** is a duty higher than the common, imposed on goods from certain countries. Thus, goods imported from Germany are subject to surtax.

133. A **Drawback** is a refund made where goods once imported, and upon which import duty has been paid, are exported.

134. A **Bonded Warehouse** is a place for the storage of goods upon which duties have not been paid.

135. Taxes are sums collected by a municipality for the payment of municipal indebtedness. The tax is a sum assessed either on the person, property, or income of an individual.

136. A **Poll Tax** or **Head Tax** is the sum collected from each male citizen liable to taxation without regard to property or income.

137. A **Property Tax** is a tax on real or personal estate, and is assessed at a given rate, generally at so many mills on the dollar.

138. An **Income Tax** is a tax assessed on the individual according to his income. Generally speaking, a certain part of the income is exempt, and above that it is liable to taxation.

139. Interest is the use of money.

140. The **Principal** is the money which is loaned, and for the interest, or use, of which a certain sum in cash has to be paid.

141. The **Amount** is the sum of the principal plus the interest.

142. A **Joint Stock Company** is an association of individuals who have obtained incorporation enabling them to transact business as a single individual.

143. Methods of Incorporation.—Joint stock companies are formed in Canada in two ways : (*a*) By special Act of Parliament, either of the Parliament of Canada or the Legislature of the Province in which business is to be conducted ; (*b*) by the letters patent issued under the Companies Act. In some of the Provinces registration takes the place of letters patent.

144. The **Charter** or **Letters Patent** is the document, issued under the seal of the Department of State or the Provincial Secretary's Department, constituting certain persons therein mentioned, and others who may become associated with them, a joint stock company.

145. The **Authorized Capital of the Company** is the amount designated in the charter as the limit to which the company may invite subscriptions for stock.

146. The **Subscribed Capital** is that part of the authorized capital which at any date has been subscribed for, or allotted to, shareholders.

147. The **Paid-up Capital** is that part of the subscribed capital which at any date has been paid for by subscribers thereto.

148. A **Share** is one of the equal divisions into which the authorized

capital is divided. Unless specially mentioned to the contrary, the face or par value of a share is $100.

149. The **Par Value of a Share** is the value fixed upon it by the charter, which mentions the amount of the authorized capital, the number of shares into which it is divided, and the value of each.

150. The **Market Value of a Share** is the figure for which the share may be sold in the stock market. Thus, a $100 share may sell for $90 or for $150. This is all a question of the standing of the company and the way in which its stock is sought for.

151. A **Stock Certificate** is a paper issued by a company to one who has bought stock, and certifies as to the number of shares to which the holder is entitled.

152. Preferred Stock is stock which is entitled to a preference over any ordinary stock. This preference usually takes the shape of a prior claim on gains for the purpose of dividend, but it may extend in any other direction, such as a right to elect a stated number of the board of directors.

153. Common Stock is the ordinary stock of the company not entitled to any preference.

154. Cumulative Preferred Stock is stock which is preferred in the matter of dividends, and which, further, is entitled to have dividends which are not paid in one year cumulate as a charge against profits in succeeding years. Thus, if the stock were a 5% preference stock it means that each year 5% dividend is to be paid. If the 5% is not forthcoming the first year, the dividend to be paid the second year would really be 10%. If not paid in the first or second year, the dividend for the third year would really be 15%.

155. The **Dividend** is simply a part of the profits of the company set aside by the directors as the amount to be divided among the shareholders as a return for the money that they invested. The dividend must not be paid unless there is a net gain from which to pay it.

156. An **Assessment** is the opposite of a dividend, and is the amount contributed by the shareholders to meet losses or expenses of the company. It will be noted that once a shareholder has paid for his stock, he cannot be forced to pay any additional amounts. The assessment is, therefore, a voluntary contribution.

157. A **Call** is an order issued by the board of directors requesting those shareholders who have not fully paid for their stock to pay a certain amount on such stock.

158. An **Instalment** is the amount paid in answer to a call.

159. A **Premium** is the amount by which the market value of stock exceeds its par value.

160. A **Discount** is the amount by which the market value is less than the par value.

161. A **Bond** or **Debenture** is a written promise to pay a sum of money, with a fixed rate of interest, at or before a specified time. Bonds are simply the promissory notes of corporations, such as joint stock companies, cities, towns, villages, etc.

162. **First Mortgage Bonds** are bonds which are secured by a first mortgage on the property of the corporation issuing the bonds. In the same way we may have Second Mortgage Bonds.

163. **Registered Bonds** are those issued to a certain person whose name is registered in the books of the company as the owner of such bonds. The interest will be paid only to the registered owner or his attorney.

164. **Coupon Bonds** are bonds having small coupons or detachable certificates attached to the bond. These coupons may be clipped off at the time the interest falls due, and, on being presented, entitle the holder to the interest amount.

SHORT METHODS

SHORT METHODS IN MULTIPLICATION

to assist with percentage

165. Aliquot Parts.

Important: The student should commit to MEMORY the following tables so thoroughly that the aliquot parts can be named without the least hesitation when the fraction is given, and vice versa.

Aliquot Parts of 10

$1\frac{1}{9} = \frac{1}{9}$	$1\frac{3}{7} = \frac{1}{7}$	$2\frac{1}{2} = \frac{1}{4}$	$5 = \frac{1}{2}$
$1\frac{1}{4} = \frac{1}{8}$	$1\frac{2}{3} = \frac{1}{6}$	$3\frac{1}{3} = \frac{1}{3}$	

Aliquot Parts of 100

$6\frac{1}{4} = \frac{1}{16}$	$25 = \frac{1}{4}$	$62\frac{1}{2} = \frac{5}{8}$ or
$8\frac{1}{3} = \frac{1}{12}$	$31\frac{1}{4} = \frac{5}{16}$ or	$\frac{1}{2} + (\frac{1}{4}$ of $\frac{1}{2})$
$12\frac{1}{2} = \frac{1}{8}$	$\frac{1}{4} + (\frac{1}{4}$ of $\frac{1}{4})$	$75 = \frac{3}{4}$ or
$14\frac{2}{7} = \frac{1}{7}$	$33\frac{1}{3} = \frac{1}{3}$	$\frac{1}{2} + (\frac{1}{2}$ of $\frac{1}{2})$
$16\frac{2}{3} = \frac{1}{6}$	$37\frac{1}{2} = \frac{3}{8}$ or	$87\frac{1}{2} = \frac{7}{8}$ or
$18\frac{3}{4} = \frac{3}{16}$ or	$\frac{1}{4} + (\frac{1}{2}$ of $\frac{1}{4})$	$\frac{1}{2} + \frac{1}{4} + \frac{1}{8}$
$\frac{1}{8} + (\frac{1}{2}$ of $\frac{1}{8})$	$50 = \frac{1}{2}$	

Aliquot Parts of 1000

$83\frac{1}{3} = \frac{1}{12}$	$375 = \frac{3}{8}$ or	$833\frac{1}{3} = \frac{5}{6}$ or
$125 = \frac{1}{8}$	$\frac{1}{4} + (\frac{1}{2}$ of $\frac{1}{4})$	$\frac{1}{2} + \frac{1}{3}$
$166\frac{2}{3} = \frac{1}{6}$	$625 = \frac{5}{8}$ or	$875 = \frac{7}{8}$ or
$250 = \frac{1}{4}$	$\frac{1}{2} + (\frac{1}{2}$ of $\frac{1}{4})$	$\frac{1}{2} + \frac{1}{4} + \frac{1}{8}$
$333\frac{1}{3} = \frac{1}{3}$		

166. Multiplication by Aliquots

To multiply any number by

10 annex a cipher. 100 annex two ciphers.
1000 annex three ciphers. 10000 annex four ciphers.
100000 annex five ciphers.

$1\frac{1}{4}$ annex a cipher and divide by 8.
$1\frac{2}{3}$ annex a cipher and divide by 6.
$2\frac{1}{2}$ annex a cipher and divide by 4.
$3\frac{1}{3}$ annex a cipher and divide by 3.
$6\frac{1}{4}$ annex two ciphers and divide by 16.
$8\frac{1}{3}$ annex two ciphers and divide by 12.
$12\frac{1}{2}$ annex two ciphers and divide by 8.
$14\frac{2}{7}$ annex two ciphers and divide by 7.
$16\frac{2}{3}$ annex two ciphers and divide by 6.
25 annex two ciphers and divide by 4.
$33\frac{1}{3}$ annex two ciphers and divide by 3.
50 annex two ciphers and divide by 2.
$66\frac{2}{3}$ annex two ciphers and subtract $\frac{1}{3}$ of the product.
75 annex two ciphers and deduct $\frac{1}{4}$ of the product.
$87\frac{1}{2}$ annex two ciphers and deduct $\frac{1}{8}$ of the product.
$37\frac{1}{2}$ annex two ciphers and take $\frac{3}{8}$ of the product.
$62\frac{1}{2}$ annex two ciphers and take $\frac{5}{8}$ of the product.

EXERCISE 52

Multiply mentally and add products :

1. 480 by $1\frac{1}{4}$
   490 by $2\frac{1}{2}$
   144 by $8\frac{1}{3}$
   680 by $12\frac{1}{2}$
   660 by $16\frac{2}{3}$

2. 375 by $1\frac{2}{3}$
   870 by $3\frac{1}{3}$
   732 by 25
   512 by $6\frac{1}{4}$
   434 by $14\frac{2}{7}$

3. 432 by $6\frac{2}{3}$
   528 by 50
   588 by $33\frac{1}{3}$
   340 by $2\frac{1}{2}$
   784 by 25

4. 464 by 125
   584 by $1\frac{1}{4}$
   564 by $8\frac{1}{3}$
   462 by $1\frac{2}{3}$
   240 by $6\frac{2}{3}$

5. 494 by $12\frac{1}{2}$
   381 by $16\frac{2}{3}$
   482 by 50
   864 by $33\frac{1}{3}$
   165 by $3\frac{1}{3}$

6. 384 by $6\frac{1}{4}$
   532 by $14\frac{2}{7}$
   326 by 125
   498 by $6\frac{2}{3}$
   342 by $11\frac{1}{9}$

7.	8.	9.	10.
360 by $7\frac{1}{2}$	1095 by $33\frac{1}{3}$	840 by 250	420 by $133\frac{1}{3}$
125 by 64	250 by 44	232 by $62\frac{1}{2}$	544 by 125
584 by 75	2163 by $3\frac{1}{3}$	320 by $18\frac{3}{4}$	384 by $37\frac{1}{2}$
240 by $13\frac{1}{3}$	432 by $6\frac{1}{4}$	360 by $87\frac{1}{2}$	468 by $11\frac{1}{9}$
440 by $37\frac{1}{2}$	351 by $66\frac{2}{3}$	450 by $83\frac{1}{3}$	364 by $14\frac{2}{7}$

167. To Multiply by Numbers from 13 to 19 inclusive.

Illustration.—Multiply 485 by 13.

(*a*) Ordinary Method—

$$\begin{array}{r} 485 \\ 13 \\ \hline 1455 \\ 485 \\ \hline 6305 \end{array}$$

(*b*) Short Method—

485	3×5	$= 15$ carry 1
13	$3 \times 8 + 1$ (carried) $+ 5$	$= 30$ carry 3
——	$3 \times 4 + 3$ (carried) $+ 8$	$= 23$ carry 2
6305	$4 + 2$ (carried)	$= 6$

Multiply each figure by 3, and to each product add the carrying figure (if any) and the figure to the right of the one multiplied. After all the figures have been multiplied by 3, complete the result by adding the carrying figure to the last figure of the multiplicand.

EXERCISE 53

Multiply the following, using the short method shown above:

1. 346725 by 13	5. 435827 by 17
2. 647386 by 14	6. 210349 by 18
3. 582630 by 15	7. 536274 by 19
4. 138629 by 16	8. 742976 by 19

Multiply the following, using the short method, and total the results:

9.	10.	11.	12.
347 by 16	158 by 14	158 by 16	329 by 15
392 by 19	376 by 16	431 by 18	176 by 17
157 by 18	457 by 11	287 by 13	384 by 12
469 by 12	386 by 19	415 by 19	910 by 18

168. To Multiply by the Factors of a Number.

Illustration.—Multiply 95 by 32.

$$\begin{array}{r} 95 \\ 8 \\ \hline 760 \\ 4 \\ \hline 3040 \end{array}$$

SOLUTION.—The factors of 32 are 8 and 4. First multiply by 8 and then multiply that product by 4.

EXERCISE 54

1. 784×36	7. 2956×35	13. 21754×96
2. 891×72	8. 2179×44	14. 17845×420
3. 794×77	9. 4754×108	15. 78941×144
4. 485×99	10. 2816×256	16. 29715×196
5. 284×56	11. 4712×324	17. 49165×98
6. 187×64	12. 2175×192	18. 97142×625

BASIS OF CALCULATION

169. The term "Per cent.," usually written %, is an abbreviation of the Latin words, *Per centum*, which signify "by the hundred."

Some values in business are estimated by the pound (sugar, for example), some by the bushel (wheat, for example), others by the hundreds (use of money, for example).

170. Commission.—An agent is hired to do a certain work. He is paid 5% commission. It means that for every \$100 worth of business done the agent is paid \$5.

171. Discount.—A man pays a debt before it is due, and he is allowed 5% discount. It means that for every \$100 that he owed he will only have to pay \$95.

172. Profit and Loss or **Loss and Gain.**—Goods are sold at 5% profit. It means that as often as goods costing \$100 are sold, \$5 more than cost is received as a selling price. If 5% were lost, \$5 less than cost would be received.

173. Insurance.—Property is insured, and a premium of 5% is paid. It means that for every \$100 worth of property insured \$5 is paid to the insurance company. These premiums form the fund from which the insurance company pay for all losses of insured property.

174. Exchange.—A draft is bought at a bank, and 5% exchange is paid. It means that for issuing a \$100 draft the bank will charge the one who buys the draft \$5 for the convenience.

175. Taxes.—A man owns a city lot, and pays a tax rate of 35 mills, which is the equivalent of $3\frac{1}{2}$%. It means that for every \$100 assessed value he must pay into the city treasury \$3.50 as his contribution to the city expenses.

176. Dividends.—Stock in a certain company yields 5% dividends.

179. SHORT METHODS IN PERCENTAGE

The principles of aliquot parts may be used to advantage in many operations in percentage, and the percentages of the following table with their equivalent fractions should be thoroughly learned.

Per Cent.	Decimal Value	Fract'nal Value	Fract'nal Value in Lowest Terms	Per Cent.	Decimal Value	Fract'nal Value	Fract'nal Value in Lowest Terms
1%	.01	$\frac{1}{100}$	$\frac{1}{100}$	$22\frac{2}{9}\%$	$.22\frac{2}{9}$	$\frac{22\frac{2}{9}}{100}$	$\frac{2}{9}$
$2\frac{1}{2}\%$	$.02\frac{1}{2}$	$\frac{2\frac{1}{2}}{100}$	$\frac{1}{40}$	25%	.25	$\frac{25}{100}$	$\frac{1}{4}$
$3\frac{1}{3}\%$	$.03\frac{1}{3}$	$\frac{3\frac{1}{3}}{100}$	$\frac{1}{30}$	$28\frac{4}{7}\%$	$.28\frac{4}{7}$	$\frac{28\frac{4}{7}}{100}$	$\frac{2}{7}$
$6\frac{1}{4}\%$	$.06\frac{1}{4}$	$\frac{6\frac{1}{4}}{100}$	$\frac{1}{16}$	$33\frac{1}{3}\%$	$.33\frac{1}{3}$	$\frac{33\frac{1}{3}}{100}$	$\frac{1}{3}$
$6\frac{2}{3}\%$	$.06\frac{2}{3}$	$\frac{6\frac{2}{3}}{100}$	$\frac{1}{15}$	$37\frac{1}{2}\%$	$.37\frac{1}{2}$	$\frac{37\frac{1}{2}}{100}$	$\frac{3}{8}$
$8\frac{1}{3}\%$	$.08\frac{1}{3}$	$\frac{8\frac{1}{3}}{100}$	$\frac{1}{12}$	$42\frac{6}{7}\%$	$.42\frac{6}{7}$	$\frac{42\frac{6}{7}}{100}$	$\frac{3}{7}$
$9\frac{1}{11}\%$	$.09\frac{1}{11}$	$\frac{9\frac{1}{11}}{100}$	$\frac{1}{11}$	50%	.50	$\frac{50}{100}$	$\frac{1}{2}$
10%	.10	$\frac{10}{100}$	$\frac{1}{10}$	$62\frac{1}{2}\%$	$.62\frac{1}{2}$	$\frac{62\frac{1}{2}}{100}$	$\frac{5}{8}$
$11\frac{1}{9}\%$	$.11\frac{1}{9}$	$\frac{11\frac{1}{9}}{100}$	$\frac{1}{9}$	$66\frac{2}{3}\%$	$.66\frac{2}{3}$	$\frac{66\frac{2}{3}}{100}$	$\frac{2}{3}$
$12\frac{1}{2}\%$	$.12\frac{1}{2}$	$\frac{12\frac{1}{2}}{100}$	$\frac{1}{8}$	75%	.75	$\frac{75}{100}$	$\frac{3}{4}$
$14\frac{2}{7}\%$	$.14\frac{2}{7}$	$\frac{14\frac{2}{7}}{100}$	$\frac{1}{7}$	$83\frac{1}{3}\%$	$.83\frac{1}{3}$	$\frac{83\frac{1}{3}}{100}$	$\frac{5}{6}$
$16\frac{2}{3}\%$	$.16\frac{2}{3}$	$\frac{16\frac{2}{3}}{100}$	$\frac{1}{6}$	$87\frac{1}{2}\%$	$.87\frac{1}{2}$	$\frac{87\frac{1}{2}}{100}$	$\frac{7}{8}$
20%	.20	$\frac{20}{100}$	$\frac{1}{5}$				

It means that for every $100 worth of stock held, a man is entitled to $5 as his share of the profits.

177. Duties.—An importer has to pay a duty of 5%. It means that in bringing goods into Canada from some other country the importer must pay to the Canadian Government $5 for every $100 worth of goods imported.

178. Interest.—Money is borrowed and interest at the rate of 5% per annum is charged. It means that for every $100 borrowed, $5 must be paid for its use every year.

EXERCISE 55

Find	$	$	$
1. 20% profit on	5,	25,	45
2. 25% loss on	4,	36,	76
3. 4% commission on	25,	75,	125
4. $12\frac{1}{2}$% interest on	64,	96,	160
5. $16\frac{2}{3}$% duty on	6,	36,	72
6. $8\frac{1}{3}$% discount on	12,	72,	60
7. $37\frac{1}{2}$% premium on	80,	32,	48
8. $66\frac{2}{3}$% advance on	9,	27,	75
9. $6\frac{1}{4}$% brokerage on	32,	64,	256
10. $31\frac{1}{4}$% assessment on	48,	80,	144
11. $87\frac{1}{2}$% dividend on	16,	72,	108
12. $22\frac{2}{9}$% tax on	27,	45,	63
13. $28\frac{4}{7}$% rebate on	21,	35,	56
14. $33\frac{1}{3}$% allowance on	26,	39,	78
15. 75% of the value of	24,	32,	28
16. 90% of the value of	70,	110,	40
17. $31\frac{1}{4}$% of the value of	86,	475,	373
18. $43\frac{3}{4}$% of the value of	374,	228,	937
19. 50% of the value of	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{2}{5}$
20. 125% of the value of	7.50	375	

In addition to the methods suggested by the table, such as dividing by 4 to get 25%, dividing by 3 to get $33\frac{1}{3}$%, and so on, a number of percentages can be rapidly calculated from the 10% base. In billing, the student should cultivate the habit of writing

the percentage on the paper direct, without carrying the work to a scratch pad and back to the bill.

180. Illustrations.

1. 10% of \$747.25 = \$74.73.

Move the decimal point one place to the left, getting \$74.725. If the mills are 5 or over, add 1c.; if less than 5 mills, drop them altogether. Thus, to the nearest cent, the answer is \$74.73.

2. 20% of \$747.25 = \$149.45.

See first what 10% would be, \$74.725. Taking twice this figure to the nearest cent, we have \$149.45.

3. 30% of \$747.25 = \$224.18.

Again, 10% = \$74.725; 30% is three times the amount, or \$224.18. In this way, 40%, 50%, 60%, 70%, 80%, or 90% of any number may be immediately written.

4. $2\frac{1}{2}$% of \$747.25 = \$18.68.

As before, 10% = \$74.725; $2\frac{1}{2}$ is $\frac{1}{4}$ of 10%, $\frac{1}{4}$ of \$74.725 = \$18.68.

In other words, see 10% first and divide by 4.

NOTE.—In placing the first figure, see if the division, if carried out another place, would give 5 mills or over. If so, follow the rule of adding an extra cent.

5. $3\frac{1}{3}$% of \$747.25 = \$24.91.

See 10% first and divide by 3.

6. What is 36% of \$2,500?

$\frac{1}{4}$ of \$3,600 = \$900

SOLUTION.—Since 36 times 25 will give the same product as 25 times 36, 36% of \$2,500 will give the same result as 25% of \$3,600. 25% is $\frac{1}{4}$ of a number; therefore, $\frac{1}{4}$ of \$3,600, or \$900, is the required result.

7. What is 16% of \$12,500?

$\frac{1}{8}$ of \$16,000 = \$2,000

SOLUTION.—16 times $12\frac{1}{2}$ will give the same product as $12\frac{1}{2}$ times 16; hence, 16% of \$12,500 is equivalent to $12\frac{1}{2}$% of \$16,000. $12\frac{1}{2}$% is $\frac{1}{8}$. $\frac{1}{8}$ of \$16,000 is \$2,000, or the required result.

8. What is 24% of \$37,500?

\$24,000 × $\frac{3}{8}$ = \$9,000

SOLUTION.—24 times $37\frac{1}{2}$ will give the same product as $37\frac{1}{2}$ times 24; hence, 24% of \$37,500 is equivalent to $37\frac{1}{2}$% of \$24,000. $37\frac{1}{2}$% is $\frac{3}{8}$. $\frac{3}{8}$ of \$24,000 is \$9.000, or the required result.

EXERCISE 56

Calculate the following mentally, giving result correct to nearest cent—

1.	10 % of $ 358.78	9.	3⅓% of $ 438.73
2.	10 % of 182.34	10.	1¼% of 8,326.75
3.	10 % of 526.45	11.	25 % of 3,827.49
4.	20 % of 346.28	12.	3 % of 1,437.52
5.	30 % of 156.97	13.	26 % of 2,500.00
6.	40 % of 358.79	14.	18 % of 12,500.00
7.	5 % of 342.87	15.	¼% of 3,742.85
8.	2½% of 1,368.74	16.	33⅓% of 3,284.95

Illustration.—What is 5% of $225?

$225
.05
$11.25

SOLUTION.—5% means 5 per hundred, or 5 out of every hundred, which is $\frac{5}{100}$ or .05. The question becomes a simple one in multiplication of decimals.

NOTE.—The circumstance which gives rise to the calculation of the percentage cannot alter the method of finding it. It may be we want to know the interest on $225 at 5%, or we may want to figure the commission on a sale of $225 at 5%, or we may want to find the duty on goods worth $225 at 5%. The process of getting the result is the same for all cases.

EXERCISE 57

INVOLVING SIMPLE PERCENTAGE

1. **Illustration.**—What is 36% of $480?

SOLUTION

100% of it equals $480
1% of it equals $4.80
36% of it equals $4.80 × 36 = $172.80
or
$480 × .36 = $172.80

2. What is 24% of 375 yards?

3. What is 76% of 793?

4. A man, who is worth $75,000, has 35% of his wealth invested in real estate. What is the value of his real estate?

5. In a certain school there are enrolled 460 pupils, of whom 45% are boys. How many girls are enrolled?

6. A rancher, who owned 640 sheep, lost 15% of them in a storm. How many sheep had he remaining?

7. A merchant bought goods for $458.75, and sold them for 36% more than he paid for them. What did he receive for the goods?

8. A farmer sold 420 bushels of wheat at $0.97 per bushel, and 20% more oats than wheat at $0.43 per bushel. What did he receive for both.

9. A man invests $17,280 as follows: 25% in real estate, 37½% in bank stock, and the remainder in city lots. How much did he invest in each?

10. What will 37½% of 480 bushels of wheat cost at $1.25 per bushel?

EXERCISE 58

1. A man, buying a house and lot, paid $1,500 for the lot and 37½% more than that for the house. What did both cost?

2. A farm contained 320 acres; 25% of it was sold at $45 per acre, 40% at $37.50 per acre, and the balance at $30 per acre. How much was received for the farm?

3. A man has a yearly salary of $2,400, and spends 33⅓% of it the first year, 45% the second, and 64% the third. How much does he save in the three years?

4. A man deposited in a bank $1,875; he withdrew 40% of the deposit, and with 9⅓% of the amount withdrawn purchased a gun. What was the cost of the gun?

5. A man, having a salary of $3,600, spends 20% of it for board, 12½% for clothing, 10% for books and lecture fees, 5% for incidentals, and deposits the remainder in the bank. How much does he deposit?

6. I owed a man $1,450, and paid him 40% of it at one time, 20% of the remainder at another time, and 10% of what then remained at another time. How much did I then owe him?

7. A rancher owned 2,070 cattle, and sold 33⅓% at one time, 40% of the remainder at another time, and 8⅓% of what then remained at another time. What is the value of the remainder at $36 each?

8. A farmer had an orchard containing 225 trees, and he paid for picking and packing the apples $3\frac{1}{3}\%$ of their value. If they averaged $8\frac{1}{2}$ barrels to the tree, and he sold them at \$1.20 per barrel, how much did he pay for picking and packing them?

9. A man, dying, left an estate of \$150,000, with instructions that after his widow received her share of $33\frac{1}{3}\%$, 10% was to be given to his brother, and the remainder was to be divided equally among his four children. How much did each child receive?

1. **Illustration.**—What number, increased by 87% of itself, is equal to 1,122?

SOLUTION

Represent the number by 100%.
87% = the increase.
187% = the number, increased by 87% of itself.
1,122 = the number, increased by 87% of itself.
Therefore, 187% of the number = 1,122.
1% = $\frac{1}{187}$ of 1,122, or 6.
100%, or the number = 100 times 6, or 600.
Hence the required result is 600.

2. **Illustration.**—What number, decreased by 35% of itself, equals 2,600?

SOLUTION

Represent the number by 100%.
35% = the decrease.
65% = the number after decrease.
2,600 = the number after decrease.
Therefore, 65% of the number = 2,600.
1% = $\frac{1}{65}$ of 2,600, or 40.
100% = 100 times 40, or 4,000, the required result.

EXERCISE 59

1. 4,186 is 15% more than what number?

2. When gold is worth 12% more than currency, what is the gold value of \$725.76 in currency?

3. A foreman, whose salary was increased 11%, receives \$1,082.25. How much did he receive before the increase?

4. An agent sold a house and lot for \$11,002.50, which was $12\frac{1}{2}\%$ more than it cost him. What did it cost him?

5. During the month of December a merchant sold goods to the value of $12,620.02, which was 9% more than his sales in November. What did his sales amount to in November?

6. Sold a horse for $170, which is 6¼% more than it cost me. How much did it cost me?

7. The number of students attending a certain school in 1895 was 357, which was 40% more than in 1894. What was the attendance in 1894?

8. What number increased by 16⅔% of itself equals 6,825?

EXERCISE 60

1. If 11⅑% of a number be added to itself, the sum will equal 2,000. What is the number?

2. A shepherd, after losing 35% of his flock, had 325 sheep remaining. How many had he at first?

3. A dealer in dry goods spent for calico $1,726.80, which was 20% less than the amount spent for muslin. How much did he spend for muslin?

4. A certain man owns two farms. The first farm contains 290 acres, which is 42% smaller than the second. How many acres does the second farm contain?

5. In the construction of a business block, 40% of the entire cost was paid for the brickwork and stonework, 20% for the carpenter work, 15% for glazing, 10% for the elevator, and the remainder, $510, for the painting. What was the whole cost?

6. What was the cost of a horse that sold for $136 at a loss of 11⅑%?

7. A collector, whose fee was 4%, returned to his principal $540.96. What sum did he collect?

8. The pressure on the surface of a steam boiler is 81 lbs. since decreasing it 10%. What was the pressure before decreasing it?

CASH DISCOUNTS

AS APPLIED TO

BILLS AND INVOICES

181. A Cash Discount, as the name implies, means a reduction from the amount of the bill for payment within a definite time.

Many business firms indicate these on their billheads, the following showing some common forms—

2%–10 dys. : net 30 dys., meaning that a reduction of 2% from the total of the invoice will be made, if paid within 10 dys., while the full term of credit will be 30 dys., without reduction of the amount.

4%–30 dys. : 2%–60 dys. : net 4 mos. : meaning that 4% will be deducted if paid within 30 dys., 2% if paid within 60 dys., while the full term of credit will be 4 mos. without reduction of the amount.

An invoice dated July 10 might have terms as follows: 2%–10 dys.: net 30 dys., as at Sept. 1. This would mean that the goods were shipped and invoiced on July 10 but the customer has 10 dys. after Sept. 1 to take advantage of the discount or 30 dys. credit after Sept. 1.

182. **Cash Discounts** should not be confused with **Trade Discounts.**

183. A **Bill** or **Invoice** is a detailed statement of merchandise sold or services rendered. A bill or invoice of goods sold usually gives the following information : Place and date of the sale ; the names of the buyer and the seller ; the terms of sale ; the identifying marks, if any, of case, package, barrel, etc., in which goods are shipped, the quantity, name, and price of each article ; the extension of each item ; and the total amount of the whole bill.

A bill or invoice is receipted by writing at the bottom the words, " Received payment," followed by the signature of the seller of the goods or someone authorized to sign for him.

184. A **Credit Note** is a bill or invoice used as an offset to a previous bill or invoice. Thus, where goods are once billed and part of the goods are returned, the credit note is intended to show the items which were returned and for which credit is being given.

185. A **Statement of Account** is an exhibit of the dates and totals of the bills or invoices for a period, say of a month. A statement may be receipted as a bill or invoice is receipted.

186. The **Work of Billing** requires good writing and quick and accurate figuring. There is no commoner introduction to the general work of an office than this work of billing. Any young man or woman contemplating engaging in office work should therefore

187. Commercial Abbreviations.

A1	First quality.
Acct.	Account.
Agt.	Agent.
Amt.	Amount.
Apr.	April.
Ass'd	Assorted.
Aug.	August.
B/L	Bill of Lading.
Bal.	Balance.
Bbl.	Barrel.
Bdls.	Bundles.
Bds.	Boards.
Bkts.	Baskets.
Blk.	Black.
Bls.	Bales.
Bot.	Bought.
Bu.	Bushels.
Bxs	Boxes.
C.I.F.	Cost, Insurance and Freight.
C.	Hundred.
c. or cts.	Cents.
Chgd.	Charged.
Co.	Company.
C.O.D.	Collect on delivery.
Com.	Commission.
Cr.	Creditor.
Cs.	Cases.
Cwt.	Hundredweight.
d.	Pence.
Dec.	December.
Dft.	Draft.
Disct.	Discount.
Do., ditto, or "	The same.
Doz.	Dozen.
Dr.	Debtor.
Ds. or da.	Days.
Ea.	Each.
E. & O.E.	Errors and omissions excepted.
Exch.	Exchange.
Ex.	Without.
Feb.	February.
F.O.B.	Free on Board.
Fol.	Folio or page.
Fr't.	Freight.
Ft.	Foot or feet.
Gal.	Gallon.
Gr. or gro.	Gross.
Hlf.	Half.
Hhd.	Hogshead.
i.e.	That is.
In.	Inches.
Ins.	Insurance.
Inst.	Present month.
Int.	Interest.
Jan.	January.
Jr. or Jun.	Junior.
Lbs.	Pounds.
Ledg.	Ledger.
M.	Thousand.
Mar.	March.
Mdse.	Merchandise.
Mem.	Memorandum.
Messrs.	Gentlemen or Sirs.
Mo.	Month.
Mr.	Mister.
Mrs.	Mistress.
N.S.F.	Not sufficient funds.
Net.	Without discount.
N.B.	Take particular notice.
No. or #	Number.
Nov.	November.
Oct.	October.
Oz.	Ounces.
P.	Page.
Pp.	Pages.
Pay't or Paym't	Payment.
Pd.	Paid.
Per, p., or ℘	By, or by the.
Pkgs.	Packages.
P.O.	Post Office.
Pr.	Pair.
Prox.	Next Month.
Ps.	Pieces.
Pts.	Pints.
Qr.	Quarter.
Qts.	Quarts.
Rec'd paym't	Received payment.
Rec't	Receipt.
R.R.	Railroad.
s.	Shilling.
Shipt.	Shipment.
S/S	Steamship.
Sunds.	Sundries.
Super.	Superfine.
Trcs.	Tierces.
Ult.	Last month.
Wt.	Weight.
Yds.	Yards.
Yr.	Year.

Commercial Characters

@	At.
a/c	Account.
¢.	Cents.
c/o	Care of.
√	Check mark.
$	Dollars.
"	Ditto, or the same.
%	Hundredths.
#	Number.
o/a	Old account.
℘	Per, or by the.
£	Pounds sterling.
×	By, as 7 × 8 inches.

make certain that the requisites for success are thoroughly mastered. The rapid figuring will require not only a thorough knowledge of the simple rules, vulgar and decimal fractions, but also the application of many short rules, which will be found to both quicken and simplify the work. A bill clerk should also see to it that in every case possible extensions are made direct, without the necessity of working the extension on a scratch pad, and then transferring the result to the bill or invoice. We commend the preceding pages on short rules and business methods to the most earnest attention of all students.

188. Commercial Expressions.

7/4 English shillings and pence are often written this way. This character stands for 7 shillings, 4 pence.

6^{1} yds., 7^{2} yds., 8^{3} yds., respectively mean $6\frac{1}{4}$, $7\frac{1}{2}$, and $8\frac{3}{4}$ yds.

15^{10} bus. means 15 bus. and 10 lbs.

20^{6} lbs. means 20 lbs. 6 ozs.

THE CANADIAN IMPORTING CO.

MANUFACTURING STATIONERS
PENCILS AND PENHOLDERS

Date Jan. 1, 19—
Sold to Mr. George Adams Our No. 427
Address Your Address Via. Delivery
Terms 30d Net

12	gro. Erasers Pat #414	4 00	48	—		
12	" " Jp #420	4 00	48	—		
50	M Paper Fasteners $2	1 60	80	—		
24	gro. Pencils #21 HB	8 00	192	—		
24	" " #21 B	8 00	192	—	560	—

Specimen Invoice

Statement

GEORGE ADAMS

CONDUCTING BUSINESS AT

Student's Address ——— CANADA

Date May 31, 19-

Name Mr. C. R. Andrews

Address 559 Bank St.
Ottawa, Ont.

		Acct. Rend.	235	50		
May	1	Mdse.	151	20		
	26	"	244	80	631	50
		Cr.				
May	1		50	—		
	15		35	—	85	—
					546	50

Statement of Account

SIR ISAAC PITMAN & SONS (CANADA), LTD.

Publishers
381-383 CHURCH STREET
TORONTO

OUR NO. 5742

DATE June 7, 19
TO Mr. George Adams,
ADDRESS Your Address, Canada.

CREDIT INVOICE

Overcharge on Invoice No. 11087 May 26 500 gross		10¢	$50 00

O.K. *am* CREDIT.....

VOUCHER No._____

Specimen Credit Invoice or Credit Note

This form is usually printed with *red* ink or printed on paper of a different color to that used in the invoice of the particular firm for whom it is intended.

15 doz., $\frac{5}{\$8}, \frac{5}{\$9}, \frac{5}{\$10}$ means 15 doz., 5 doz. at $8 a dozen, 5 doz. at $9 dozen, and 5 doz. at $10 a dozen.

2 bbl. $\frac{240 - 18}{250 - 20}$ or 2 bbl. $\frac{240}{18}, \frac{250}{20}$ means 2 bbl., one of 240 lbs. gross weight, from which 18 lbs. is taken for weight of barrel, and one of 250 lbs., from which is taken 20 lbs. for barrel.

BILLING

EXERCISE 61

NOTE.—If the student is not familiar with lumber measurements, full explanations are given on page 279 of this text.

THE CUSHING LUMBER CO., of Calgary, Alta., sold the following goods to Hudson's Bay Co., Lethbridge, Alta.—

1. 20 pcs. 2 × 6—18 Hem., at $19.00 a M.; 18 pcs. 2 × 6—16 Hem., at $17.50; 36 pcs. 2 × 4—10 Hem., at $18.00; 26 pcs. 2 × 4—16 Hem., at $18.00; 10 pcs. 2 × 4—12 Hem., at $17.50; 800 ft. 1-in. Hem., at $17.50; 10 8-in. Cedar Posts, at 30c. each.

2. To Imperial Oil Co., Regina, Sask.

3 pcs. 2 × 6—16 Hem., at $17.50 a M.; 12 pcs. 2 × 4—16 Hem., at $18.00; 1 pc. 2 × 10—12 Hem., at $17.50; 16 ft. 3 × 4 Hand-rail Pine, at 8c. a foot; 16 ft. Bottom-rail Pine, at 5c. a foot; 24 ft. 3 × 4 Hand-rail Birch, at 12c. a foot; 70 ft. Parting Stop, at $\frac{3}{4}$c. a running foot; 1000 B.C. Shingles, at $3.40 a M.; 100 ft. $\frac{7}{8} \times 1\frac{3}{8}$ Balusters, at 2c. a running foot; 24 ft. Bottom-rail Birch, at 7c. a running foot; 100 ft. $\frac{7}{8}$ Cove, at $1\frac{1}{2}$c. a running foot.

Canada Paint Co., Ltd., 572 William Street, Toronto, Ont., sell the following goods—

3. To Messrs. Evans & Co., Oshawa, Ont.

100 lbs. Imp. Green, Lt. Dry, at 9c.; 1 28-lb. box " D " Ultramarine Blue, Dry, at 11c.; 100 lbs. Yell. Ochre, Dry, at 2c.; 1 B.B.C. B.B. Ven. Red. 400 lbs., at 3c.

4. To Messrs. Johnstone & Co., Yonge Street, Toronto, Ont.

2 bbls. Boiled Oil, 468 lbs.–69, 504 lbs.– 81, at 60c. per gal.

Oil is figured at 9 lbs. to the gallon.

5. J. G. McIlwraith & Co., Hamilton, bought of Gordon, McKay & Co., Ltd., Toronto. Terms, 3 mos. 30 dys., 5%.

10 pcs. R. Gingham, 61^2, 50^1, 51, 61^3, 50, 55, 51^3, 50, 55, 60, at 8c.; 5 doz. C. Silk, at 80c.; 15 gr. G. Buttons, at $\$1.12\frac{1}{2}$; 10 pcs. L. Cotton, 56, 61, 60^1, 65^3, 55, 52, 62, 55^2, 65, 51^3, at $8\frac{1}{4}$c.; 500 lbs. S. Warp, at 18c.; 10 pcs. B. Checks, 41, 45, 40^2, 45^2, 50, 55, 42, 55, 55^1, 45^2, at 25c.; 6 pcs. E. Lining, 50^1, 52, 40, 55^1 52, 41, at 5c.; 5 doz. L. L. Gloves, at $2.95; 5 pcs. A. Cashmere, 62^2, 60, 51^3, 55, 60, at $18\frac{3}{4}$c.; 6 pcs. A. L. Cotton, 40, 46^2, 55, 51^2, 42^1, 40, at 4c.; 2 pcs. G. Flannel 60, 65, at 30c.

6. Joseph Mickelborough, St. Thomas, bought of W. R. Brock & Co., Toronto. Terms, draft 30 dys.

10 pcs. B. Checks, 52^1, 41^2, 40, 55^3, 50^2, 45, 40, 50^3, 51^1, 50^2, at 25c.; 15 pcs. A. Flannel, 65^1, 62^2, 58, 60^1, 56^3, 63, 60^2, 62, 65^3, 63^1, 62^3, 65^1, 61, 58^2, 55, at $33\frac{1}{3}$c.; 10 pcs. B. Velvet, 29^3, 26^2, 25, 27^3, 21^2, 23^2, 20^3, 21, 24^3, 22^2, at $6.50; 20 pcs. B. Sheeting, 41^2, 35, 38^2, 35, 37^3, 39^2, 40^2, 51, 44^3, 44^1, 40, 37^2, 32^1, 32^3, 46^3, 49^2, 38^1, 41^3, 38^2, 36^1, at $6\frac{1}{4}$c.

EXERCISE 62

THE TORONTO WHOLESALE CO. sells to—

1. Geo. Wenino, Sarnia, Ont. Terms, 3% discount allowed for cash.

			Quan.	Price.	Amt.
1 tin	40	Sodas, ½ lbs.	2½	48	
2 "	100	Fig Drops	17	11½	
1 "	50	Map. Creams	5¾	13½	
1 "	50	F. G. Bread	7¼	10	
1 "	100	Rich Traveller	15¾	14	
2 "	50	Gin. Nut	7½	11	
1 "	50	Sultana	14	13½	
1 "	50	Pilot	8	7	
2 "	80	Sodas	33½	7	

N.B.—The figures in the second column from the left represent cents to be added to amount for cost of the tin.

JOHN SLOAN & Co., Wholesale Grocers, Winnipeg, Man., sell the following goods—

2. To W. Turnbull, 540 Main Street, Winnipeg, Man.

	Quan.	Price.	10 dys.	Net Cash
1 cs. Robt. Scotch Marm.	4	1.55	.....	
1 " Shell Walnuts	55	.26	.....	
10 lbs. " Almonds	10	.33	.....	
1 hlf. cs. Curr. 78–9	69	.08	.....	
1 bbl. Common Salt	1	1.40	.....	
1 cs. E. Army Blkg.	3	.75	.....	
10 lbs. Rape Seed	10	.08	.....	
1 cs. Def. Syrup, 2's	1	2.40		.....
3 " Magic Bak. Powder, less 5%	12	.75	.....	
2 " " " " "	8	1.65	.....	
3 bbls. No. 1 St. L. Gran. Sugar, 346–21, 322–19, 342–19, 1010–59	951	.04½	.....	
1 cs. Silent Matches, 500's	1	5.20		.....
1 " cr. of Wheat	1	5.75	.....	
1 bag Pot Barley	100	.02½	.....	
" Prl. "	100	.03½	.....	
1 bx. G. & S. 70/80 Prunes	25	.08	.....	
5 cs. Def. Map. Syrup Mxt., pts.	5	2.50	.....	
5 " " " " 5's	5	3.90	.....	
25 " H. S. Salmon, ½'s	200	.23½	.....	
1 " Gusto	1	2.85	.....	
1 " Q. A. Root	3	.90	.....	
1 " King Ed. Matches	1	3.60		.....
1 " Upton's H. M. Blk. C. Jam	2	2.00	.....	

3. To Johnston & Co., Brandon, Man.

	Quan.	Price.	10 dys.	Net Cash
6 bags Yellow Sugar	600	4.25	.....	
6 doz. Def. Marmalade	.....	1.15	.....	
1 " K. Cust. Powder	.....	.90	.....	
1 box Baker's Cocoa, ¼'s	12	.43	.....	
1 " " " ½'s	6	.43	.....	
1 cs. Rob. Marmalade	4	1.55	.....	
1 doz. " Barley	.....	2.40	.....	
1 box Durham C. Starch	40	.06¾		.....
1 " Bens. Corn	40	.07½		.....
5 cs. Quaker Oats, Family	.....	4.50	.....	
1 " Comfort Soap	.....	3.85	.....	
1 " Upt. Marmalade, 1's	2	1.00	.....	
2 " Sift Peas, Quaker	4	1.27½	.....	
1 bbl. Common Salt	.....	1.40	.....	
1 cs. Bee Syrup, 5's	.....	2.75		.....
1 " " " 2's	.....	2.40		.....
1 " Def. Syrup, "	.....	2.40		.....
3 " Pearline, 1's	.....	3.80	.....	
2 " " 100's	.....	3.70	.....	
1 doz. Cell. Starch	.....	1.00		.....
2 cs. Shred. Wheat	.....	5.15	.....	
1 " Gusto	.....	2.85	.....	
1 bbl. Wind Salt, 100's	.....	2.85	.....	
1 cs. King Edward Matches	.....	3.60		.....
1 box Reck. Blue	12	.16	.....	
1 cs. Candles, 12's	36	.08½	.....	
1 box Keen's Blue	12	.16	.....	
1 cs. Olives, stuffed	4	1.40	.....	
1 box Clay Pipes	.....	.80	.....	
1 cs. Wetley's Mincemeat	3	1.00	.....	

4. Robertson Bros., Ltd., Wholesale Confectioners, Calgary, Alta., sell to J. Gibson, Winnipeg, Man. Terms, 3% discount for cash.

	Weight.	Price.		
1 T. T. Mix	5	.12		
1 Coco. Taffy	18½	.07½		
2 Valentines	.....	.50		
1 Peanut Taffy	5¼	.07½		
1 Ruby Mandarines	5	.22		
1 " Pept.	5	.22		
1 Sc. Mints	5	.15		
1 Ell. Almonds	5	.20		
1 " Crisp	5	.20		
1 1c. Jujubes	.....	.70		
1 Quakers	.....	.55		
1 Grab Bags	.....	.35		
1 Drum No. 1 Choc. Culls	18	.14		
1 " Cry. Mix.	20	.07½		
1 Swiss Milk Patties	.....	.50		
6 Cameo Bon-bons	.....	.35		

5. LUMSDEN BROS., Hamilton, Ont., sent the following goods to George Harris, 346 St. Paul Street, St. Catharines, Ont. Terms, 30 dys. net, 2% 10 dys.

3 gal. Pineapple Syrup, at $2.50; 3 gal. Lemon Syrup, at $2.50; 2 gal. Orange Syrup, at $2.50; 5 gal. Ginger Syrup, at $2; 3 gal. Strawberry Syrup, at $2.50; 1/12 doz. Crushed Strawberry, at $15; 1-12 doz. Crushed Cherry, at $16; 3-12 doz. Crushed Pineapple, at $14; 3-12 doz. Crushed Peach, at $14; 5 lbs. Ext. Vanilla, at $1.50; 25 lbs. Powdered Chocolate, at 38c. Less 15%.

TRADE DISCOUNTS
AS APPLIED TO
BILLS AND INVOICES

189. Manufacturers, jobbers, and wholesale dealers usually have a fixed price, or catalogue for their goods. In order to overcome the necessity of changing these prices from time to time, because of market fluctuations, they make use of trade discounts. As the prices rise or fall, they raise or lower the rates of discount. This saves the expense of getting out new lists or new catalogues.

190. Frequently two or more discounts are used as 10%, 10% and 5% off the list price. This means 10% off the list price, another 10% off the remainder and then 5% off the second remainder.

$100.00 list price less 10%, 10% and 5% would equal
$100 less 10% leaving $90.00
$ 90 „ 10% „ $81.00
$ 81 „ 5% „ $76.95

191. These discounts may be interchanged without affecting the final result, as—

$100.00 list price less 20%, 10% and 5% would equal
$100.00 list price less 10%, 5%, and 20%.

192. Again, two or more discounts may be combined as 10%, and 10% is equivalent to 19% off; 20% and 10% = 28%.

193. Illustration.—Find mentally a single discount equivalent to 20% and 10%.

SOLUTION.—From the sum of the discounts subtract $\frac{1}{100}$ of their product and the result will be the required discount.

Thus $20 + 10 - \frac{1}{100}$ of $20 \times 10 = 30 - 2 = 28$

$\therefore$ the single discount is 28%.

When a third discount is given, combine it with the result obtained from the other two.

EXERCISE 63

By inspection find a single rate of discount equivalent to the following discount series—

1. 30% and 10%
2. $33\frac{1}{3}$% and 6%
3. 10% and $12\frac{1}{2}$%
4. 40%, 20%, and 10%
5. 25%, 8%, and 5%

6. RICE LEWIS & Co., Toronto, sold Ingram & Davey, St. Thomas, Apr. 2, 1941, on account 30 dys., 2% 10 dys. : 15 cultivators listed at \$7.50 each, less 20% and 10%; 25 doz. table knives listed at \$9.75, less 10%; 5 doz. pocket knives, 3 doz. at \$6.50 and 2 doz. at \$7.50, less $33\frac{1}{3}$% on each; $\frac{3}{4}$ doz. cheese knives at \$9.75, less $16\frac{2}{3}$%. Find the net amount due on the bill 9 days after date.

7. JOHN J. DOANE, St. Thomas, bought of Heintzman & Co., Toronto, Apr. 1, 1941, on account 60 dys., 5% 10 dys. : 5 pianos at \$450 each, 6 pianos at \$575 each, 4 pianos at \$250 each, less a discount of 40% from each list price; 10 organs at \$125 each, less 25% and 10% from the list price. Find the amount of the bill to render, also the amount to be remitted if the bill is paid Apr. 8, 1941.

194. Illustration.—Goods are listed at \$720, with discounts of 25%, 20%, and 10% off. Find the net selling price.

SOLUTION

		Or
$720		
180	= 25% of $720	100% − 25% = 75%
$540		100% − 20% = 80%
108	= 20% of $540	100% − 10% = 90%
$432		.75 × .80 × .90 = .54
43.20	= 10% of $432	$720 × .54 = $388.80
$388.80	= Net price	

NOTE.—The order in which the discounts of any series are considered is not material, a series of 25%, 20%, and 10% being the same as one of 20%, 10%, and 25%, or 10%, 25%, and 20%, etc.

EXERCISE 64

Find the net amount of the following bills—

1. $625, less 20% and 10%.
2. $432, less 33⅓% and 25%.
3. $327.85, less 50% and 12½%.
4. $45.50, less 60%, 20%, and 2½%.
5. $316.80, less 40%, 25%, and 10%.
6. $421, less 37½%, 5%, and 2½%.
7. $360, less 50%, 20%, and 5%.
8. $324, less 33⅓%, 20%, and 10%.
9. $243.50, less 5%, 2½%, and 2½%.
10. $325.50, less 25%, 12½%, and 10%.
11. $348.20, less 20%, 25%, and 10%.
12. $127, less 66⅔%, 10%, and 5%.
13. $850, less 30%, 20%, and 10%.
14. $426.25, less 33⅓%, 10%, and 8⅓%.
15. One merchant offers to sell neckties for $12 a dozen, with discounts of 20%, 12½%, and 10%; another offers the same grade for $12 a dozen, with discounts of 25% and 16⅔%. Which is the better offer, and how much would be saved on 35 doz. neckties?
16. Bought 75 carpet sweepers at $2.40 each, less 25%, 20%, and 10%, and sold them at the same price, less 20%, 15%, and 10%. What was my profit?

17. The catalogue price of hats is $3.60, subject to discounts of 25%, 10%, and 10%. How many hats can be bought for $349.92?

195. Illustration.—What single discount is equivalent to 25%, 33⅓%, and 10% off?

SOLUTION

$1.00
.25 = 25% of $1.00

$ .75 = Net after first discount.
.25 = 33⅓% of .75

$ .50 = Net after second discount.
.05 = 10% of .50

$ .45 = Net after third discount.
$1.00 − .45 = .55, or 55% = single discount.

Or

$1.00 × .25 × .33⅓ × .10 = $ .75 × .66⅔ × .90 = .45

$1.00 − .45 = .55

EXERCISE 65

What single discounts are equivalent to the following discount series?

1. 20% and 10%
2. 25% and 20%
3. 33⅓% and 10%
4. 30%, 20%, and 10%
5. 50%, 20%, and 5%
6. 20%, 25%, and 10%

196. Illustration.—What must be asked for goods that a discount of 20% and 10% may be allowed, and net $2.88 to the seller?

SOLUTION

Let 100% = asking price.
20% and 10% = allowance.
72% = asking price after allowance.
72% of asking price = $2.88

$$1\% \text{ of asking price} = \frac{2.88}{72}$$

$$100\% \text{ of asking price} = \frac{100 \times 2.88}{72}$$

$$= \$4.00$$

EXERCISE 66

At what price must goods be marked

1. To net $2.72, after allowing a discount of 15%?
2. To net $12, after allowing a discount of 10% and 10%?

3. To net \$3.60, after allowing a discount of 25%, 20%, and 10%?

4. To net \$30, after allowing a discount of 10%, 10%, and 10%?

5. To net \$14.58, after allowing a discount of 40%, 10%, and 10%?

COMMERCIAL ARITHMETIC

TEST PAPER—No. 5

PERCENTAGE

Your work for this paper is to be submitted to the instructor for correction and criticism.

1. Multiply the following decimals by the contracted method—

(*a*) 1·6288946 × 2·0789282 correct to 7 decimal places.

(*b*) 5·6721 × ·0064 to 3 places of decimals.

2. Divide 1,000 by 1·06458 correct to 3 places of decimals.
Divide $500.00 by 2·0156025 correct to the nearest cent.

3. Two firms, A and B, give you quotations on fixtures. A's quotation is $1,500 less 20%, 10%, and 5%. B's quotation is $1,600 less 25% and 15%. Which is the better quotation, and by how much ?

4. A merchant buys goods at a discount of 25% off the list price, and a further discount of 3% for cash, and sells them at a discount of 10% off the list price. Find the rate per cent of his profit.

5. (*a*) By selling apples at $2.50 a barrel, I gained $200. Had I sold them at $2.75 a barrel, my rate of gain would have been 37½%. How many barrels did I sell ?

(*b*) A man sold a horse at 33⅓% profit. He put with the sum received $50 and bought a piano, which he sold at 20% gain. If his total gain was $100, what was the cost of the horse ?

6. A builder pays three times as much for material as for labour. Had he paid 10% more for material, and 6% less for labour, his contract would have cost him $3,637.92. What was his contract price ?

7. On April 1, 1926, Y bought goods on the following terms from a wholesale firm: 5/10, 3/30, n/60 days. On April 27, 1926, he purchased a demand bank draft, and remitted in full settlement. If the face value of the draft was $1,245.48, what was the amount of the invoice?

8. Joseph Mickelborough, St. Thomas, bought of W. R. Brock & Co., Toronto, terms, draft 30 ds.

10 pcs. B. Checks, 52^1, 41^2, 40, 55^3, 50^2 45, 40, 50^3, 51^1, 50^2, at 25c.; 15 pcs. A. Flannel, 65^1, 62^2, 58, 60^1, 56^3, 63, 60^2, 62, 65^3, 63^1, 62^3, 65^1, 61, 58^2, 55, at $33\frac{1}{3}$c.; 10 pcs. B. Velvet, 29^3, 26^2, 25, 27^3, 21^2, 23^2, 20^3, 21, 24^3, 22^2, at $6.50; 20 pcs. B. Sheeting, 41^2, 35, 38^2, 35, 37^3, 39^2, 40^2, 51, 44^3, 44^1, 40, 37^2, 32^1, 32^3, 46^3, 49^2, 38^1, 41^3, 38^2, 36^1, at $6\frac{1}{4}$c.

Make out the bill for the above.

COMMERCIAL PAPER

197. Exchange is a system by which debts are paid in distant places without the transmission of money. This is done by means of written orders called Bills of Exchange, Express Orders, Telegraphic Money Orders, and Post Office Money Orders.

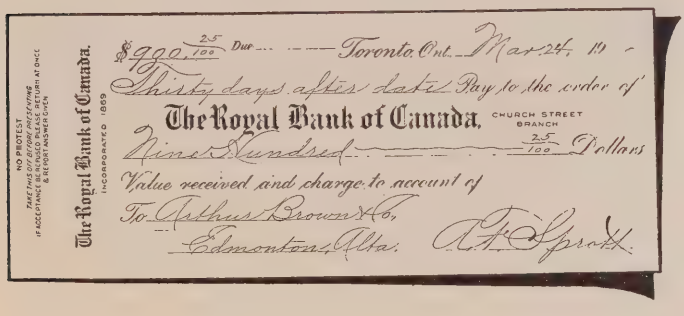
In Canada the law concerning Bills of Exchange is the same for all Provinces, and is embodied in a Dominion statute, "The Bills of Exchange Act."

Bills of Exchange

198. A **Bill of Exchange** is an unconditional order in writing addressed by one person (the drawer) to another (the drawee), signed by the person giving it, requiring the person to whom it is addressed to pay, on demand or at a fixed or determinable future time, a sum certain in money to or to the order of a specified person (the payee) or to bearer.

199. An **Inland Bill** is a bill which is, or on the face of it purports to be, (*a*) both drawn and payable within Canada, or (*b*) drawn

Inland Bill or Draft


NO PROTEST
TAKE THIS BILL TO BE PRESENTED
IF ACCEPTANCE IS REFUSED PLEASE RETURN AT ONCE
& REPORT REASON GIVEN
The Royal Bank of Canada.
INCORPORATED 1869
$990 25/100 Due ... Toronto Ont. Mar 24, 19 -
Thirty days after date Pay to the order of
The Royal Bank of Canada. CHURCH STREET BRANCH
Nine Hundred 25/100 Dollars
Value received and charge to account of
To Arthur Brown & Co,
Edmonton, Alta.
A. E. Sprott.

within Canada upon some person resident therein. Any other bill is a foreign bill.

200. The **Acceptance** of a bill is the signification by the drawee of his assent to the order of the drawer. It is written on the bill (usually across the face), and for the bill shown would read—

"Accepted.

ARTHUR BROWN & CO."

201. A bill is payable on demand—

(*a*) Which is expressed to be payable on demand, or on presentation.

(*b*) In which no time for payment is expressed.

202. A bill is payable at a determinable future time, within the meaning of this Act, which is expressed to be payable—

At sight or at a fixed period after date or sight.

203. Days of Grace. Where a bill is *not payable on demand,* the day on which it falls due is determined as follows—

Three days, called days of grace, are, in every case where the bill itself does not otherwise provide, added to the time of payment as fixed by the bill, and the bill is due and payable on the last day of grace.

Provided that—

Whenever the last day of grace falls on a legal holiday or non-juridical day in the Province where any such bill is payable, then the day next following, not being a legal holiday or non-juridical day in such Province, shall be the last day of grace.

204. Counting Days. Where a bill is payable at sight, or at a fixed period after date or after sight, the time of payment is determined by *excluding* the day from which the time is to begin to run and by *including* the day of payment.

205. Dating Acceptance. Where a bill is payable at sight, or at a fixed period after sight, the time begins to run from the date of acceptance. The acceptance of all such bills must, therefore, show date.

206. The term "Month" in a bill means the calendar month.

207. Due Date. Every bill which is made payable at a month or months after date becomes due on the same numbered day of the month in which it is made payable as the day on which it is dated—unless there is no such day in the month in which it is made payable, in which case it becomes due on the last day of that *month*—with the addition, in all cases, of the days of grace.

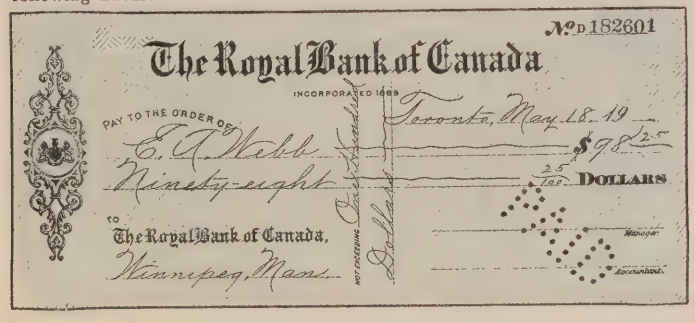
Cheques

208. A **Cheque** is a bill of exchange drawn on a bank by one with funds deposited there, payable on demand.

Bank Drafts

209. Business men do a great deal of business with banks in the way of buying and selling exchange. In buying from the bank they generally purchase what are known as bank drafts, which enable them to pay debts in distant places without transmitting the actual money.

For instance, should E. A. Webb, a merchant of Toronto, desire to remit $98.25 to Winnipeg, he could purchase from his bank a paper of the following nature—



He could endorse this paper to anyone to whom he owed the money in Winnipeg, and remit it by mail, or he might have had it made out in the name of the person he owed and then have sent it directly to him.

Of course, it is to be expected that the bank, in thus accommodating its customers, will charge for the accommodation. This charge is known in business as exchange, and is reckoned as a percentage of the face of the draft. Thus, should the bank charge E. A. Webb one-quarter of one per cent. exchange, it means that it would cost him twenty-five cents in addition to the $98.25 to make his remittance to Toronto.

In practice, these bank drafts are always drawn payable on demand, and exchange is always an additional item in the cost. In other words, exchange is at a premium.

210. Collection Charges. In selling to a bank, business men generally dispose of notes or acceptances of other people or notes of their own. This is called discounting negotiable papers, and, of course, is done for the purpose of raising money on the papers thus sold. In this connection, the following sections from our "Bank Act" regarding collection and agency fees should be of interest—

"PERCENTAGE CHARGEABLE FOR COLLECTION.—When any note, bill, or other negotiable security or paper, payable at any of the bank's places or seats of business, branches, agencies or offices of discount and deposit in Canada, is discounted at any other of the bank's places or seats of business, branches, agencies or offices of discount and deposit, the bank may, in order to defray the expenses attending the collection thereof, receive or retain in addition to the discount thereon, a percentage calculated upon the amount of such note, bill, or other negotiable security or paper, not exceeding one-eighth of one per cent.; provided that the bank may make a minimum charge of fifteen cents.—*Revised Statutes, Chapter* 12, *Section* 93.

"AGENCY CHARGES.—The bank may, in discounting any note, bill or other negotiable security or paper, *bona fide* payable at any place in Canada, other than that at which it is discounted, and other than one of its own places or seats of business, branches, agencies or offices of discount and deposit in Canada, receive and retain, in addition to the discount thereon, a sum not exceeding one-fourth of one per cent. on the amount thereof; provided that the bank may make a minimum charge of twenty-five cents." —*Revised Statutes, Chapter* 12, *Section* 94.

211. Canadian Post Office Money Orders are drawn by the postmaster, or his clerk, at one office, directing the postmaster of another

office to pay to the person named in his private letter of advice, the sum specified in the order.

Applications for money orders must be in writing, and must state the amount of each order wanted, the name and address of the person to whom the order is to be paid, and the name and address of the remitter. Application blanks may be obtained at any money order office.

Money Orders

212. The rates of commission for Money orders issued in Canada for payment in Canada, Antigua, Bahamas, Barbados, Bermuda, British Guiana, Cayman Islands, Dominica, Grenada, Jamaica, Montserrat, Nevis, Newfoundland, St. Christopher (St. Kitts), St. Lucia, St. Vincent, Tobago, Trinidad, Turks Islands, Virgin Islands are as follows—

On Orders from—	Commission	Revenue Tax	Total Fee
	Cents	Cents	Cents
$ 0.01 to $ 5.00 inclusive	.05	.03	.08
5.01 " 10.00 "	.07	.03	.10
10.01 " 30.00 "	.12	.03	.15
30.01 " 50.00 "	.15	.03	.18
50.01 " 60.00 "	.20	.03	.23
60.01 " 100.00 "	.25	.03	.28

Owing to the unsettled state of New York Exchange, the rates of commission on Money Orders issued for payment in the United States (including Guam, Hawaii, Panama Canal Zone, Philippine Islands, Puerto Rico, Tutuila (Samoa) and Virgin Islands of the United States), Cuba, including Isle of Pines, and Mexico, cannot be furnished in advance.

The rates of commission on Orders payable in Great Britain, British Possessions and all other countries upon which Money

Orders may be obtained, except those mentioned above, are as follows, viz.—

On Orders from—	Commission	Revenue Tax	Total Fee
	Cents	Cents	Cents
$ 0.01 to $ 5.00 inclusive	.05	.03	.08
5.01 " 10.00 "	.10	.03	.13
10.01 " 20.00 "	.20	.03	.23
20.01 " 30.00 "	.30	.03	.33
30.01 " 40.00 "	.40	.03	.43
40 01 " 50.00 "	.50	.03	53
50.01 " 60.00 "	.60	.03	.63
60.01 " 70.00 "	.70	.03	.73
70.01 " 80.00 "	.80	.03	.83
80.01 " 90.00 "	.90	.03	.93
90.01 " 100.00 "	$1.00	.03	$1.03

Duplicate orders issued free of charge.

In case of removal of payee from one place to another, transfer of payment can be arranged without additional charge.

Changes of names of persons to whom payment is to be made can also be effected without additional cost.

Postal Notes

213. The Postal Note System affords a cheap and convenient means of remitting small sums of money to places in Canada and Newfoundland. The rates of commission plus revenue tax for the various denominations are as follows: One cent for the 10 cent denomination; two cents for the 20c., 25c., 30c. and 40c. denominations; three cents for the 50c., 60c., 70c., 75c., 80c., 90c., and $1 denominations; five cents for the $1.50 denominations and six cents for the $2, $2.50, $3, $4 and $5 denominations. Also at the smaller Post Offices at which Money Orders are not issued, Postal Notes of the $10 denomination (commission plus tax 8c.) are sold. Odd cents (up to, but not exceeding 9 cents) may be provided for by affixing Postal Note stamps to a postal note. The purchaser of Postal Notes must fill in the name of paying office, the payee,

and his own name in the space provided for on the face of all Postal Notes.

Canadian Postal Notes are payable only in Canada and Newfoundland and should not be remitted to other countries.

214. The above and following tables or rates are given to aid the student in working the examples relating to money orders in this subject. These rates, while current at the time of the writing of this chapter, are subject to change, and hence should not be regarded as authentic by the business man, until verified.

215. An Express Money Order is an order drawn by the agent of an express company at the office where the order is bought, directing another agent of the company, at some place designated, to pay to the person named therein a certain sum of money.

Express orders are transferable by indorsement the same as cheques.

Express Money Order Rates

Payable in Canada or United States

	$ 2.50 and under		5 cts.	Over $ 50.00 to		$ 60.00	18 cts.
Over	2.50 to	$ 5.00	7 "	" 60.00 to		80.00	20 "
"	5.00 to	10.00	10 "	" 80.00 to		100.00	24 "
"	10.00 to	30.00	12 "	" 100.00 at same rates.			
"	30.00 to	50.00	15 "				

Telegraphic Money Orders

216. Telegraphic Money Orders represent a system of exchange by which the remitter sends a message to the payee, directing him to call at a certain telegraph office for the sum named therein. The telegraph agent at the sending office instructs the agent at the receiving office to pay to the person named in the message the sum specified, upon his personal application and proper identification.

Reasons for Telegraphic Remittance

217. Sometimes it becomes necessary to send money immediately for some special purpose, say one of the following—

1. To banks to meet maturing obligations.

2. To fire and life insurance companies for premiums.
3. To travelers and traveling salesmen.
4. To students and pupils at schools, seminaries, colleges, etc.
5. To guarantee purchases.
6. To accompany bids for contracts.
7. For payment of bills.
8. For the purchase of railroad, steamship, and theater tickets.
9. For purchases of all kinds.
10. For holiday gifts and other remembrances.
11. For memorial occasions and anniversaries.
12. For payment of taxes and assessments, and for all other purposes requiring the quick remittance of money.

218. Below is given a sample of Telegraphic Money Order rates based on $25.00.

From Toronto, Ont., to—			
British Columbia points	$2.50 plus	25c.=	$2.75
Alberta points	2.50 "	25c.=	2.75
Northern Saskatchewan points	2.20 "	25c.=	2.45
Southern Saskatchewan points	1.90 "	25c.=	2.15
Other Ontario points	.60 "	25c.=	.85
Quebec points	.80 "	25c.=	1.05
Maritime Provinces points	.90 "	25c.=	1.15

For Sums over $25.00, *add* 1%.

219. Illustration.—$125.00 remitted to any British Columbia point would cost $2.50 plus 1% of the amount, $1.25 = $3.75.

To United States points the cost varies according to rise and fall in the exchange rates.

220. To Find the Cost of Domestic Exchange

Example.—A hardware merchant of Winnipeg, Man., owes Morgan & Co., of Montreal, Que., an account of $720. He buys

of his banker a draft on the Bank of Toronto, at a cost of 1/8 of 1%. How much does the draft cost ?

Operation.		*Explanation.*
Draft, face . . .	= $720.00	The cost of the draft will be its face,
Exchange $\frac{1}{8}$ of 1% . .	= .90	$720, plus the cost of exchange, 90c.
Cost of draft . . .	= $720.90	or $720.90. Credit will be given by Morgan & Co., however, for only $720, for should they deposit this draft with their banker, or transfer it to any one on account, the draft would be worth only its face, $720.

221. Rule.—Add to the face of the draft, the charge for drawing it ; the sum will represent the cost of the draft.

INTEREST

222. Interest is that which is paid for the use of money.

223. Terms Used.—The essential elements in interest calculations are the principal, the time, the rate, the interest, and the amount.

224. Terms Defined.—The sum upon which interest is charged is termed the *principal*; the period for which the principal bears interest, the *time*; the annual rate charged for the use of the principal, the *rate* of interest; the product of the rate of interest and the time, the *per cent.* of interest; the result obtained by taking a per cent. of interest of the principal, the *interest*; the sum of the principal and interest, the *amount.*

SIMPLE INTEREST

225. Simple Interest is interest allowed for the use of the principal only.

226. The Term Interest is always understood to mean simple interest. If other forms of interest are meant they are specifically designated ; as, compound interest, periodic interest.

227. Illustration.—Find the interest on \$650 for 2 years at 4%.

SOLUTION (*a*)

\$650 Principal
.04

\$26.00 Int. for 1 yr.
2

\$52.00 Int. for 2 yrs.

SOLUTION.—Interest for 1 yr. is 4% of the principal, \$650 = \$650 × .04 = \$26.00, and the interest for 2 yrs. is twice the interest for 1 yr., or \$26.00 × 2 = \$52.00

SOLUTION (*b*)

The interest on \$100 for 1 year at 4% = \$4.00.

The interest on \$650 for 2 years at 4% = $\frac{650}{100} \times \frac{2}{1} \times 4.00 =$ \$52.00.

EXERCISE 67

Find the interest on the following—

1. \$120 for 4 yr. at 5%
2. \$72 for 5 yr. at 6%
3. \$84 for 3 yr. at 5%
4. \$150 for 2 yr. at 6%
5. \$320 for 5 yr. at 4%
6. \$325 for 5 yr. at 4%
7. \$320 for 3 yr. at 6%
8. \$800 for 2 yr. at 7½%
9. \$120 for 4 yr. at 5%
10. \$32 for 1 yr. at 5%
11. \$60 for 2 yr. at 6½%
12. \$75 for 1 yr. at 6%

228. Illustration.—Find the interest on \$850 for 62 dys. at 5%.

SOLUTION (*a*)

\$850
.05

\$42.50 Int. for 1 yr.
62

365)2635.00(\$7.21 +
or
\$7.22.

SOLUTION.—Sixty-two days is $\frac{62}{365}$ of 1 yr. The interest for 62 dys. is, therefore, $\frac{62}{365}$ of the interest for 1 yr., and this may be found by multiplying the interest for 1 yr. (\$42.50) by 62 and dividing the result by 365.

SOLUTION (*b*)

The interest on \$100 for 365 days at 5% = \$5.00.

The interest on \$850 for 62 days at 5% = $\frac{850}{100} \times \frac{62}{365} \times 5.00 =$ \$7.22

EXERCISE 68

Compute the interest on the following—

1. \$840 for 63 dys. at 5%
2. \$960 for 75 dys. at 6%
3. \$320 for 96 dys. at 7%
4. \$1,260 for 123 dys. at 8%
5. \$2,480 for 85 dys. at 9%
6. \$1,244 for 54 dys. at 10%
7. \$2,360 for 59 dys. at 11%
8. \$9,200 for 36 dys. at 12%

229. Illustration.—Find the interest on \$3,250 from April 16, 1943, to June 18, 1945, at 6% per annum.

(From April 16, 1943, to June 18, 1945, is 2 yrs. and 63 dys.)

SOLUTION 1

\$3,250
.06
\$195.00
$2\frac{63}{365}$
\$423.66

SOLUTION 2

\$3,250 × $\frac{6}{100}$ × $\frac{63}{365}$ = \$33.66 Int. for 63 dys.
\$3,250 × $\frac{6}{100}$ × 2 = \$390.00 Int. for 2 yrs.
\$423.66 Int. for 2 yrs. 63 dys.

EXERCISE 69

Find the interest on—

1. \$800 from Jan. 15, 1943, to June 18, 1943, at 8%.
2. \$2,400 from Feb. 11, 1944, to June 3, 1945, at 4½%. At 7½%.
3. \$650 from June 17, 1943, to Jan. 21, 1945, at 7%. At 9%.
4. \$565.25 from Jan. 29, 1944, to Jan. 1, 1946, at 6%. At 6½%.
5. \$940 from June 15, 1944, to Jan. 15, 1947, at 6½%. At 7½%.

230. Illustration.—What principal will yield \$400 interest in 2 yrs. at 8%.

SOLUTION

Interest on \$100 for 2 yrs. at 8% = \$16
\$16 is the interest on \$100
\$1 is the interest on $\frac{\$100}{16}$
\$400 is the interest on $\frac{400 \times 100}{16}$
= \$2,500

EXERCISE 70

Find the principal—

	RATE.	TIME.	INTEREST.		RATE.	TIME.	INTEREST.
1.	$3\frac{1}{2}\%$	1 yr.	$\$45\frac{1}{2}$	7.	5%	7 yrs.	$29.75
2.	$5\frac{1}{2}\%$	1 yr.	$\$41\frac{1}{4}$	8.	$3\frac{1}{3}\%$	$4\frac{1}{2}$ yrs.	$94.50
3.	$4\frac{1}{2}\%$	$\frac{1}{2}$ yr.	$\$25\frac{1}{2}$	9.	4%	$1\frac{3}{4}$ yrs.	$68.25
4.	$3\frac{3}{4}\%$	$\frac{1}{2}$ yr.	$\$3\frac{3}{5}$	10.	$4\frac{1}{2}\%$	$1\frac{1}{4}$ yrs.	$47.25
5.	8%	$\frac{3}{4}$ yr.	$18	11.	6%	$5\frac{2}{3}$ yrs.	$170.00
6.	$2\frac{1}{2}\%$	6 yrs.	$\$52\frac{1}{2}$	12.	$3\frac{1}{5}\%$	$4\frac{1}{4}$ yrs.	$136.00

231. Illustration.—What principal will amount to $508 in $4\frac{1}{2}$ yrs. at 6%?

SOLUTION

Let $1 represent the principal.

$1.27 = the amount of a dollar for $4\frac{1}{2}$ yrs.

$508 = the amount of a certain principal for $4\frac{1}{2}$ yrs.

$508 ÷ $1.27 = 400.

Since the given amount is 400 times the assumed amount, the required principal must be 400 times the assumed principal.

400 times $1 = $400, the required principal.

EXERCISE 71

What sum must be put out at interest for

1.	2 yrs. at 4%	to amount to	$540.00
2.	4 yrs. at 6%	to amount to	$2,480.00
3.	6 yrs. at $2\frac{1}{2}\%$	to amount to	$2,760.00
4.	3 yrs. at 3%	to amount to	$87.20
5.	10 yrs. at 7%	to amount to	$342.00
6.	8 yrs. at 5%	to amount to	$616.00
7.	102 dys. at 5%	to amount to	$4,441.20
8.	318 dys. at 3%	to amount to	$2,246.70
9.	75 dys. at 6%	to amount to	$2,586.50
10.	150 dys. at $2\frac{1}{2}\%$	to amount to	$3,318.75
11.	200 dys. at $6\frac{1}{4}\%$	to amount to	$755.00
12.	85 dys. at 5%	to amount to	$516.95
13.	174 dys. at $3\frac{1}{3}\%$	to amount to	$593.28
14.	312 dys. at $6\frac{2}{3}\%$	to amount to	$462.96

232. The 6% Method of Calculating Interest

This method is called the 6% method from the fact that, no

matter what the given rate is, the interest is computed first at 6%, and then changed to what it would be at the given rate.

Also, in computing interest by the 6% method, the amount is calculated first on the basis of a 360-dy. year, and then changed to the basis of a 365-dy. year.

Then, the interest for 360 dys. being 6% of the sum, the interest for 60 dys. is 1% of the sum, and is obtained by dividing by 100, which simply shifts the decimal point two places to the left.

The interest for 60 dys. is taken as the basis of the operation.

233. Illustration.—Find the interest on $540 for 49 dys. at 6%, calculated on the basis of 360 dys. to the year.

The interest for 60 days =	$5.40	
The interest for 30 days =	$2.70	(Half as much, since time is halved).
The interest for 10 days =	$0.90	(One-third as much as for 30 days).
The interest for 6 days =	$0.54	(One-fifth as much as for 30 days).
The interest for 3 days =	$0.27	(One-half as much as for 6 days).
The interest for 49 days =	$4.41	(Sum of amount for 30 days, 10 days, 6 days, and 3 days).

It will be noted that in the illustration above we start with the interest for 60 days, and from it build up the interest for 49 days by taking, in each case, a number of days which is an exact factor of some preceding number of days. Thus, 30 is $\frac{1}{2}$ of 60, 10 is $\frac{1}{3}$ of 30, 6 is $\frac{1}{5}$ of 30, and 3 is $\frac{1}{2}$ of 6.

The number 60 has so many factors that any number whatever can be built up in this way. Thus, had the time been 28 days, we would have got the sum of the interest for 20 days ($\frac{1}{3}$ of 60 days), 6 days ($\frac{1}{10}$ of 60 days), and 2 days ($\frac{1}{3}$ of 6 days). Had it been 57 days, we would have got the sum of the interest for 30 days ($\frac{1}{2}$ of 60 days), 15 days ($\frac{1}{2}$ of 30 days), and 12 days ($\frac{1}{5}$ of 60 days), and so on.

Of course, in building up the amount, a great many different groupings of the number of days may be formed. Thus, in the last case we might have taken 30 days ($\frac{1}{2}$ of 60 days), 20 days ($\frac{1}{3}$ of 60 days), 6 days ($\frac{1}{5}$ of 30 days or $\frac{1}{10}$ of 60 days), and 1 day ($\frac{1}{6}$ of 6 days); or we might have taken 30 days, 20 days, 5 days ($\frac{1}{4}$ of 20 days), and 2 days ($\frac{1}{5}$ of 10 days), and so on.

The facility in performing the operation lies in the wisdom displayed in making the groupings. This is got only from practice.

If the time is over 60 days, we would first take out the largest multiple of 60 days possible. Thus, had the time in our illustration been 267 days, we would have taken the sum of the interest for 240 days (4 times 60 days), 20 days ($\frac{1}{3}$ of 60 days), 6 days ($\frac{1}{10}$ of 60 days), and 1 day ($\frac{1}{6}$ of 6 days).

Having got the interest at 6%, the next step is to change it

to what would be at the given rate. Thus, in our illustration, had the rate been 4%, we would proceed as follows—

Interest at 6% =	$4.41	
Interest at 3% =	2.205	($\frac{1}{2}$ as much as at 6%).
Interest at 1% =	.735	($\frac{1}{3}$ as much as at 3%).
Interest at 4% =	2.94	(Sum of interest at 3% and at 1%).

Or

Interest at 6% =	$4.41	
Interest at 2% =	1.47	($\frac{1}{3}$ as much as at 6%).
Interest at 4% =	2.94	(Interest at 6% – interest at 2%).

It will be noted that the interest at the given rate is built up from the interest at 6% in the same manner as the interest for the given number of days is built up from the interest for 60 days.

Having got the interest for the given time at the given rate, on a basis of 360 days to the year, it remains to change it to what it would be on the basis of a 365-day year.

Now, the amount, $2.94, could be got by simplifying the following expression—

$$\$540 \times \tfrac{4}{100} \times \tfrac{49}{360}$$

The expression necessary to give the correct amount of interest reckoned at 365 days to the year, is

$$\$540 \times \tfrac{4}{100} \times \tfrac{49}{365}$$

The latter expression can be obtained from the former one by multiplying it by $\frac{360}{365}$. The 360 in the numerator will cancel the 360 in the denominator, throwing the 365 into the denominator. Thus—

$$\$540 \times \tfrac{4}{100} \times \tfrac{49}{\cancel{360}} \times \tfrac{\cancel{360}}{365}$$

From which we see that, to change the amount of interest reckoned on the basis of a 360-day year to the amount reckoned on the basis of a 365-day year, we must multiply the amount by $\frac{360}{365}$; or, which is the same thing, multiply it by $\frac{73}{73}$.

Now, the easiest way to multiply any amount by $\frac{73}{73}$ is to subtract $\frac{1}{73}$ of the amount ($\frac{73}{73} - \frac{1}{73} = \frac{72}{73}$).

234. Illustration.—Find the interest on $397.85 for 223 dys. at 7%.

SOLUTION.—It will be observed from the foregoing that there are three steps to the solution—

(*a*) Get the interest on the amount for 223 days at 6%, reckoned on the basis of a 360-day year.

(*b*) Change the amount of interest to what it would be at the given rate.

(*c*) Change the amount of interest to what it would be reckoned on the basis of a 365-day year.

(*a*) Interest for 60 days at 6% = $ 3.9785

Interest for 180 days at 6% = 11.9355 (3 times amt. for 60 days).
Interest for 30 days at 6% = 1.9892 + ($\frac{1}{2}$ of amt. for 60 days).
Interest for 10 days at 6% = .6631 + ($\frac{1}{3}$ of amt. for 30 days).
Interest for 3 days at 6% = .1989 + ($\frac{1}{10}$ of amt. for 30 days).

Interest for 223 days at 6% = 14.7867 +

(*b*) Interest at 6% = 14.7867 +
Interest at 1% = 2.4644 + ($\frac{1}{6}$ of amt. at 6%)

Interest at 7% = 17.2511 + (amt. at 6% + amt. at 1%).

(*c*) $17.2511 − $\frac{1}{73}$ of $17.2511 = $17.2511 − $.2363
= $17.0148 = $17.01.

In calculating the amount, where the numbers do not divide evenly, allow two decimal places past the cents' figures, but no further, as that is enough to give the amount correct to the nearest cent.

NOTE 1.—In reckoning interest for an exact number of months, or for a number of months and days where the particular months are not known, allow 30 days to the month, and do not take off the $\frac{1}{73}$ of the amount; since if a month is considered as 30 days, the year would be 360 days.

NOTE 2.—In reckoning interest for one or more years, together with an exact number of days, allow 365 days for the whole year or years, and deduct $\frac{1}{73}$ from the whole amount.

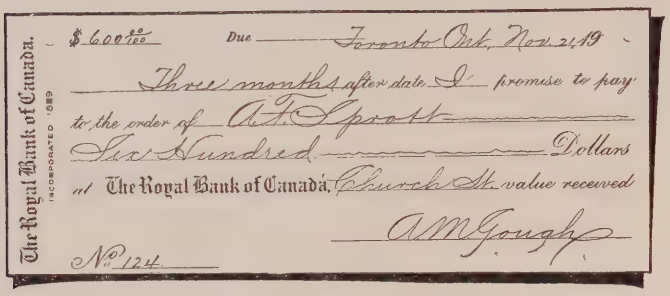
EXERCISE 72

By the 6% method find the interest on

	Principal.	Time in Dys.	Rate.		Principal.	Time in Dys.	Rate.
1.	$ 450	33	6 %	15.	$ 385.60	32	8 %
2.	920	48	6 %	16.	438.15	206	6 %
3.	630	58	6 %	17.	926.50	111	4 %
4.	385	226	5 %	18.	457.29	87	3 %
5.	875	152	4 %	19.	837.45	289	3½ %
6.	1,450	47	6 %	20.	187.28	156	9 %
7.	1,385	269	7 %	21.	1,368.19	43	5 %
8.	429	88	6 %	22.	436.73	251	4 %
9.	1,500	59	5 %	23.	15.28	117	8 %
10.	325	125	3 %	24.	187.65	17	5 %
11.	1,280	143	8 %	25.	348.27	46	3½ %
12.	850	79	4 %	26.	1,425.36	325	5½ %
13.	338	148	3 %	27.	129.37	293	6½ %
14.	540	329	4½ %				

28. $650 from May 13, 1946, to Sept. 18, 1946, at 6%
29. $3,260 from Jan. 9, 1943, to Aug. 1, 1943, at 7%
30. $346.75 from Sept. 3, 1944, to May 10, 1945, at 4%
31. $293.26 from April 12, 1944, to May 28, 1945, at 5%
32. $914.68 from July 27, 1945, to April 13, 1947, at 3%
33. $536.27 from Nov. 23, 1942, to July 11, 1946, at 3½%
34. $1,368.45 from Dec. 14, 1943, to May 25, 1947, at 4%
35. $459.38 from Aug. 17, 1945, to Jan. 3, 1950, at 5%
36. $98.67 from Oct. 30, 1944, to July 14, 1947, at 6½%

235. A **Promissory Note** is an unconditional promise in writing made by one person (the maker) to another (the payee), signed by the maker, engaging to pay, on demand or at a fixed or determinable future time, a sum certain in money to, or to the order of, a specified person or to bearer—


The Royal Bank of Canada
INCORPORATED 1869

$ 600 00/100 Due Toronto Ont. Nov 24 19

Three months after date I promise to pay

to the order of A. T. Sproat

Six Hundred Dollars

at The Royal Bank of Canada, Church St. value received

A M Gough

No. 124

Interest-Bearing Notes

236. The Legal Rate of interest in Canada is 5%. By this it is not meant that a higher rate cannot be agreed upon or collected, but that this is the rate that may be collected where interest is considered by law to be collectible, and still the parties have not fixed upon a definite rate.

237. Where nothing is said about interest in the note, the note does not bear interest unless it should happen that it is not paid at maturity. In such a case it begins at maturity to bear the legal rate, five per cent., and will continue to do so until paid. This point is worth remembering, in view of the fact that a great many seem to think that a note must necessarily bear interest from the date of its issue, which is manifestly a mistake. In the

note which we have just shown, the amount that is payable on the due date is just the face amount, $600.00. If it were desired to make this an interest-bearing note, it would have to read as follows—

> 600^{00}/_{100}$ *Toronto, Ont., Nov.* 21, 1944
>
> *Three months after date for value received..... I.....promise to pay to A. F. Sprott.....or order at the office of The Royal Bank of Canada here, the sum of..... Six Hundred* $^{00}/_{100}$ *Dollars with interest at six per cent. per annum.*
>
> A. M. Gough.

In the case of this note, the amount payable at maturity is $600.00, plus the interest on $600.00 for ninety-five days at six per cent. Should this note not be paid at maturity, the rate of interest for any time after maturity would be five per cent., the legal rate. That is, suppose the note were not paid until March 24th, the amount that could be collected would be $600.00, plus the interest on $600.00 for ninety-five days at six per cent., plus the interest on $600.00 for twenty-eight days at five per cent.

238. Rate after Maturity. Where an interest-bearing note has a rate which is less than five per cent., and such a note is not paid at maturity, the interest would rise to five per cent. for any time after maturity.

If the rate is over or under five per cent., and it is desired that it should remain at that rate after maturity, the interest clause should be written as shown below—

> $500.00 *Toronto, Ont., May* 29, 1944
>
> *Thirty days after date..... I.....promise to pay to..... A. G. McKay.....or order at The Royal Bank of Canada, Toronto..... Five Hundred* $^{00}/_{100}$ *Dollars for value received, with interest at six per cent. per annum until maturity, and thereafter at the same rate until paid.*
>
> *No.* 3. *Due July* 2, 1944 R. Brown.

Simply inserting the clause, " With interest at six per cent. until paid," would not answer, as the six per cent. would not be collectible beyond the date of maturity.

239. Stating the Rate. The following section from our Canadian law should also be carefully noted: " Whenever any interest is by the terms of any contract, whether under seal or not, made payable at a rate per day, week, month, or for any period less than a year, no interest exceeding the rate of five per cent. per annum shall be recoverable, unless the contract

contains the express statement of the yearly rate of interest to which such other rate is equivalent."

By this we understand that, if a note were drawn with interest at twelve per cent. per annum, it could be collected. If it read "one per cent. per month," only five per cent. per annum could be collected.

In the last-mentioned case, where it is desired to charge one per cent. per month, a note should, in addition to this statement, give the rate per annum to which one per cent. per month is equivalent.

This section does not apply to mortgages on real estate.

Compound interest cannot be collected, unless it is agreed upon in the contract.

240. Book accounts differ from notes. A book account overdue will not draw interest unless it is expressly stated on invoices and statements that interest will be charged after a certain date. Even then only five per cent. can be collected, unless the debtor has agreed to pay more. Simply having eight or ten per cent. printed on the invoice and statement forms does not make the charge binding, and the debtor may refuse to pay anything over five per cent.

Calculating Interest Periods

241. Finding Due Date.—Notes are generally given at a certain number of days or months after date. Drafts may be given at a certain number of days or months after date or after sight. This may give rise to difference in the method of getting the due date. There is no difference, however, in any case, in getting the period of time for which interest or discount is to be calculated. It is always a question of fixing two dates, and then calculating the time between these dates in actual days.

For instance, an interest-bearing note is given June 15th, at thirty days. It falls due July 18th. The term for which the interest is reckoned is the term from June 15th to July 18th, reckoned by the actual counting of the days. In other words, the interest will be for thirty-three days.

Again, an interest-bearing note is given July 8th, at two months. It will fall due September 11th. This due date is reckoned by taking two calendar months from July 8th, and adding the three days of grace. The interest on this note will be for the term between July 8th and September 11th, reckoned in actual days. In other words, the interest term will be sixty-five days.

242. Interest Term. Note that, having once fixed the date of the note and the due date, the interest term in each case is computed in exactly the same way. It is always the exact number of days between the two dates. Let us caution the student against such mistakes as saying that the interest on the first note will be for thirty days and on the second note for two months.

EXERCISE 73

Find the date of maturity of the following notes. Find also the number of days between date and due date.

	Date of Note.	Year.	Time.		Date of Note.	Year.	Time.
1.	Jan. 4,	1943	30 dys.	9.	Dec. 29,	1942	2 mos.
2.	Jan. 4,	1943	1 mo.	10.	Dec. 30,	1942	2 mos.
3.	Feb. 9,	1943	30 dys.	11.	Dec. 31,	1942	2 mos.
4.	Feb. 9,	1943	1 mo.	12.	July 31,	1943	1 mo.
5.	Feb. 9,	1944	30 dys.	13.	July 31,	1943	2 mos.
6.	Feb. 9,	1944	1 mo.	14.	Dec. 1,	1943	1 mo.
7.	Dec. 28,	1942	60 dys.	15.	Nov. 29,	1943	1 mo.
8.	Dec. 28,	1942	2 mos.				

$500\frac{00}{100}$ *Toronto, Ont., April* 9, 1944.

Two months after date..... I.....promise to pay.....
J. F. Nunn.....or Bearer
at The Royal Bank of Canada, Church Street Branch,.....
Five Hundred $^{00}/_{100}$ *Dollars for value received.*

No..... *Due.....* H. Denton.

16. How much would it require to redeem the above note on June 12, 1944 ; on July 4, 1944 ; on Sept. 9, 1945 ?

17. Adding the following interest clause to the note, "with interest at seven per cent. per annum," how much would it require to redeem the note on June 12, 1944 ; on July 4, 1944 ; on Sept. 9, 1945 ?

18. Adding the following interest clause to the note, "with interest at four per cent. per annum," how much would it require to redeem the note on June 12, 1944 ; on July 4, 1944 ; on Sept. 9, 1945 ?

19. Adding the following interest clause to the note, " with interest at seven per cent. per annum, both before and after maturity until paid," how much would it require to redeem the note on June 12, 1944 ; on July 4, 1944 ; on Sept. 9, 1945 ?

20. Adding the following interest clause to the note, "with interest at one per cent. per month," how much would it require to redeem the note on June 12, 1944 ; on July 4, 1944 ; on Sept. 9, 1945 ?

$425\frac{00}{100}$ *Toronto, Ont., Jan.* 14, 1944.

Ninety days after date.....pay to the Order of
..... John Henderson .....
Four Hundred and Twenty-five $^{00}/_{100}$ *Dollars, value received and charge the same to the account of*

To Herman Long.....
No. ..... Due .....

JAMES BALSDON.

21. The above draft is accepted as follows—

"Accepted.
Jan. 17, 1944.
Payable at the Bank of Toronto, Toronto
HERMAN LONG."

When is it due? What amount will redeem it on its due date? On June 18, 1944?

22. Suppose the above draft were drawn at 90 dys. after sight and accepted as above, when would it fall due? What amount would redeem it on its due date? On June 18, 1944?

EXERCISE 74

243. Illustration.—At what rate will $4,380, in 76 dys., produce $45.60 interest?

SOLUTION.—Let 1% equal the rate. Then interest on $4,380 for 76 dys. at 1% = $9.12. $45.60 ÷ $9.12 = 5. The interest at 1% is contained in the given interest 5 times. Therefore, the required rate is 5 times 1%, or 5%.

Find the rate of interest—

	Principal.	Interest.	Time.		Principal.	Interest.	Time.
1.	$600	$72	2 yrs.	6.	$100	$24	8 yrs.
2.	$500	$60	3 yrs.	7.	$584	$9.28	174 dys.
3.	$300	$60	5 yrs.	8.	$511	$5.95	85 dys.
4.	$200	$24	4 yrs.	9.	$1460	$10.80	45 dys.
5.	$400	$16	6 mos.				

COMMERCIAL ARITHMETIC

TEST PAPER—No. 6

SIMPLE INTEREST

Your work for this paper is to be submitted to the instructor for correction and criticism.

1. A gave B a note dated March 1, 1926, for $1,200, payable 70 days after date. The interest on the note when due was $13.20. What was the rate of interest? (Three days of grace to be added.)

2. Find the interest on $375.40 from January 10, 1925, to August 31, 1925, at 5% per annum.

3. In what time will $212 amount to $245.39 at 7% per annum? (Simple interest.)

4. A bank in Ottawa makes a collection of commercial paper, amounting to $12,800.00, on its branch in Toronto, charging $\frac{1}{8}$ of 1% for collection. Find the amount of collection charges and the amount received by the client.

5. Using the 6% method of calculating interest, calculate the interest on—

	Principal	Days	Rate
(*a*)	$287.50	114	6%
(*b*)	563.47	214	8%
(*c*)	497.86	314	4%
(*d*)	986.20	14	3%
(*e*)	568.92	326	5%
(*f*)	714.76	129	7%
(*g*)	586.47	215	9%

Hold Fast to Your Ideal

"Ideality," said Horace Mann, "is only the avant-courier of the mind; and where that in a healthy and normal state goes, I hold it to be a prophecy that realization can follow."

What you think you are or can be, you will become. Your thought of yourself is the pattern which your life follows. Every time you think you are a failure, you are helping yourself to become one. Every time you doubt your ability to do what you have undertaken, you are putting another stumbling block in your way and making it less likely that you will win out.

Never think of yourself as anything but what you want to be. Hold fast to your ideal.

BANK DISCOUNT

Discounting a negotiable paper, such as a note of acceptance, simply means selling it or realizing on it before its maturity.

Banks, naturally, do the greater part of this business of buying such paper, and the term "bank discount" is given to the deduction from the maturity value that is made by the bank in finding the present value it places on a paper offered to it.

Bank Discount is figured as the simple interest on the maturity value for the time the paper has to run before the date of maturity is reached.

The **Present Worth** or **Present Value** or **Proceeds** of the paper is the difference between the maturity value and the discount.

In finding bank discount, these questions must be answered—

(1) When does the paper fall due?

(2) What is it worth at maturity?

(3) What is the number of days from the proposed date of discount till the date of maturity?

If we can answer these and can work a question in simple interest, we know how to find bank discount.

244. Illustration 1

200^{00}/_{100}$ *Toronto, Ont., July 7, 1944.*

Three months after date..... I.....promise to pay to John Wildfong .....or order at The Royal Bank of Canada..... Two Hundred $^{00}/_{100}$ *Dollars for value received.*

No..... *Due.....* GEO. CAMPBELL.

This note is discounted at the Bank of Montreal, Toronto, on July 29th. If the rate of discount is 6%, what are the proceeds of the note ?

SOLUTION

(1) The due date is Oct. 10th.
(2) The value of the note on Oct. 10th is $200.
(3) The number of days from July 29th to Oct. 10th is 73 dys.
(4) The interest on $200 for 73 days at 6% is $2.40.
(5) The proceeds of the note are $200 – $2.40, or $197.60.

Illustration 2

300^{00}/_{100}$ *Toronto, Ont., May 27, 1944.*

Sixty days after date..... I .....promise to pay to Wm. Colville.....or order at The Royal Bank of Canada, Church Street Branch ..... Three Hundred $^{00}/_{100}$ *Dollars for value received, with interest at five per cent. per annum.*

No..... *Due.....* W. MILLER.

This note is discounted June 4th, at the Bank of Commerce, Stratford. If the rate of discount is 6%, what are the proceeds ?

SOLUTION

(1) The due date is July 29th.
(2) The value of the note on July 29th is $302.59 ($300 + interest on $300 for 63 dys. at 5%).
(3) The number of days from June 4th to July 29th is 55 dys.
(4) The interest on $302.59 for 55 dys. at 6% is $2.74.
(5) The proceeds of the note are $302.59 – $2.74, or $299.85.

EXERCISE 75

Find the date of maturity, term of discount, bank discount, and proceeds of the following ten notes and acceptances—

1. $367\frac{50}{100}$ St. Thomas, Ont., Jan. 5, 1944.
Three months after date I promise to pay to Clare Crawford, or order, at the Imperial Bank, here, Three Hundred and Sixty-seven and $\frac{50}{100}$ Dollars, value received.
Discounted Jan. 5, 1944, at 6%. JOHN SMITH.

2. $423\frac{70}{100}$ Toronto, Ont., Dec. 30, 1943.
Two months after date I promise to pay to Fred. L. Sanders, or order, at the Merchants Bank, here, Four Hundred and Twenty-three and $\frac{70}{100}$ Dollars, value received.
Discounted Feb. 1, 1944, at 5%. R. LEARN.

3. $126\frac{45}{100}$ Toronto, Ont., Dec. 30, 1943.
Sixty days after date I promise to pay to D. B. Brown, or order, One Hundred and Twenty-six and $\frac{45}{100}$ Dollars, value received.
Discounted Jan. 10, 1944, at 5½%. WM. OSTRANDER.

4. $537\frac{25}{100}$ Montreal, Que., Feb. 29, 1944.
Six months after date I promise to pay to W. H. Simpson, or order, Five Hundred and Thirty-seven and $\frac{25}{100}$ Dollars, value received, with interest at six per cent. per annum.
Discounted Feb. 29, 1944, at 6%. T. MEDCRAFT.

5. $162\frac{90}{100}$ Aylmer, Ont., Dec. 13, 1943.
Four months after date I promise to pay to F. D. White, or order, One Hundred and Sixty-two and $\frac{90}{100}$ Dollars, value received, with interest at five per cent. per annum.
Discounted Feb. 16, 1944, at 5%. G. P. BROWN.

6. $75\frac{30}{100}$ Windsor, Ont., Nov. 15, 1943.
One month after date I promise to pay to J. E. Norman, or order, Seventy-five and $\frac{30}{100}$ Dollars, value received, with interest at four per cent. per annum.
Discounted Nov. 15, 1943, at 6%. JOHN GLOIN.

7. 240\frac{00}{100}$ Aylmer, Ont., Oct. 31, 1943.

Four months after date I promise to pay to D. H. Price, or order, Two Hundred and Forty and $\frac{00}{100}$ Dollars, value received, with interest at seven per cent. per annum.

Discounted Dec. 5, 1943, at 5%. A. A. LESLIE.

8. 346\frac{75}{100}$ Ingersoll, Ont., Dec. 18, 1943.

One year after date I promise to pay to Emerson McTaggert, or order, Three Hundred and Forty-six and $\frac{75}{100}$ Dollars, value received, with interest at five per cent. per annum.

Discounted May 16, 1944, at 6%. P. GILLETT.

9. 168\frac{75}{100}$ Toronto, Ont., Mar. 15, 1943.

Ninety days after sight pay to the order of T. Tanton, One Hundred and Sixty-eight and $\frac{75}{100}$ Dollars, value received, and charge to the account of A. F. SPROTT.

To W. W. AUSTIN, Toronto, Ont.

Accepted Mar. 16, 1943; discounted May 1, 1943, at 6%.

10. 243\frac{25}{100}$ Ottawa, Ont., Aug. 1, 1943.

Thirty days after date pay to the order of ourselves, Two Hundred and Forty-three and $\frac{25}{100}$ Dollars, value received, and charge to the account of BENNER & LINDSAY.

To R. ANGER, Toronto, Ont.

Discounted Aug. 1, 1943, at 6%.

11. Find the proceeds of a draft for $580 at 60 dys., discounted at 6%.

12. A note for $1,500, with interest at 5% per annum, dated May 1, 1943, at 3 mos., was discounted June 3rd. Discount rate 5% per annum. Find the proceeds.

13. I bought a lot for $1,200 cash, and sold it at an advance of 12½%, on a 90-dy. note, which I immediately discounted at 6%. Find my gain or loss.

14. C. H. Good & Co.'s bank account is overdrawn $7,564.19. They discount at 6%: a 90-dy. note for $3,975.21, a 60-dy. note for $1,546.19, and a 20-dy. note for $2,546.85; proceeds of all to their credit at the bank. What is the condition of their bank account after they receive credit as above?

15. A dealer bought 100 bbl. apples at $4 per bbl. for cash. He sold 25 bbls. for cash at $5 per bbl., 25 bbls. at $6 per bbl. for a 30-dy. note without interest, and the remaining 50 bbls. at $5,75 per bbl. for a 60-dy. note drawing interest at 5%. He discounted the notes at 6% the same day as drawn, getting cash for the proceeds. How much did he make on the transaction?

16. A farmer purchases a machine for $62, and gives in settlement two notes, one for $31, due Oct. 1st following (1943), and one for $31, due Oct. 1st one year later. On July 15th the company proposes, and farmer accepts, a proposition to pay off both notes on basis of discount at 6% per annum. What is the net amount paid by the farmer to retire his notes?

245. **Illustration**

$..... *Toronto, Ont., January 7,* 1944.

Three months after date..... I .....promise to pay to The Royal Bank of Canada .....or order at their office, Toronto, Ont.....

.....100 *Dollars for value received.*

No..... *Due.....* J. P. MURRAY.

It is desired to fill in the above note for such a sum that, when discounted at 5%, the note will realize $500 in cash. What should be the face value of the note?

SOLUTION

(1) The due date is April 10th.
(2) Assume a value of $1 for the face of the note on April 10th.
(3) The number of days from Jan. 7th to April 10th is 93 dys.
(4) The interest on $1 for 93 dys. at 5% is $.012741.
(5) The proceeds of a $1 note are $1.00 −.012741, or $.987259.
(6) If $.9873 is proceeds of note for $1.

Then $500 is proceeds of note for $\frac{\$500}{.987259}$, or $506.45.

The note should have a face value of $506.45.

EXERCISE 76

1. I wish to borrow $900 at a bank. For what sum must I issue a 90-dy. note to obtain the amount, discount being at 6%?

2. A note, dated Mar. 15, 1944, payable in 3 mos., with interest at 5%, was discounted April 10, 1944, at 6%. If the proceeds were $1,342.27, what must the face have been?

3. The proceeds of a 3-mos.' note, dated Sept. 20, 1943, and discounted on Oct. 15th, at 5%, were $426.89. What was the face of the note?

4. A 30-dy, 5% interest-bearing note was discounted 10 dys. after it was drawn up. If the rate of discount was 6% and the bank discount $13.40, what was the face of the note?

5. Owing a man $615, I gave him my 60-dy. note. What should be the face of the note to pay him the exact debt, if discounted at 7%?

6. You have $650.80 to your credit at a bank; you give your cheque for $1,872.40, after which you discount a 30-dy. note for $850.80, proceeds to your credit at the bank. You then discount a 90-dy. note, made by F. D. White, proceeds to your credit, when you find yourself indebted to the bank $24.74. If discount be at 6%, what must have been the face of the note made by White?

246. Illustration.—What is the cost of a sight draft of $960 at $\frac{5}{8}$% premium?

SOLUTION

Face of draft ($960) × rate of premium ($\frac{5}{8}$%) = premium ($6).
Face of draft ($960) + premium ($6) = cost of draft ($966).

What is the cost of a sight draft

7. Of $7,200 at $\frac{1}{2}$% premium?

8. Of $8,500 at $\frac{3}{4}$% premium?

INTEREST ON PARTNERS' ACCOUNTS

247. Where partners have not the same amounts invested in a business, it is customary for the articles of agreement to contain an interest clause.

The interest clause provides for interest to be allowed to each partner on all investments. If withdrawals are made, a clause may also be inserted providing that each partner is to be charged interest on all sums withdrawn.

248. Illustration.—J. R. Coulan and N. Miller become partners under agreement to share gains and losses equally. It is also agreed that interest at the rate of 10% is to be allowed on all investments and charged on all withdrawals. From the following statement of the partners' accounts find the net credit interest due to each at the end of the year—

Dr. J. R. COULAN *Cr.*

1943.	Withdrawals.	1943.	Investments.
Apr. 4	$3,000 00	Jan. 1	$6,000.00
		Sept. 19	5,000.00

Dr. N. MILLER *Cr.*

1943.	Withdrawals.	1943.	Investments.
Mar. 9	$1,000.00	Jan. 1	$4,000.00
Oct. 1	1,200.00	Oct. 6	1,500.00

SOLUTION

Dr. J. R. COULAN *Cr.*

Date. 1943.	Sums.	Days.	Int.	Date. 1943.	Sums.	Days.	Int.
Apr. 4	$3,000	272	$223.56	Jan. 1	$6,000	365	$600.00
				Sept. 19	5,000	104	142.47
							$742.47
	Total credit interest						$742.47
	Total debit interest						223.56
	Net credit interest						$518.91

Dr. N. MILLER *Cr.*

Date. 1943.	Sums.	Days.	Int.	Date. 1943.	Sums.	Days.	Int.
Mar. 9	$1,000	298	$81.64	Jan. 1	$4,000	365	$400.00
Oct. 1	1,200	92	30.25	Oct. 6	1,500	87	35.75
			$111.89				$435.75
	Total credit interest						$435.75
	Total debit interest						111.89
	Net credit interest						$323.86

NOTE.—Under the agreement, J. B. Coulan is entitled to receive from the firm $518.91, and N. Miller to receive $323.86. That is, the firm of Coulan & Miller pays Coulan $518.91 and the same firm pays Miller $323.86. Each partner is, therefore, in the position of receiving money from a firm

in which he is a partner. In other words, the payment of these two amounts ($518.91 and $323.86, or $842.77) is a loss to the business, of which loss each partner is under agreement to share one-half. Dividing $842.77 into two parts, we have $421.38 to be charged to Coulan and $421.39 to be charged to Miller.

If Coulan is entitled to draw out $518.91, he must understand that he is really paying half of the amount required for this interest settlement, or $421.38. He nets or clears only $518.91 – $421.38, or $97.53. Miller, while he receives from the business $323.86, has a portion of loss to sustain of $421.39. In other words, he owes the business $421.39 – $323.86, or $97.53.

Whenever any allowance for interest, salary, or the like is made to a partner, this interesting fact of dual-relationship must not be overlooked.

EXERCISE 77

1. On Jan. 1, 1942, Secord & Caldwell became partners under agreement to share gains and losses equally. Secord invests $3,000 and Caldwell $2,500. On April 1 Secord invests $1,000 more and Caldwell $2,000 more. On July 1 Secord withdraws $1,500, and on Sept. 1 Caldwell withdraws $1,000. It is agreed that interest at the rate of 6% is to be allowed on all investments and charged on all withdrawals. Find the net credit interest due each at the end of the year. If they decide to adjust the interest privately, how much should the one pay the other to make an equitable settlement?

2. Mar. 1, 1942, Leeson, Martyn & Baker engaged in business. Leeson invested $8,000, Martyn $7,000, and Baker $6,000. On Aug. 1 Leeson invested $2,000 additional, but on Dec. 1 withdrew $1,500. On July 1 Martyn withdrew $2,000, but on Sept. 1 invested $1,500 additional. On June 1 Baker invested $2,000 additional, but on Oct. 1 withdrew $1,000. If it was agreed to pay interest at the rate of 8% on net investments, find the net credit interest due each on Mar. 1, 1943. If they agreed to adjust the interest without the use of the books, how would it be managed? Compute the time in even months.

3. On July 1, 1943, White & Johnson became partners under agreement to share gains and losses in proportion of $\frac{2}{3}$ and $\frac{1}{3}$. White invested $7,000 and Johnson $4,000. On Jan. 1 White invested $2,000 additional, but on Feb. 1 withdrew $2,500, and on April 1 withdrew $500. On Nov. 1 Johnson invested $3,000

additional, but on Jan. 1 withdrew $2,000. If they agreed to pay interest at 7% on net investments, find the net credit interest due each on July 1, 1944. How much should one pay to the other to adjust the interest privately?

4. On Jan. 1, 1941, Brown & James formed a partnership, agreeing to share gains and losses in the proportion of $\frac{3}{5}$ and $\frac{2}{5}$. Brown's investments were: On Jan. 1, $9,000; on Mar. 10, $2,000; on Oct. 23, $2,000. Brown's withdrawals were: On May 15, $1,200; on Sept. 10, $1,500; and on Nov. 3, $1,000. James' investments were: On Jan. 1, $6,000; on May 17, $2,000; and on Sept. 4, $3,000. James' withdrawals were: On Feb. 7, $1,000; on Oct. 1, $3,000. If they agreed to pay interest at the rate of 5% on all investments and be charged 5% on all sums withdrawn, find the net credit interest due each on Jan. 1, 1942. How might they adjust the interest without the use of the books?

5. On May 1, 1941, Kent, Todd & Tufford became partners, agreeing to share gains and losses in proportion of $\frac{4}{9}$, $\frac{3}{9}$, and $\frac{2}{9}$. Kent invested on May 1, $8,000, and on Nov. 10, $3,000, but on Feb. 17 withdrew $4,000. Todd invested on May 1, $6,000, but on Nov. 3 withdrew $500, and on Feb. 20, $1,000. Tufford invested on May 1, $4,000, and on Oct. 25, $2,000, but on Dec. 1 withdrew $1,400, and on Mar. 12, $1,200. Find the net credit interest due each on May 1, 1942, if interest at the rate of 9% were allowed on all investments and charged on all withdrawals. With what single amount might each partner's account be debited or credited to make the adjustment?

COMMERCIAL ARITHMETIC

TEST PAPER—No. 7

BANK DISCOUNT

Your work for this paper is to be submitted to the instructor for correction and criticism.

1. Two notes were issued on October 1, 1925, for $3,000 each, one at 3 months and the other at 6 months. Each was to bear interest at 6%. Both were discounted on October 1, 1925, at 6%. Find the proceeds of each note.

2. $750.00. Kingston, January 3, 1926.

Three months after date, I promise to pay James Baird, or order, the sum of Seven Hundred and Fifty 00/Dollars, with interest at 5% per annum, value received.

(Signed) JOHN KEARNS.

The above note was discounted February 1, 1926, at the rate of 6% per annum. Find the proceeds.

3. I want to raise $1,300 to-day. What must be the face of a 60-day note which, when discounted at 7% will give the required amount?

4. A note dated March 15, 1926, payable in 3 months, with interest at 5%, was discounted April 10, 1926, at 6%. If the proceeds were $1,342.27, what must the face have been?

5. On May 1, A gives B a note at 90 days for $400.00. On May 21, B discounts the note at the bank, the bank charging discount at the rate of 7% per annum. Find (*a*) due date of note, (*b*) time for which discount must be calculated, (*c*) Discount, (*d*) Proceeds.

6. J. Wilson and W. Johns formed a partnership, agreeing to share gains and losses equally. January 1, each invests $4,000, on April 1, Wilson invests $1,000 more and Johns $2,000 more. On June 1, Wilson invests $1,000 and Johns withdraws $2,000. On July 1, Wilson withdraws $2,000. On October 1, Wilson invests $3,000 more and Johns $4,000. On November 1, Wilson withdraws $1,000 and Johns invests $1,000.

It is agreed that interest at the rate of 6% per annum is to be allowed on all investments and charged on all withdrawals. Find the net credit interest due each at the end of the year. How much should the one pay the other to make an equable settlement?

7. If, in the previous question, the partners had shared gains or losses according to average net investment instead of equally, what would be the net effect on the partners' accounts of the introduction of the interest feature? Show how you arrive at this result.

"The longer I live the more I am certain that the great difference between men—between the feeble and the powerful, the great and the insignificant—is energy, invincible determination, a purpose once fixed, and then death or victory."

—Sir Fowell Buxton.

Most people would say that the difference between the great and the insignificant is due to difference of ability or of opportunity, and some would make it a mere matter of luck. But Sir Fowell Buxton attributes it to a difference in men's wills. One man arrives at an "invincible determination" while another never gets beyond a feeble wish for a goal that he is too indolent to work for. There can be no question which of the two men will win.

If your interest in your studies sometimes flags, revive it by considering that you are deciding now your status during future years. Where will you belong twenty years from now—among the successes, those whose minds will stay made up, or among the failures, those who were content to wish feebly where they should have willed strongly? And remember that you will decide this question, in all probability, within the next few years.

Inspiration *vs.* Perspiration

Mr. Edison, when asked if he did not think that genius was inspiration, replied, "No! Genius is perspiration."

Many people fail to do anything of much account because they undervalue their talents. They mistakenly suppose that the achievements of exceptionally successful men and women are due to a power vaguely called genius, whereas in most cases they are due to an exceptional ability for patient, sustained effort towards a definite and clearly-conceived goal.

You are not going to fail through lack of ability, nor, we hope, through lack of persistence.

PROFIT AND LOSS

249. The profits and losses arising from business transactions are frequently computed as percentages of the cost.

250. The first cost of goods is called the prime cost. The prime cost, increased by all direct outlays incident to the purchase and holding of the goods to the date of sale, such as packing, freight, cartage, storage, commission, etc., is called the gross or full cost.

251. The actual amount received from the sale of goods is called

the gross selling price. The gross selling price, less all charges incident to the sale of the goods, is called the net selling price.

252. The difference between the net selling price and the gross cost of the goods is the gain or loss—a gain if the selling price is the larger, and a loss if the gross cost is the larger.

253. Many business firms, however, to-day figure the profit or loss on the basis of their sales instead of on the cost price, since the sales are usually more easily determined at any time or date than is the cost price of the goods sold.

254. Then, again, if an article invoiced at \$1.00 were sold at \$1.25, the profit is not 25c., because in every business there are fixed charges for doing business, such as taxes, insurances, salaries, general expenses, depreciation, etc., that would amount to, say, 25% of the cost price or 20% of the sale price of all the goods bought and sold. Thus a business with a turnover (total sales) in a year of \$100,000.00 would have fixed operating expenses of, say, \$20,000. In the illustration given above it will therefore be seen that an article invoiced at \$1.00 and sold at \$1.25 would practically yield no profit to the seller at all. Because of this many firms price all their goods to give a profit based on the sales.

255. Illustration.—During the previous year, or from estimates, they find results as follows at the close of the year—

Gross Sales . . .	\$100,000.00 =	100%
Cost of Goods sold . .	\$ 65,000.00 =	65%
Gross Trading Profit .	\$ 35,000.00 =	35%
Less Fixed Expense .	\$ 25,000.00 =	25%
Net Profit . .	\$ 10,000.00 =	10%

This illustration shows 53.85% gross gain on cost $\left(\frac{35,000}{65,000} \times 100\right)$

and 15.38% net gain on cost $\left(\frac{10,000}{65,000} \times 100\right)$

256. The student will thus see the danger to the merchant who marks his goods at an advance of, say, 40% on invoice cost as in the

above case, feeling that he is making sufficient profit, with the following result—

Gross Sales (40% advance on cost) . . .	$91,000.00
Cost of Goods sold	$65,000.00
Gross Trading Profit (40% on cost) . . .	$26,000.00
Less Fixed Expense	$25,000.00
Net Profit	$ 1,000.00
Net Profit on invoice cost (about) . . .	$1\frac{1}{2}\%$

257. **Table for Computing Profit**

If an article costing $1.00 were sold for $1.50 it would be seen quite readily that the Profit, 50 cents, is equal to one-half the cost, and at the same time it is one-third of the selling price.

From this a table can be outlined, as shown below—

If 50	% or $\frac{1}{2}$	is added to the cost,	then	$33\frac{1}{3}$	% or $\frac{1}{3}$	of S.P. is Profit.	
" $33\frac{1}{3}$	% or $\frac{1}{3}$	"	"	" 25	% or $\frac{1}{4}$	"	"
" 25	% or $\frac{1}{4}$	"	"	" 20	% or $\frac{1}{5}$	"	"
" 20	% or $\frac{1}{5}$	"	"	" $16\frac{2}{3}$	% or $\frac{1}{6}$	"	"
" $16\frac{2}{3}$	% or $\frac{1}{6}$	"	"	" $14\frac{2}{7}$	% or $\frac{1}{7}$	"	"
" $14\frac{2}{7}$	% or $\frac{1}{7}$	"	"	" $12\frac{1}{2}$	% or $\frac{1}{8}$	"	"
" $12\frac{1}{2}$	% or $\frac{1}{8}$	"	"	" $11\frac{1}{9}$	% or $\frac{1}{9}$	"	"
" $11\frac{1}{9}$	% or $\frac{1}{9}$	"	"	" 10	% or $\frac{1}{10}$	"	"
" 10	% or $\frac{1}{10}$	"	"	" $9\frac{1}{11}$	% or $\frac{1}{11}$	"	"
" $9\frac{1}{11}$	% or $\frac{1}{11}$	"	"	" $8\frac{1}{3}$	% or $\frac{1}{12}$	"	"

Oral Exercise

1. If an article costing $50 is sold for $60, what is the gain per cent. on the cost? What is the gain per cent. on the selling price?

2. An article cost $25 and was sold for $30. What is the per cent. of the gain on the cost? On the selling price?

3. An article is sold for $60. If the gain on the selling price is 25%, what is the gain? What is the cost? What is the gain per cent. on the cost?

4. A man sold an article for $8 and made 12½% on the selling price. What was the per cent. of profit on the cost?

5. A merchant sold an article for 15c., which cost him 12c. What per cent. profit is that on the cost? On the selling price?

6. If I buy an article for $1, and then mark it to sell for $1.25, how much must it be reduced to sell at cost? What per cent. is this on the marked price? On the cost?

EXERCISE 78

1. A man sells goods for $15,000. The overhead charges are 15% of sales, and the profits 10% of sales. The freight inward is $125. Find the amount of the purchase invoice of the goods.

2. An automobile is invoiced at $1,140. The freight charges are $50. If we allow 15% of selling price for overhead and 15% for profit, what should be the selling price of the automobile?

3. An invoice of merchandise amounts to $1,575.25. If the overhead charges are 20% of selling price, the gain 10%, and the freight $125, find the selling price.

4. The invoice cost of a lady's coat is $30, the overhead charges are 25% of selling price, the profit is 15%, and the freight is $2. Find the selling price.

5. A merchant marked goods at 20% above the cost. Owing to the fact that these goods did not sell well he reduced them 20% and claimed that he was selling them out at cost. Find the amount of his error in per cent. If he had reduced them 25%, how much would he have lost?

6. If a man buys some articles and marks them so as to gain 25%, and then reduces them to cost in order to move them, what per cent. must he reduce them on the marked price?

7. The invoice cost of an article was $3.50. The freight is $.25, the overhead charges are 20%, and the profit is computed at 15%. Find the selling price.

8. A retailer buys tables at $40, which he marks to sell at a profit of 40% on the cost. On account of slow business he decides to retail them at 25% less on the marked price. At what price does he sell them? Does he gain or lose and how much? What per cent. is this on the selling price?

9. An automobile is invoiced at $3,000. The freight is $50. If we allow 10% for overhead, and 20% for profit, what should be the selling price of the automobile?

Computation of Loss or Gain

258. In figuring the profits of a business, whether by departments or for the business as a whole, the deduction of the goods bought from the goods sold or net sales, gives the gain for the period of time covered by the figures. To determine the cost of the goods sold it is necessary to deduct the cost of the goods unsold at the end of the month or year from the purchases made during the same period. Assuming that there are no goods on hand, i.e. no opening inventory at the beginning of the period, the computation would be as follows—

Goods sold (Sales)		$70,000.00
Purchases	$75,000.00	
Cost of Goods unsold (inventory end of year)	$10,000.00	
Cost of Goods sold		$65,000.00
Gain		$ 5,000.00
Gain % on cost of goods sold	$5,000 ÷ $65,000 = 7.7%	
Gain % on sales	$5,000 ÷ $70,000 = 7.14%	

Accounting for Opening Inventories

EXERCISE 79

Assuming an opening inventory, then the gain for the period may be computed as shown below. Given the sales, inventories, and the cost of the purchases, this plan may be followed by any business to arrive at gain or loss.

Illustrative Example—

Sales		$232,314.26
Opening Inventory beginning of year	$ 36,756.65	
Purchases for year	214,643.53	
Total	$251,400.18	
Closing Inventory end of year	75,654.78	
Difference (cost of goods sold)		$175,745.40
Gain		$ 56,568.86

1. A merchant had goods on hand January 1, 1940, $46,756.87. During that year he purchased goods to the amount of $314,567.89, and sold goods to the amount of $298,654.65. His inventory at the end of that year showed goods on hand, $29,675.76.

Find his gain or loss for the year, and the per cent. of loss or gain on the cost of goods sold.

2. Inventory at the beginning of the year . $ 56,765.45
Inventory at the end of the year . . 44,643.44
Purchases during the year . . . 347,124.96
Sales during the year 357,649.85

Find the gain or loss.

Find the gain or loss per cent. on sales.

3. Purchases during the year . . . $250,669.25
Sales during the year 276,040.34
Beginning inventory 32,675.26
Inventory at end of year 31,575.45

Find the gain or loss.

Find the gain or loss per cent. on sales.

Find the gain or loss per cent. on cost of goods sold.

259. Illustration.—A farmer, having 480 bus. of wheat, sold 120 bus. What per cent. of his wheat did he sell?

(1)
$\frac{120}{480} = \frac{1}{4}$
$\frac{1}{4}$ of 100% = 25%

SOLUTION 1.—120 is $\frac{120}{480}$, or $\frac{1}{4}$ of 480, and since 480 is 100% of itself, 120, which is $\frac{1}{4}$ of 480, must be $\frac{1}{4}$ of 100%, or 25%. Or,

(2)
120 ÷ 4.80 = 25
25 times 1% = 25%

SOLUTION 2.—Since 480 is 100% of itself, 1% of 480 would be $\frac{1}{100}$ part of it, or 4.80. Since 4.80 is 1% of 480, 120 would be as many times 1% as 4.80 is contained times in 120, which is 25 times; 25 times 1% is 25%. Or,

SOLUTION 3.—Out of each 480 bus. he sold 120 bus.

Out of each 100 bus. he sold $\frac{100}{480} \times 120 = 25$ bus. or 25%

EXERCISE 80

1. I bought 152 yds. of muslin and sold 19 yds of it. What per cent. of it did I sell?

2. A dealer put up 8,000 tons of ice. During the summer 15% of it melted, and he sold 65% of the remainder. What per cent. of the whole amount did he still have?

3. The base is 25.8; the percentage is 2.58. What is the per cent.?

4. What per cent. does a man make on his money, who rents a house that cost $7,500, for $600 a year?

5. A certain school enrolled 60 pupils. If the attendance is 54 pupils, what is the per cent. of attendance?

6. In a battle in which 9,000 men were engaged, 2,000 men were killed. What per cent. of the number engaged was not killed?

7. A man's income is $1,500 a year, and his expenses for the same are $1,095. What per cent. of his income does he save?

8. The assets of a firm are $20,280, and the liabilities are $24,000. What per cent. of its debts can the firm pay?

260. Illustration.—A reaper that cost $350 was sold at a profit of $105. Find the rate per cent. of gain.

Cost = $350
Gain = $105
$105 gained on $350 = $\frac{105}{350}$
$\frac{105}{350}$ of 100% = 30%

SOLUTION.—105 is $\frac{105}{350}$ of 350, and as 350 is 100% of itself, 105, which is $\frac{105}{350}$ of 350, must be $\frac{105}{350}$ of 100%, or 30%.

Find the rate per cent. of gain or loss when the cost is

9. $300 and the loss is $50
10. $500 and the gain is $75
11. $400 and the loss is $8
12. $750 and the gain is $150
13. $800 and the loss is $600
14. $450 and the gain is $2.25
15. $900 and the loss is $4.50
16. $800 and the gain is $2.00
17. $1,750 and the gain is $750
18. $1,000 and the loss is $250

19. What is the per cent. of profit on a piano which cost $350 and is sold for $420?

20. What is the rate per cent. of profit on merchandise which cost $573 and is sold for $647.49?

21. A stationer bought 9 reams of paper at $2.40 per ream, and retailed it at 1c. per sheet. What was his per cent. of gain?

22. A grain dealer bought 735 bus. of wheat at $1.14 per bushel, and sold the entire quantity for $977.55. What was his per cent. of gain?

23. A drover bought 15 horses at $125 per head, sold 2 of them at $127.75 per head, 8 at $140 per head, and the remainder

at $150 per head. If his expenses in taking them to market amounted to $5 per head, what was his per cent. of gain?

24. Oats purchased at 45⅝c. per bushel were sold for 54¾c. per bushel. What was the rate per cent. of gain?

25. A grocer bought 536 gal. of vinegar for $150.08, sold 175 gal. at 30 c. per gallon, 124 gal. at 35c. per gallon, and the remainder for $91.70. What was his rate per cent. of gain?

261. Illustration.—375 is 12½% of what number?

(*a*) 12½% of the number = 375
1% of the number = 30
100% of the number = 3000

SOLUTION (*a*).—If 12½% of a number is 375, 1% of the number is $\frac{2}{25}$ of 375, or 30, and 100%, or the whole of the number is 100 times 30, or 3000. Or,

(*b*) 12½%, or $\frac{1}{8}$, of a number = 375
$\frac{8}{8}$ = 8 times 375, or 3000

SOLUTION (*b*).—12½% of a number is $\frac{1}{8}$ of it. If $\frac{1}{8}$ of a number is 375, $\frac{8}{8}$, or the whole number is 8 times 375, or 3000. Therefore, the required result is 3000.

EXERCISE 81

1. A house rents for $360 per year, which amount represents 8% of its value. How much is the house worth?

2. A grocer sold 55 bus. of potatoes in 1 month, which amount represented 5½% of his stock of potatoes. How many bushels of potatoes had he at first?

3. From a cask of oil there leaked out 5.04 gal., which was 8% of what it originally contained. How many gallons were there in the first place, and how many gallons remained in the cask?

4. A farmer sold 17 bus. 2 pk. and 1 qt. of tomatoes, which were 3% of his whole stock of tomatoes before selling. How many bushels of tomatoes had he at first?

5. From his bank account a man checked out $75.50, which amount was 30% of his account. How much had he in the bank at first?

262. Illustration.—By selling a farm at a gain of 20%, I realized a profit of $850. Find the cost of farm.

SOLUTION

20% of cost = $850

1% of cost = $\frac{\$850}{20}$

100% of cost = $\frac{100}{20}$ of $850 = $4,250

6. A bill of goods was sold for $13 less than cost, by which a loss of 5% was sustained. What was the cost of the goods?

7. A gentleman gained 24% by selling land for $195.60 more than he paid for it. What did he obtain for the land?

8. What must have been the cost of a stock of goods if owner, by selling at a gain of 12½%, received $450 more than the cost?

9. By selling a lot of delaine for 37½% more than I paid for it, I gained $135. What did it cost me?

10. A merchant lost 5% by selling gingham at 9½c. a yard. Find the cost per yard.

Find cost—

	GAIN.	GAIN %.		LOSS.	LOSS %.
11.	$3.00	10 %	15.	$2.50	30%
12.	60c.	12½%	16.	$4.80	25%
13.	37½c.	16⅔%	17.	$1.20	8⅓%
14.	$5.60	40 %	18.	$3.00	6¼%

263. Illustration.—A horse and carriage which sell for $240 net a gain of 20%. Find the cost.

SOLUTION.— 100% = cost.
20% = gain.

120% = sale = $240.
1% = $240 ÷ 120 = $2.
100% = 100 × $2 = $200.

EXERCISE 82

1. A set of jewelry was sold for $126.30. What was the cost if the seller gained 25%?

2. A music dealer sold a piano for $413 and gained 18%. What was his profit?

3. A grocer sold a quantity of sugar for $330 and thereby lost 12%. How much did he lose?

4. A man sold two lots for $1,800 each, gaining 25% on one lot and losing 25% on the other. Did he gain or lose, and how much?

5. A merchant sold a bill of goods for $89.30, and thereby lost 6%. How much did he lose?

6. A house was sold for $5,616 at a gain of 8%. What was the gain?

7. A horse was sold for $753.75, which was $\frac{1}{2}$ of 1% more than it cost. Had it been sold for $700, what per cent. would have been gained or lost?

8. Sold goods for $4,026.75, at a loss of $3\frac{1}{4}$%. What would they have had to sell for to yield a profit of $3\frac{1}{4}$%?

EXERCISE 83

1. A grocer bought 10 bbls. of sugar, each weighing 330 lbs., at $4\frac{1}{2}$c. per pound, and sold them so as to gain $16\frac{2}{3}$%. Find the gain and the selling price.

2. A stock of goods, consisting of $25,000 worth of groceries, was sold at a loss of $12\frac{1}{2}$%, and 15% of the selling price was in uncollectible accounts. What was the total loss sustained?

3. A produce dealer paid $320 for apples, $90 for onions, and $120 for potatoes. He sold the apples at a gain of 25%, the onions at cost, and the potatoes at 95% of their cost. Did he gain or lose, and how much?

4. A man bought 3 horses, paying respectively $240, $300, and $500. He sold the first at 125% of its cost, the second at a loss of 10%, and the third at a gain of 15%. Did he gain or lose, and how much?

5. A dry-goods merchant bought a bill of goods amounting to $175. He sold $14\frac{2}{7}$% of the bill, and realized a gain equal to 50% of the cost of the whole bill. If the remainder of the stock was sold for $100, what was the gain or loss?

6. A merchant bought a stock of goods amounting to $9,500.00, and after disposing of a part of it for $8,000.00, he took account of the stock remaining unsold and found that at cost prices it was worth $3,000.00. What per cent. of gain of the sales is the gain?

7. A merchant bought goods and paid freight on them equal to 12% of their cost. He then sold them at 25% profit on the full cost of the goods, receiving 40% of the price in cash, the remainder to be paid in 30 days. If the amount of the cash was $1,792.00 what was the first cost of the goods?

8. I sold a house to B at $12\frac{1}{2}$% profit. B sold it to C, gaining

10%, and C, by selling it to D for $11,880.00, gained 20% on his purchase. How much did the house cost me?

9. I sold 2 watches at the same price. On one I gained 25%, and on the other I lost 25%. If my total loss was $5.00, what was the cost of each?

10. An article marked to gain 62½% is sold less 25% and 20%. If a collector was afterwards paid 20% for collecting the account, what was the gain or loss per cent?

11. If 25% of the selling price is gain, what is the gain per cent.?

12. Having paid a retailer $138.60 for a set of furniture, I ascertain that by selling to me he gained 12½%, that the wholesaler of whom he bought gained 10%, that the jobber by selling to the wholesaler gained 16⅔%, and that the manufacturer sold to the jobber at 20% above its first cost. How much more than its first cost did I pay?

13. A merchandise account shows that the cost of a stock of goods was $15,000, that the sales to date aggregate $12,000, and that the goods on hand, estimated at cost prices, amount to $4,500. Find the per cent. of gain or loss on the sales.

14. A merchant purchased a bill of goods as follows: 100 gross Faber pencils, $3.10 a gross. He paid freight and cartage amounting to $3.85, and duty amounting to 35% of the invoice price. Not considering fixed charges, what per cent. would he gain or lose by selling at 60c. a dozen?

COMMISSION AND BROKERAGE

264. Commission is a compensation allowed by one person, called the principal, to another, called the agent, for the transaction of business. It is usually a percentage of the money involved in the transaction.

Thus, in the purchase of goods, it is usually a percentage of the prime cost; in the sale of goods, a percentage of the gross selling price; in the collection of a debt, a percentage of the amount collected.

265. The person for whom business is transacted is called the **principal**, and the person authorized to transact business for

another, the **agent broker, commission merchant,** or **collector,** according to the nature of the business transacted.

266. Guaranty is a percentage charged by an agent for assuming the risk of loss from sales made by him on credit, or for giving pledge of the grade of goods bought.

267. Account Sales is an itemized statement rendered by an agent to his principal, showing in detail the sales of goods and charges thereon, together with the net proceeds remitted or credited.

268. Illustration.—An agent sold goods to the amount of $1,580. Find his commission at 2%.

$1580
.02
$31.60

SOLUTION.—The value of the sales is $1,580. The rate is .02. Then, $1,580 × .02 = $31.60, the commission.

269. Illustration.—What are the net proceeds on a sale of goods amounting to $200, at 3% commission?

$200
.97
1400
1800
$194.00

SOLUTION.—The sale is $200. The rate is .03. 1 − .03 = .97. Then $200 × .97 = $194, the net proceeds.

The commission may be found and then deducted from the value of the sales. It will give the same result as the method just explained.

EXERCISE 84

1. A real estate agent sold a farm of 90 ac. at $125 per acre on a commission of 2%. What was the amount of his commission? How much did he turn over to his principal?

2. An agent sold 450 bbls. of flour at $6.25 per barrel on a commission of $3\frac{1}{3}\%$. What was his commission?

3. A collector succeeded in collecting 80% of a doubtful account of $1,500. If he charged $7\frac{1}{2}\%$ commission, how much did he turn over to his principal?

4. My Montreal agent buys for me 4,500 bus. of wheat at $83\frac{1}{3}$c. per bushel. How much should I remit him to cover the cost of the wheat and his commission of 5%?

5. Rule a piece of paper, copy the following Account Sales, and make the necesary extensions, footings, etc.—

TORONTO, ONT., Aug. 3, 1943.

Sold for Account of S. L. BOWLING. By EDWARD HESS & Co.

1943								
July	6	150 bbls. "Snowflake" . . .	$5\frac{60}{}$	***				
"	11	80 " "New Carlisle" . .	$5\frac{25}{}$	***				
"	25	100 " "Snowflake" . . .	$6\frac{00}{}$	***				
Aug.	3	70 " "New Carlisle" . .	$5\frac{50}{}$	***		****		
		CHARGES						
July	5	Transportation, 400 bbls.	@ 31c. . .	***				
"	5	Cartage, 300 "	@ 6c. . .	**				
Aug.	3	Storage, 300 "	@ 3c. . .	*				
"	3	Commission and Guaranty, 4%	. .	**	**	***	**	
		Net Proceeds				****	**	

6. Account Sales of 5,000 bus. wheat by Rutherford & Marshall for account of the Produce and Provision Co., of Stratford; Jan. 1st, 2,000 bus. at $1.05; Jan. 20th, 2,000 bus. at $1.10; Jan. 30th 1,000 bus. at $1.02. Charges: cartage, $25; storage, $12.50; guaranty, ¼%; commission, 2%. Find the net proceeds.

EXERCISE 85

270. Illustration.—An agent received $50 commission for selling $2,500 worth of goods. What is his rate of commission?

SOLUTION

On $2,500, commission is $50

On $1, commission is $\frac{\$50}{2500}$

On $100, commission is $100 \times \frac{50}{2500}$

or $2

The commission is 2%.

271. NOTE.—An agent is paid for what he does. If he buys $500 worth of goods and ships them to his principal, his commission is on $500. Other costs, such as freight, may bring the cost to the principal up to, say, $600, but the agent gets his commission on $500. Again, an agent may sell goods for $500. His commission is on $500. The principal may receive only $400, after all expenses are deducted from the sales money, but the agent is paid on what he sold, $500 worth of goods.

What is the rate per cent. of commission if

1. The sales are $960 and the commission $15.60?
2. The cost is $3,264 and the commission for buying $3.06?
3. The sales are $3,200 and the commission for selling $6?
4. The cost is $460 and the commission for buying $23?
5. The first cost of the purchase is $275 and the gross cost $286?
6. The total sales are $105 and the net sales $102.90?
7. The total sales are $380 and the net sales $379.05?
8. The prime cost is $124 and the gross cost $128.34?
9. My agent sold queensware for me, and, retaining his commission, $90, remitted me $1,910. What per cent. commission did I allow him?
10. A commission merchant sold corn for $6,000; he charged $80 commission and $135 for guaranteeing payment. What was his per cent. of commission and of guaranty?
11. The total cost of a church was $12,646.40, which includes the architect's commission, $296.40. Find the per cent. of commission.
12. I sold real estate through an agent for $12,750; I received $765 commission and paid him $573.75. What was the per cent. of commission of each?
13. My commission for selling furniture was $591.50; the net proceeds, $16,308.50. What per cent. did I charge for my services?
14. I bought sugar for a grocer: the whole cost, including expenses, $69.45, and commission, $170.50, was $6,439.95. What per cent. commission did I get?

EXERCISE 86

272. Illustration.—An agent received \$170 for buying a house. If his commission was 5%, what was the value of the house?

SOLUTION 1

$$5\% \text{ of value} = \$170$$

$$1\% \text{ of value} = \frac{\$170}{5}$$

$$100\% \text{ of value} = \frac{100}{5} \times 170 = \$3{,}400$$

Or

.05) \$170.00
\$ 3,400

SOLUTION 2.—The commission is \$170. The rate .05. Then, \$170 ÷ .05 = \$3,400, value of house.

1. I paid a grain dealer $1\frac{1}{2}\%$ for buying corn for me at 62c. a bushel. If his commission amounted to \$83.70, how many bushels did he buy?

2. An agent charged \$433.60 for selling a consignment of canned fruit. If his rate of commission was $2\frac{1}{2}\%$, what were the net proceeds?

3. What must an agent's sales for one year aggregate in order that at 3% commission his yearly income may be \$2,700?

4. A mobile factor earned \$99.75 by selling cotton at $2\frac{3}{8}\%$ commission. How many bales, averaging 560 lbs., did he sell, the price being 15c. per pound?

5. An agent charged \$13.40 for selling a consignment of merchandise. What were the gross proceeds of the sale if his rate of commission for selling was $2\frac{1}{2}\%$.

6. I paid $\frac{3}{4}\%$ brokerage to a grain broker for effecting the sale of a quantity of wheat at \$1.15 per bushel. How many bushels of wheat were sold, if the total brokerage was \$63.48?

273. Illustration.—I sent my agent \$4,100 with instructions to deduct his commission of $2\frac{1}{2}\%$ and invest the balance in wheat. How much did he invest, and what was his commission?

274. SOLUTION

Represent the actual investment by 100%.
$2\frac{1}{2}\%$ = the charges for buying.
$100\% + 2\frac{1}{2}\% = 102\frac{1}{2}\%$, the cost of the investment to the principal.
\$4,100 = the cost of the investment to the principal.

Therefore, $102\frac{1}{2}\%$ of the cost = \$4,100.
1% = \$40.
100% = \$4,000, the actual investment in wheat.
\$4,100 − \$4,000 = \$100, the commission for buying.

EXERCISE 87

1. An agent received \$6,180 to buy cotton. After deducting his commission at 3%, how much did he invest in cotton?

2. How many yards of muslin at 5c. a yard can my agent buy with \$609 after deducting his commission at $1\frac{1}{2}\%$?

3. I bought coffee at $1\frac{7}{8}\%$ commission and charged $2\frac{1}{4}\%$ for guaranteeing payment. What did I pay for the coffee if the whole cost was \$1,832.60?

4. Having sent a Toronto agent \$1,835.46 to be invested in sugar, after allowing 3% on the investment for his commission, I received 32,400 lbs. of sugar. What price per pound did the sugar cost the agent?

5. An agent in Hamilton received \$828 to invest in prints, after deducting his commission of $3\frac{1}{2}\%$. If he paid $7\frac{1}{2}$c. per yard for the prints, how many yards did he buy?

6. A Stratford fruit dealer sent a Grimsby agent \$1,946.70, and instructed him to buy apples at \$1.40 per barrel. The agent charged 3% for buying, and shipped the purchase to his principal in six car loads of an equal number of barrels. How many barrels did each car contain?

275. Illustration.—What is the amount of sales when the net proceeds are \$975 and the commission $2\frac{1}{2}\%$?

SOLUTION.—Let 100% = sales
$2\frac{1}{2}\%$ = commission

Then $97\frac{1}{2}\%$ = net proceeds
$97\frac{1}{2}\%$ of sales = \$975
1% of sales = \$10
100% of sales = \$1,000

EXERCISE 88

1. The net proceeds of a consignment are $675.50, and the rate of commission 3½%. What is the amount of the sale?

2. An agent collected part of an account for me, and, after deducting his commission of 5%, he sent me $427.50. What per cent. of the original debt of $900 remains unpaid?

3. A farmer received from his city agent $496.80 as the net proceeds of a shipment of butter. If the agent's commission is 3%, and 5% charge is made for guaranty of quality to purchasers, how many pounds at 27c. per pound must have been sold, and how much commission was allowed?

4. Sold clothing at 1¾% commission and 2½% for guaranteeing payment. What did the clothing sell for if the net proceeds due the principal is $2,711.64?

5. What was received for a farm the net proceeds of which amounted to $6,993.82, after paying 1⅜% commission and $263.18 other charges?

EXERCISE 89

1. A house and lot were offered for sale at an advance of 50% above their cost, but the agent sold the house at 20% below the asking price, receiving $500.00 more than the cost. What was the owner's gain per cent., after allowing the agent 2% for his services?

2. A commission merchant sold fruit for $5,040.00 on a 4% commission, and on the instructions of the consignor invested the proceeds of the fruit in manufactured goods on a 5% commission. Find the value of the goods he bought for the consignor.

3. At what price shall an agent be ordered to buy potatoes at 2% commission that, after paying 7 cents a barrel for transportation, they can be sold at $1.76 a barrel and net 10% profit?

4. An agent receives a commission of 3% for buying, 2% for selling, and an additional commission of 2% for guaranteeing the quality of goods purchased. I make him a shipment of wheat, with instructions to sell and invest the proceeds in a shipment of cotton. What was the selling price of the wheat, and what was the cost of the cotton, his total commission and quaranty charges being $1,031.25?

5. A commission merchant bought for his principal 12,000 bushels of wheat at $1.20 per bushel, and immediately sold it at $1.30, investing the proceeds in the purchase of oats, in carload lots of 1,000 bushels each, at 35 cents a bushel, his commission being 2% for buying and 2½% for selling. Find the following: Number of carloads, total commission, and unexpended balance.

6. A merchant failed, with liabilities amounting to $50,000.00; his assets in merchandise are $20,000.00; in real estate, $8,000.00; in accounts receivable, $12,000.00. If the assignee received 10% of the assets for making a settlement, how many cents on the dollar will the creditors receive?

7. An attorney collected an account, and after deducting his commission of 7½% and exchange 75c. on the draft, he remitted his principal $548.70. What was the amount of the collection?

8. An agent receives a certain percentage of the gross rental for letting a house, and assumes responsibility for collecting the rent. On a house rented at $38.00 a month for three years, he fails to collect one month's rent, and in the three years his net commission is $37.24. Find the percentage allowed him.

COMMERCIAL ARITHMETIC

TEST PAPER—No. 8

PROFIT AND LOSS—COMMISSION—BROKERAGE

Your work for this paper is to be submitted to the instructor for correction and criticism.

1. X sells a house for $2,000, his profit being 25% on the selling price. What rate per cent was his profit on the cost? What was the cost?

2. I sent a Montreal agent $3,670.92 to be invested in sugar. After allowing 3% on the investment for his commission, I received 64,800 lb. of sugar. What price per pound did the sugar cost the agent?

3. A grocer buys 10 barrels of apples, each barrel containing $2\frac{1}{2}$ bus., at $2 a barrel. If the loss by decay amounts to 20%, at what price per peck must he retail them in order to clear 20% on cost?

4. A manufacturer sells to a wholesaler at 20% gain; the wholesaler to the retailer at 25% gain; and the retailer to the consumer at 60% gain. Find the cost to the manufacturer of an article for which the consumer pays $2.40 more than twice the cost to the manufacturer.

5. What price per pound must be asked for coffee costing 18c. per pound, in order that the seller may deduct 10% from the asking price for bad debts, allow $16\frac{2}{3}$% for loss in roasting, and still gain 20% on the cost?

6. An agent sold wheat on 5% commission and invested the proceeds in barley, at 75c. per bushel, on a commission of $5\frac{5}{9}$%. If

his total commission was $1,200, how many bushels of barley did he buy ?

7. An agent received $288 to invest in apples after deducting all expenses. How many barrels can he buy at $1.75 per barrel, his charges being 4% commission for buying, 3c. per barrel for drayage, 10c. per barrel for freight, $3.25 for cooperage, and $4.00 for advertising. What was the unexpended balance of the remittance ?

8. M sold a shipment of 10,094 lb. of butter at 20c. per lb., on a commission of 5%. With the proceeds, M bought Morse's 25c. tea, having first deducted his purchasing commission of 3%. How many pounds of tea did he buy ?

9. A speculator bought 2,000 barrels of flour and had it insured for 80% of its cost, at 3½%, paying a premium of $429. At what price per barrel must he sell the flour to make a profit of 10% on the total cost ?

10. An article is bought and disposed of to gain 5%. If it had been bought at 5% less and sold at $1 less he would have gained 10%. Find the cost price.

Just a Little—Day by Day

"There's no man who feels so sure of himself as the man who has mastered his work. There's no man who enjoys his holidays like the man who has worked himself to the limit. There's no man who can laugh like the man who has topped the hill and sees his way clear before him."

We know that this course you are taking eats into your leisure. We know that sometimes it is a physical as well as a mental strain to keep at it. We know that sometimes it may all seem to be of no use. But stay with it.

Just a little day by day, but make it a practice to get that little done regularly. We want you to know what a pleasure it is for us to see your lessons coming in and to see your record card growing in its lists of completed exercises and good percentages.

TORONTO STOCK EXCHANGE
MONTREAL STOCK EXCHANGE & CURB MARKET
Industrial Stocks
Commission Rates

On Shares Selling—	Per Share
Under 50 cents a share	3 cents
At 50 cents and under $1.00	5 cents
At $1.00 and under $2.00	7 cents
At $2.00 and under $3.00	8 cents
At $3.00 and under $4.00	9 cents
At $4.00 and under $5.00	10 cents
At $5.00 and under $15.00	15 cents
At $15.00 and under $25.00	20 cents
At $25.00 and under $50.00	25 cents
At $50.00 and under $75.00	30 cents
At $75.00 and under $100.00	35 cents
At $100.00 and under $125.00	50 cents
At $125.00 and over	½%
Each $100.00 Bond or Debenture	30 cents

Mining Stocks

Commission Rates

On Shares Selling—	
Under 5 cents per share	$1.50 per M.
At 5 cents and under 10 cents	3.00 per M.
	Per Share
At 10 cents and under 25 cents	½ cent
At 25 cents and under 50 cents	¾ cent
At 50 cents and under 75 cents	1 cent
At 75 cents and under $1.00	1½ cents
At $1.00 and under $2.00	2 cents
At $2.00 and under $3.00	3 cents
At $3.00 and under $4.00	4 cents
At $4.00 and under $5.00	5 cents
At $5.00 and under $10.00	10 cents
At $10.00 and under $15.00	15 cents
At $15.00 and under $25.00	20 cents
At $25.00 and under $50.00	25 cents
At $50.00 and under $75.00	30 cents
At $75.00 and under $100.00	35 cents
At $100.00 and under $125.00	50 cents
At $125.00 and over	½%

Minimum Commission

The minimum commission shall be not less than $2.00, except on transactions where the amount involved is less than $50.00, in which case the minimum commission shall be at the Member's discretion if the amount involved is less than $10.00 and shall be not less than $1.00 in other cases.

FEDERAL OR PROVINCIAL TAX

(Levied on the Seller)

Stocks selling at—	Per Share
Less than $1.00	1/16th of 1% of value
$ 1.00 to $ 5.00	¼ cent
5.01 to 25.00	1 cent
25.01 to 50.00	2 cents
50.01 to 75.00	3 cents
75.01 to 150.00	4 cents
Over $150.00	4 cents a share plus 1/16th of 1% of value above $150.00
Bonds or Debentures	3 cents per $100.00 par value

STOCKS AND BONDS

276. **Stocks** is a general term applied to shares in the capital stock of banks, insurance, railroad, and other incorporated or joint stock companies.

277. A **share** represents simply a certain component part of the capital stock. It is commonly $25, $50, or $100. The stock certificate represents the number of shares specified thereon.

278. The **Capital Stock** of a company is the sum of all the shares issued at their par value.

279. The **Common Stock** of a corporation is the stock which is ordinarily issued to the incorporators.

280. **Preferred Stock** is usually stock on which dividends are paid before any allowance is made for dividends on the common stock.

281. The **Par Value** of stocks is their face value; their market value is the sum at which they are quoted in the market. The Dominion Companies Act also permits the issuing of shares having "no par value"; that is, stock which has no nominal value, as is the case of a stock with a par value.

282. Stocks are above par or at a premium when they are worth more than their face value; below par or at a discount when they are worth less than their face value.

283. Stock quotations are published prices or rates per share that stocks sell for.

284. Thus, when stock is 3% above par it is quoted at 103; when it is 2% below par it is quoted at 98.

285. A **Dividend** is a pro rata division of profits among the stockholders of a company or corporation.

286. The income from stocks is in the nature of dividends, and is dependent upon the prosperity of the company or corporation. Dividends are declared at a certain per cent. on the par value of the capital stock of the company, either quarterly, semi-annually, or annually. The dividend on preferred stock is often at a different rate from that on common stock. The dividend on preferred stock is frequently a fixed rate per cent. per annum, and usually must be paid before any dividend may be declared on the common stock.

287. **Stock Brokers** are persons who act for others in buying and

selling stocks at a stock exchange. For this service they charge a commission, called **Brokerage**, based on the market value of the stocks dealt in.

288. The commission charged by brokers varies according to the market value of the stock bought or sold. See table of commission rates on page 143.

289. A Margin is a deposit made with a broker by a person who wishes to speculate in stocks, such deposit being used by the broker to protect himself against loss. The margin varies from 20% to 50% of the market value of the stock dealt in.

290. A wishes to speculate, and deposits with B, his broker, $1,000 as a margin, directing B to buy 50 shares of a stock quoted at 90. B would pay for the stock $4,500, $1,000 of which is the margin furnished by A; B furnishes $3,500 and charges the usual rate of interest on that sum for "carrying" the stock. In case the quoted value of the stock drops below 90, the margin must be made good by A's depositing an additional amount. If A fails to make good his margin B may sell the stock to protect himself from losing any of the money he has furnished.

Toronto Stock Exchange

Complete record of transactions

INDUSTRIAL

19— High	Low		Div. in $	Open	High	Low	Close	Net Chge.	Clos. Bid	Qtns. Ask	Sales
8½	5	Beatty Bros. "A"		5	5	5	5	Un.	†3	5	100
5	2	Beauharnois		4⅜	4⅜	4⅜	4⅜	+ ⅛	4⅜	4½	100
176½	165	Bell Telephone	8	175½	176	175½	176	+ ½	175	176	65
32½	25	Blue Ribbon pfd	2	31	31	31	31	+ 1/20	†31	32	10
12½	7¼	Brazilian		10½	10½	10½	10¼	+ ¼	10½	10⅝	2245
23⅛	19⅞	B.A. Oil	1	22½	22⅞	22½	22¾	+ ¼	22⅝	23	1385
17	14	Build. Prod. A	.70+.10*	16	16	16	16	Un.	†15¾	16	225
56	49	C. Br'd pf. B	2.50+.75*m*	50	50	50	50	Un.	†50	55	19
10¼	7	Canada Cement		8	8	8	8	Un.	†8½	9	50
101½	89½	do pfd.	(*c*) 2.50	93½	93½	93½	93½	− 1½	†93¾	95	20
35⅔	32	Canada Malting	1.50	35½	35⅔	35½	35⅔	+ ½	35	36	310
77	66	Canada Packers	3	73	75	73	75	+ 3	75	76	160
2⅝	1·70	Canada S.S. Lines		200	200	200	200	Un.	1.85	2½	100
10½	7⅞	do pfd.	.50*b*	9⅜	9⅜	9⅜	9⅜	+ ⅛	†9½	9¾	25
23	18	Can. Breweries pfd.	2	22	22	22	22	Un.	†22	22½	125
19	16½	Can. Can'rs new pfd.	1	17	17	17	17	Un.	†16½	17	20
8	6	do Conv. pfd.	.60	6	6	6	6	Un.	6	6½	100
34½	19½	Canadian Car pfd.	1.75	21	21½	21	21¼	− ¼	21	22	100
6⅝	3½	Canadian Pacific		4¼	4¼	4⅛	4¼	+ ⅛	4¼	4⅜	585
15	9	Can. Wallpaper 'A'	.50*b*	9	9	9	9	Un.	† —	14	20
13	9	do 'B'	.50*b*	9	9	9	9	Un.	† —	12	15
10	5	Cockshutt Plow		7⅛	7⅛	7⅛	7⅛	+ ⅛	†6⅞	7⅛	30

INDUSTRIAL (*continued*)

19— High	Low		Div. in $	Open	High	Low	Close	Net Chge.	Clos. Bid	Qtns Ask	Sales
17	14	Con. Bakeries	1.00+.15*	15¼	15⅓	15⅓	15⅝	+ ⅜	†15	15½	25
61⅓	37⅞	Con. Min. & Sm.		41	41	40⅝	40⅞	+ ¼	40⅓	40⅝	645
183	175	Consumers' Gas	10	182	182	182	182	− ⅜	†181	182	22
22	17	Cosmos Imp. Mills	1.00	17	18	17	18	− ⅜	†17	19	200
105	104	do pfd.	5	101	101	101	101	Un.	103	—	4
20⅜	15⅓	Dist. Corp. Seagrams	2	18⅞	19	18⅝	19	+ ⅓	18⅜	19	110
25	19	Dom. Foundries new	.65*b*	23	23	23	23	Un.	23	23⅓	120
12⅓	7⅓	Dom. S. & C. "B"		11⅞	12	11⅞	12	+ ⅛	11⅞	12	577
7⅞	4⅞	Dominion Stores		7¼	7⅜	7¼	7⅜	− ⅛	7⅓	7⅞	200
77	72	Dom. Tar pfd.	5.50	76	76	76	76	+ 4	†76	77	10
24	19	Fanny Farmer	1.50	23	23	23	23	+ ½	22½	23½	86
23½	16½	Ford Co. of C "A"	1	20	20½	20	20	Un.	†19⅞	20	57
16½	11	Gatineau Power	.40	14½	14½	14½	14½	Un.	14⅜	14½	77
94⅔	87	do pfd.	5	93	94⅔	93	94⅔	+ 2⅓	†93⅔	95	65
6	2½	do rights		4½	4½	4½	4½	Un.	†4½	5	5
7⅔	5	Gen. Steel Wares		5⅔	5⅔	5⅔	5⅔	Un.	†6	6⅔	100
78	66	Goodyear common	2.50	68	68	68	68	Un.	†67⅓	69	12
58	55	Goodyear Tire pfd.	2.50	58	58	58	58	+ 1	†57	58	60
17	10⅓	Gt. Lakes pfd.		13½	13½	13½	13½	+ ¼	13	13⅓	50
6½	3⅔	Gyp. L.-Al.		5⅜	5⅜	5½	5½	− ⅛	5⅓	5½	400
17½	14⅔	Imperial Oil	.50+.75*	16⅜	16½	15⅞	15⅞	− ⅛	15⅔	16	2353
56⅜	42⅔	Int. Nickel	(*c*) 2	49½	49⅔	49½	49⅔	− ⅛	49⅔	49⅔	1160
27½	22⅓	Int. Petrol'm.	1.50+.50*	25⅜	26	25⅜	26	+ ⅛	25⅜	26	1875
13⅓	10⅓	Laura Secord Candy	.80	12	12⅓	12	12⅛	− ⅛	12	12⅓	65
25	22	Loblaw "A"	1.00+.12½*	24	24	23⅞	23⅞	− ⅛	23⅞	24	90
23	21⅓	do "B"	1.00+.12½*	22	22	22	22	Un.	†21⅔	22	275
4⅔	2⅓	Maple Leaf Mill pfd.		3	3	3	3	Un.	3⅓	5	2
7⅓	4⅓	Massey-Harris		5⅓	5⅜	5⅓	5⅜	− ⅛	†5⅜	5⅓	160
60	29⅓	do pfd.		43⅜	45	42⅓	42⅓	+ 1⅓	42⅓	44	215
40	35	Moore Corp.	1.60+.40*	38	38	38	38	Un.	37⅓	38	60
25	23	Nat. Grocers new pf.	1.50	24⅓	24⅓	24⅓	24⅓	+ ⅓	†24	25	140
61⅓	43⅓	Nat. Steel Car	2	49	49⅓	48⅔	49⅓	+ ⅔	†49⅓	50	105
3.00	1.50	Orange Crush		3	3	3	3	Un.	†3	—	100
7⅜	4⅓	do pfd.	.70	6⅔	6⅔	6⅔	6⅔	Un.	6⅔	7⅓	5
104⅓	94	Page-Hersey	4	97	97	97	97	Un.	†96⅓	97	10
7⅔	4⅔	Pantepec Oil		6	6	6	6	− ⅜	—	—	50
12⅓	9	Power Corp.	.30*b*	9⅞	10⅓	9⅞	10⅓	+ ⅓	10⅜	11	65
28	22⅓	Riverside Silk 'A'	2	28	28	28	28	Un.	28	29⅓	115
90	78	Simpson's Ltd. pf.	5	85½	85½	85½	85½	+ 1⅛	84⅓	88	10
77⅔	66⅓	Steel of Canada	1.75+2*	75	75⅔	75	75⅔	+ ⅔	75⅔	76⅓	95
74	65	do pfd.	1.75	73⅓	73⅔	73⅓	73⅓	− ⅔	73⅓	74⅓	35
12⅓	11	Tamblyn	.80	12	12	12	12	− ¼	†11⅔	12⅓	25
16⅓	10	Toronto Elevators		12	12⅓	12	12⅓	+ ⅓	†12⅓	13⅓	25
14	11	Union Gas	.80+.20*	13⅔	13⅞	13⅔	13⅔	Un.	13⅓	14	825
5	2⅔	United Fuel 'B' pfd.		4	4	4	4	Un.	†4	4⅔	125
7	3⅓	United Steel		4⅓	4⅓	4⅓	4⅓	+ ¼	†4⅓	4⅔	595
51	38	Walkers	4	41	41	41	41	Un.	40⅓	41	825
20⅓	19	do pfd.	1	20	20	20	20	⌣ ⅜	20	20⅓	50
24	15	West. Can. Flour pf.		23	23	23	23	+ 3	†23⅓	35	10
12⅓	9⅔	Weston, new	.80	12	12	12	12	+ ⅔	11⅓	12⅓	100
2.00	1.50	Winnipeg Electric 'B'		1.50	1.50	1.50	1.50	Un.	†1.55	2.00	5

FINANCIAL INSTITUTIONS

310	300	Bank of Nova Scotia	12	300	300	300	300	Un.	†300	303	1
192	178	Royal Bank	8	185	186	185	186	− 1	†185	190	32

CURB

4	3	Bruck Silk		3	3	3	3	Un.	2⅞	3	290
7⅜	3⅓	Cons. Paper		4⅜	4⅓	4⅜	4⅓	+ ¼	4⅜	4⅜	385
37¼	23⅜	Dominion Bridge	1.20	30	30	30	30	+ 1½	†29	30	30
32	29¼	Montreal Power	1.50	32	32	31⅜	31⅜	− ¼	31⅜	32	92

References: (*m*) Paid on arrears. (*) Extra dividends paid in 19—. (*a*) Stock dividends. (†) Less than quotation lots. (*b*) Interim dividends paid in 19—. (*c*-1) Funding rights. (*c*) Ex-dividend yesterday.

MINING

19— High	Low			Div. in $	Open	High	Low	Close	Net Chge.	Clos. Bid	Qtns. Ask	Sales
.09½	.04½	Acme	(P)		7½	7½	7½	7½	Un.	7½	8	12) 0
.04	.02	Afton			2¼	2¼	2¼	2¼	Un.	2¼	2⅝	500
.52	.29⅜	Aldermac	(P)		37	37	37	37	− 1	35	37	1170
.17	.09½	Amm. Gold	(P)		10	10	10	10	Un.	10	10½	2000
1.52	.82	Anglo. Can. Oils	(P)		1.05	1.14	1.05	1.12	+.07	1.11	1.13	22600
.17½	.09⅜	Arntfield	(P)		14	14	14	14	Un.	14	14½	600
.72	.38	Augite			58½	61	58	61	+2½	60	61	64450
.38	.18	Bankfield	(P)		28	28	28	28	−	27	28	200†
.32	.11	B.E.A.R.			19	19	19	19	+2	17	20	2600
1.40	1.00	Beattie	(P)	.05*a*	1.26	1.27	1.26	1.27	+.01	1.26	1.27	1100
.30	.17	Bidgood-Kirkland	(P)		20½	22⅜	20½	22⅜	+2½	22½	22⅞	25332
.30	.15	Big Missouri	(P)		16½	16½	16½	16½	+ ½	16	16½	700
.22	.09	Bobjo			12	12	12	12	Un.	11	12	1500
11.75	9.55	Bralorne	(P)	.80+.20*	11⅝	11⅝	11⅝	11⅝	+ ½	11½	11⅝	565
.75	.28	Broulan			35	36	35	35½	−1½	36	37	4500
.33	.18	Brown Oil	(P)		20½	24	20½	24	+4	23½	24	12300
15.25	10.50	Buffalo Ankerite	(P)	1.00	12½	12⅝	12⅝	12⅝	Un.	12⅝	12½	575
.05	.02½	Buffalo Can.			3⅜	3⅜	3⅜	3⅜	Un.	3	3½	1000
2.80	1.81	Calgary & Ed	(P)		2.15	2.25	2.15	2.20	+.05	2.20	2.24	5050
.65	.30½	Calmont	(P)		36	39	36	38	+4	36½	38	4500
1.03	.69	Can. Malartic	(P)	.03*a*	82	82	82	82	− 3	78	82	800
2.75	2.05	Cent. Pat.	(P)	.16+.04*	2.55	2.58	2.55	2.58	+.03	2.58	2.59	2600
.12	.06	Central Porcupine			9⅛	9⅛	9	9	Un.	9	9½	2500
.70	.27½	Chem. Research	(P)		28½	32	28½	32	+4½	32	35	2650
1.39	.85	Chesterville			1.15	1.15	1.12	1.12	−.03	1.12	1.13	13250
.88	.50	Chromium	(P)		65	65	65	65	Un.	63	65	800
.36	.21⅝	Commonwealth	(P)	.02*z*	25	25	25	25	−	26	28	200†
1.74	1.20	Coniaurum	(P)	.05*a*	1.53	1.55	1.52	1.55	+.01	1.52	1.57	1100
.27	.14	Cons. Chibougamau			14	14	14	14	− 1	14	15	600
.60	.27	Davies Pete	(P)	.01*a*	30	35	30	35	+6	34½	35	8800
.16	.09½	Denison			12	12	12	12	+ ½	11	12	500
34.00	30½	Dome	(P)	2.00	32	32	32	32	+ ¼	31⅝	33½	220
.09½	.05	Dorval Siscoe			5	5½	5	5	Un.	5	6	2200
.27	.14	Duquesne			18	18½	18	18	−1½	17½	19	2000
.13	.06½	East Crest	(P)		8	8½	8	8	+ ⅜	8	8½	7500
2.79	2.07	East Malartic	(P)		2.67	2.70	2.65	2.70	+.03	2.70	2.71	9915
2.36	1.05	Eldorado	(P)		1.53	1.53	1.48	1.52	+.02	1.51	1.53	3385
6.00	4.50	Falconbridge	(P)	.30	5.25	5.25	5.25	5.25	Un.	5.25	5.40	300
.08½	.04	Federal Kirkland			4	4	4	4	Un.	4	4½	500
.17	.04	Fernland			4½	4½	4½	4½	+ ⅜	4½	5	1700
.13	.08	Firestone Pete	(P)	.00½*a*	7⅞	7⅞	7⅞	7⅞	Un.	8	8½	400
.16½	.09½	Found'n-Pete	(P)	.00½*a*	9⅜	10	9⅜	10	+1	10½	11	1500
.26	.15	Francoeur	(P)		23	26	23	26	+6	25	28	5300
.11⅝	.04	Gillies Lake			8⅝	9	8⅝	9	Un.	8⅝	9	2300
.40	.20	God's Lake	(P)		38	38	37	37	−1⅛	37	38	3800
.28	.15	Goldale		.02*a*	21	21	21	21	Un.	20	21½	1600
.13½	.05	Gold Eagle	(P)		7	7¼	7	7¼	+ ¼	6⅞	7⅞	2000
.03½	.02	Goodfish			2	2	2	2	Un.	1⅝	2	1000
.04	.02⅝	Graham-Bousquet			3	3	3	3	− ⅜	3	3¼	500
.13	.04½	Granada			5½	5½	5	5	Un.	5	6	3000
.07	.04½	Grandora			4⅝	4⅝	4½	4½	− ¼	4⅝	5⅞	1000
.64	.40	Gunnar Gold	(P)	.03*a*	51	52	50	52	Un.	51	53	800
.03½	.01	Halcrow			3	3½	3	3	+1	2⅜	3	8000
.06	.02½	Halliwell			4½	6	4½	5⅜	+1⅜	6	7	14430
1.95	.94	Hardrock	(P)	.03*a*	1.20	1.21	1.18	1.18	−.02	1.17	1.19	7400
.10	.06	Harker G.M.			9½	9⅝	9	9	Un.	9	9⅛	11000
.35	.15	Highwood-Sar.	(P)		19	21	19	20	+4	20	21	4000
15.00	13.50	Holl. Cons.	(P)	.65+.10*	14⅝	14⅞	14⅝	14⅝	Un.	14½	14⅞	760
3.75	1.98	Home Oil	(P)		2.20	2.43	2.20	2.43	+.23	2.43	2.44	27200
.26½	.08	Homestead	(P)		10	13	10	13	+5	12½	13	18260
.36½	.24	Howey	(P)	.03*a*	34⅜	36½	34⅜	35	+ ⅛	34½	35	23150
35.25	25.50	Hudson Bay	(P)	.75*a*	32½	32½	32	32	−	32⅜	32½	120†
1.26	1.18	Island Mt.	(P)	.08*a*	1.20	1.20	1.20	1.20	Un.	1.18	1.23	100
.45	.22	Inspiration			35	35	35	35	+5	26	35	500
.34	.16⅛	Jack Waite	(P)		19½	19½	19½	19½	−	20½	23	200†
.11½	.03	Jacola			5½	6¼	5½	6	+ ¼	5½	6⅜	10000
.13⅜	.05	Jellicoe			7	9	7	9	+2	8½	9	11500
.208	1.47	Kerr Addison	(P)		1.85	1.92	1.83	1.90	+.05	1.91	1.92	20710

MINING (*continued*)

19— High	Low			Div. in $	Open	High	Low	Close	Net Chge.	Clos. Bid	Qtns. Ask	Sales
.73	.30	Kirkland-Hudson			35	35	35	35	—	31	35	100†
1.75	1.15	Kirkland L.	(P)	.08+.01*	1.55	1.58	1.53	1.58	+.01	1.58	1.59	9700
.08½	.07	Laguna		(P)	8½	8½	8½	8½	Un.	8½	9	800
50.00	32.75	Lake Shore		(P) 4.00	41½	41½	41½	41½	+ ½	41½	41½	400
6.85	5.50	Lamaque	(P)	.40+.05*	6.35	6.60	6.35	6.60	+.35	6.50	6.60	1214
.54	.25	Lapa Cadillac		(P)	28½	28½	28½	28½	− ½	28½	29½	2100
.08½	.03½	Lebel Oro		(P)	4½	5½	4½	5½	+ ½	5	5½	16500
.87	.65	Leitch Gold		(P) .04*a*	82	87	82	87	+2	86	87	8800
3.60	2.60	Little Long Lac		(P) .10*a*	3.15	3.15	3.10	3.10	−.10	3.10	3.15	400
5.90	3.85	Macassa	(P)	*c* .20+.02½*	4.80	4.80	4.70	4.75	−.15	4.70	4.80	2640
3.00	1.70	Mac-L-Cockshutt		(P)	2.30	2.30	2.20	2.22	−.05	2.23	2.25	3300
.55	.30	Madsen R. Lake		(P)	45	45	43	43	−1	43	44	2300
.75	.33	Malartic G. Fields			75	75	72	72	−3	71	72	9200
.11½	.00½	Man. & Eastern			1½	1½	1½	1½	Un.	1⅜	1⅝	1500
.24	.12	McDougal Segar		(P) .01*a*	10	10	10	10	−	—	16	50†
59.00	49.75	McIntyre		(P) 2.00	57½	58	57½	58	Un.	58	59	610
1.38	1.03	McKenzie Red L.		(P) .12	1.35	1.38	1.35	1.36	+.01	1.36	1.37	7800
.20	.06	McVittie			19½	19½	18	19	− ½	18½	19	2300
.75	.38	McWatters		(P)	50	53	50	53	+4	50	53	2700
2.05	1.10	Mining Corp.			1.39	1.39	1.39	1.39	+.04	1.33	1.40	275
1.45	.89	Moneta		(P) .06*a*	1.21	1.22	1.18	1.21	Un.	1.21	1.23	3300
.20	.08	Morris Kirkland			9⅜	9⅜	8	8½	−1½	8⅜	9	1100
.02½	.01½	Murphy			1½	1½	1½	1½	Un.	1½	1⅝	1000
.57½	.16	Naybob		(P)	20	20	19	19	−1	18⅜	19	6000
.04	.02½	Newbec			3½	4	3½	4	+ ½	3⅞	4	4200
82.50	70.00	Noranda		(P) 1.00*a*	79½	80	79½	80	−	79¾	80	85†
.08	.06	Nordon Oil		(P)	6½	6½	6½	6½	+ ½	6	6½	500
.75	.38	Normetals		(P)	40	47	40	47	−	45	50	120†
3.75	2.01	O'Brien		(P) .10*a*	2.60	2.60	2.50	2.50	−.10	2.49	2.54	1900
1.13	.96	Okalta Oils		(P)	1.11	1.25	1.11	1.25	+.14	1.24	1.25	8550
.02½	.01	Olga Gas & Oil		(P)	1½	1½	1½	1½	+ ⅛	1	1½	1000
.53	.25	Omega		(P)	35	35	35	35	−2	34	40	1200
.52	.23	Oro Plata			36	36	36	36	Un.	36	38	1200
.12	.05	Pacalta		(P)	5½	6⅜	5½	6½	+1⅜	6½	6½	8000
4.75	2.76	Pamour		(P) .08*a*	3.10	3.10	3.10	3.10	Un.	3.05	3.10	750
.07½	.03½	Partanen Malartic			5½	5½	5½	5½	+½	5	5½	500
.07½	.02½	Paulore			4½	4½	4½	4½	+½	4	4½	1200
.61	.35	Paymaster		(P) .01*a*	47	50	47	48	+2	49	49⅜	24000
1.94	1.45	Perron		(P) .16	1.85	1.88	1.85	1.88	+.03	1.82	1.88	667
5.60	4.50	Pickle Crow		(P) .40	4.90	4.90	4.85	4.85	Un.	4.85	4.90	3975
2.70	2.25	Pioneer Gold		(P) (*c*) .40	2.50	2.50	2.50	2.50	−.03	2.46	2.50	175
2.45	1.18	Powell-Rouyn		(P)	1.88	1.89	1.88	1.89	−.05	1.86	1.89	300
2.40	1.80	Premier Gold		.12	1.91	1.95	1.91	1.95	Un.	1.95	1.97	610
1.75	1.17	Preston E. Dome		(P)	1.63	1.65	1.60	1.61	−.02	1.60	1.61	10740
.56	.20	Reno Gold		.01⅛	51	51	51	51	−1½	51	52½	2500
.11½	.05½	Roche Long Lac			8	8½	8	8½	+ ½	8	8½	1600
44.25	32.00	Royalite	(P)	1.00+.25*	37½	38	37½	37½	+1	37	38	280
1.70	1.18	San Antonio G.		(P) .07*a*	1.58	1.62	1.58	1.61	+.01	1.61	1.62	2041
.17	.10	Sand River		(P)	10⅞	12	10⅞	12	+1½	11⅞	12	4300
.04⅜	.02	Shawkey			2	2	2	2	Un.	2	2⅜	1000
1.14	.92	Sheep Crk	(P)	.12+.02*	1.10	1.14	1.10	1.14	+.02	1.14	1.15	2500
1.43	.92	Sherritt-Gordon		(P)	1.10	1.10	1.08	1.10	Un.	1.10	1.11	5100
1.65	.95	Siscoe		(P) .12	1.38	1.38	1.32	1.34	−.04	1.32	1.35	2050
.80	.41	Sladen Malartic		(P)	49	49	49	49	−1	49	49¾	1750
.13	.05½	Slave Lake			6½	6½	6½	6½	+1	6½	7	500
1.03	.41	Stadacona		(P)	56	57	55½	57	+1	56	57	11101
.15½	.08	St. Anthony		(P)	11	11½	11	11½	+ ½	11½	12	1500
.11	.04½	Straw Lake		(P)	6	6	6	6	Un.	5⅝	6	3400
24⅜	12	Sturgeon River		(P)	16	16	16	16	+1	15½	16½	1500
.14⅜	.07	Sudbury Contact			9	9	9	9	+1	8½	9⅜	1500
1.01	.77	Sullivan		(P) .03*a*	87	88	87	87	−1	85½	88	1100
3.55	2.78	Sylvanite	(P)	.20+.05*	3.35	3.35	3.30	3.30	Un.	3.30	3.35	500
7.20	5.50	Sigma			6.60	6.75	6.60	6.75	+.15	6.65	6.85	515
4.70	3.80	Teck-Hughes		(P) .40	4.25	4.25	4.25	4.25	+.05	4.20	4.25	1115
1.30	.65	Texas Canadian		(P) .12	91	91	90	90	−	90	97	200†
2.30	1.74	Toburns	(P)	.08+.04	1.85	1.85	1.85	1.85	−.10	1.80	1.97	100
1.65	1.03	Uchi Gold			1.44	1.45	1.40	1.41	−.01	1.42	1.44	6000
.15⅜	.07	United Oils		(P)	9	9	9	9	+1½	8	9⅜	2000

MINING (*continued*)

19— High	19— Low		Div. in $	Open	High	Low	Close	Net Chge.	Clos. Bid	Qtns. Ask	Sales
5.80	4.30	Ventures	.10*a*	5.10	5.15	5.10	5.10	+.05	5.00	5.25	605
8.25	5.50	Waite-Amulet (P)		6.80	7.05	6.80	7.00	+.20	7.00	7.05	3268
.08½	.03½	Wiltsey Coughlan		4	4½	4	4½	+ ½	4½	5	5000
.18½	.09	Wood Cadillac		14	14	14	14	Un.	13½	14½	700
8.90	7.30	Wr-H. (P)	.40+.15*+.10*a*	8.10	8.15	8.10	8.15	+.05	8.15	8.20	2860
			CURB								
.02½	.01½	Brett-Trethewey		1⅝	1⅝	1⅝	1⅝	Un.	1⅝	1¾	1000
.75	.38	Dalhousie Oil (P)		40	45	40	45	+5	40	45	3240
1.45	.55	Foothills (P)		65	65	65	65	+5	75	—	1700
.17	.10	Mandy		16	16	16	16	—	15	16	300†
.03½	.01½	Oil Selections (P)		1½	1½	1½	1½	Un.	1¼	2	1000
1.94	1.28	Pend Oreille (P)		1.48	1.50	1.47	1.47	−.05	1.48	1.51	950
.01½	.00½	Robb. Mont.		1	1	1	1	+ ⅛	⅛	1	5000
.14½	.06½	Temiskaming		8	8	8	8	—	8	9	100†

Total sales, 702,000 shares. (P) Producing mine. (†) Less than quotation lots. (*a*) Interim dividends paid in 19—. (*) Extra dividends paid in 19—. (*c*) Ex-dividend yesterday. (*z*) Stock dividend.

CANADIAN BONDS

(By the Investment Dealers' Association of Canada)

DOMINION OF CANADA

	Bid	Asked
Dom. 4½ 1 Sep. 1940	103.30	104.30
Dom. 5 15 Nov. 1941	107.125	108.125
Dom. 3 15 Oct. 1942	103.375	104.375
Dom. 5 15 Oct. 1943	111.25	112.25
Dom. 4½ 15 Oct. 1944	111.125	112.125
Dom. 4 15 Oct. 1943–45	107.125	108.125
Dom. 4½ 1 Feb. 1946	111.875	112.875
Dom. 3¼ 1 Jun. 1946–49	103.50	104.50
Dom. 3½ 15 Oct. 1944–49	104.00	105.00
Dom. 3¼ 15 Nov. 1948–51	102.75	103.75
Dom. 4 15 Oct. 1947–52	107.75	108.75
Dom. 3 1 Jun. 1950–55	99.625	100.625
Dom. 4½ 1 Nov. 1946–56	110.625	111.625
Dom. 4½ 1 Nov. 1947–57	111.00	112.00
Dom. 4½ 1 Nov. 1948–58	111.75	112.75
Dom. 4½ 1 Nov. 1949–59	112·75	113.75
Dom. 3 1 Jun. 1953–58	98.75	99.75
Dom. 3¼ 1 Jun. 1956–66	101·50	102·50
Dom. 3 15 Jan. 1967*w*	102.25	103.25
Dom. 3 Perpetuals	94.25	95.25

DOMINION GOVT. GUARANTEED

C.N.R. 3 1 May 1944	103.50	105.00
C.N.R. 3 15 Dec. 1950	99.75	101.25
*x*C.N.R. 4½ 1 Sep. 1951	114.75	116.25
C.N.R. 5 1 Feb. 1954	118.75	120.25
*x*C.N.R. 4¾ 15 Jun. 1955	117.50	119.00
*x*C.N.R. 4½ 1 Feb. 1956	115.75	117.25
*y*C.N.R. 4½ 1 Jul. 1957	116.00	117.50

xC.N.R. 5 1 Jul. 1969	115.50	117.00
xC.N.R. 5 1 Oct. 1969	119.00	120.50
xG.T.P. 3 1 Jan. 1962	99.75	101.25
xG.T.P. 4 1 Jan. 1962	109.50	111.50

PROVINCIAL AND PROVINCIAL GUARANTEED

Ontario 5½ 1 Jul. 1946	114.50	116.50
Ontario 5 15 Oct. 1948	114.00	116.00
Ontario 4½ 1 Jun. 1950	111.50	113.50
Ontario 3 1 Dec. 1951	99.00	101.00
xOntario 5 1 May 1959	121.50	124.00
xOnt. Hydro 4 1 Aug. 1957	108.50	110.50
Ont. Hydro 4¾ 1 Jan. 1970	117.00	120.00
zN Bruns 5 2 Jul. 1963	109.00	111.00
N Bruns 5½ 1 Jan. 1952	112.00	114.00
N Scotia 4½ 15 Sep. 47–52	106.50	108.50
Quebec 4¼ 1 Aug. 1958	105.50	107.50
Br Col 5½ 1 Feb. 1945	104.00	106.00
Br Col 6 15 Feb. 1947	105.00	107.50
Br Col 4½ 1 Oct. 1953	100.00	102.00
Manitoba 6 1 Apr. 1947	96.50	99.00
Manitoba 5½ 1 Jul. 1958	93.00	96.00
Sask. 6 15 Mar. 1952	78.00	82.00
Sask. 5½ 1 Oct. 1952	74.00	78.00
Sask. 5 1 Aug. 1958	72.00	75.00

w—Payable New York only.
x—Payable London, N.Y., or Canada.
y—Payable New York or Canada.
z—Payable Canada or London.

CORPORATION

(Supplied by Investment Dealers' Association of Canada)

	Bid	Asked
xAbitibi P & P 5s, 1953	49½	52½
Algoma Steel 5s, 1948	100	103
xAssoc Tel & Tel 5¼s, 1955	82½	85½
Beauh Pwr 5s, 1978	100½	103½
xBr Col Pr 4¼s, 1960	100	103
B.C. Telephone 4½s, 1961	107½	110½
xBrown Co. 5½s, 1946	35½	38½
xCalgary Pwr 5s, 1960	104	107
Can Canners 4s, 1951	101½	104½
Can Cement 4¼s, 1951	104	107
xCan Int Ppr 6s, 1949	96½	99½
xCan Nor. Pwr 5s, 1953	104	107
C.P.R. 4½s, 1944	91	94
C.P.R. 3s, 1945	81½	84½
C.P.R. 4s, 1949	84½	87½
C.P.R. 4½s, 1951	80	83
Can S.S. Lines 5s, 1957	86½	89½
xCan Vickers 6s, 1947	83	86
xCons Paper 5½s, 1961	38	41
Dom Gas & El 6½s, 1945	95½	98½

Dom Stl & Cl 6½s, 1955	103½	106½
Dom Tar & Chem 4½s, 1951	101½	104½
Famous Players 4½s, 1951	102	105
Gatineau Pr 5s, 49 ex wts	104	107
Gatineau Pr 3¾s, 1969 B	99	102
Gen Steel Wrs 4½s, 1952	96½	99½
⁎Gr Lakes Ppr 5s, 1955	82½	85½
⁎Gr Lakes Pr 4½s, 1956	100	103
⁎Gypsum, L & Al 5½s, 1948	99	102
Ham By-Prod Coke 5s, 1955	100½	103½
⁎Inter Hydro-El 6s, 1944	85½	88½
⁎Inter Pwr 6½s, 1957	97½	100½
⁎Kingston Elev 6s, 1950	102	105
L St John P & P 5½s, 1961	73	—
Maclyaren-Que Pr 4s, 1959	97½	100½
Massey-H. Co. 4½s, 1954	97½	100½
Massey-H. Co. 4½s, 1954	91½	94½
⁎Minn & Ont Ppr 6s, 1945	30	33
Montreal LH & P 3½s, 1963	101	104
⁎Montreal Trams 5s, 1941	95½	98½
⁎McColl-Fron Oil 4¾s, 1949	101½	104½
⁎McCormick's Ltd. 4s, 1952	87½	90½
⁎N Scotia Stl & Cl 3½s, 1963	81½	84½
Ottawa Valley Pow 5s, 1959	103½	106½
⁎Power Corp 4½s, 1959	104	107
⁎Price Bros. & Co. 5s, 1957	90	93
⁎Preston E. Dome 6s, 1943	101½	104½
⁎Quebec Power 4s, 1962	103½	106½
⁎Restigouche Ltd 6s, 1948	88	91
Saguenay Pow 4½s, 1966 B	105½	108½
Shawgn W & P 4s, 1961	104	107
Simpson's Ltd 4½s, 1951	101½	104½
Smith, Howard 4½s, 1951	103½	106½
Uchi Gold Mines 6s, 1941	95	98
Union Gas 4½s, 1950	102½	105½
⁎United Gas & Fuel 5½s, 1948	102½	105½
Winnipeg Elec A 4s, 1965	72	75
Winnipeg Elec B 4s, 1965	53	56

⁎—Payable New York funds.

291. Illustration.—What income will be derived from 60 shares of Imperial Oil, Ltd., paying an annual dividend of 75c.?

SOLUTION

1 share of stock yields 75c.
60 shares of stock yield $60 \times .75 = \$45.00$.

292. If the shares of stock mentioned in No. 291 were purchased at \$15½ plus 12½c. per share for brokerage, what would be the yield per cent. to the purchaser?

SOLUTION

1 share Imperial Oil, Ltd., costs \$15.50 plus brokerage, $12\frac{1}{2}$c. = \$15.62$\frac{1}{2}$

∴ \$15.62$\frac{1}{2}$ invested yields an income of 75 cents

\$100.00 invested yields an income of $\frac{100.00}{15.625} \times .75 = 4.8\%$

EXERCISE 90

(For Brokerage Rates—Buying or Selling—turn to pages 143–4.)

1. What dividend would Brown receive, who purchased 50 shares of Loblaw "A" stock at \$22.50 a share, plus brokerage, if the dividend paid is \$1.00 per share annually? What per cent. would he receive on his investment (correct to 3 decimal places)?

2. What income would be derived from the purchase of 100 shares of Lake Shore Mines, Ltd. at \$27.50, plus brokerage, if the annual dividend is \$3.75? What per cent. would be earned on the investment (correct to 3 decimal places)?

3. Williams bought:

25 shares of Imperial Oil, Ltd., at \$15.00 a share, plus brokerage. The annual dividend amounts to \$1.00.

40 shares International Petroleum Co., Ltd., at \$23$\frac{7}{8}$, plus brokerage, paying an annual dividend of \$1.75.

20 shares Steel Co. of Canada, Ltd., Preferred, at \$80, plus brokerage, paying an annual dividend of \$1.75.

Find the total investment and average percentage on this investment (correct to 3 decimal places).

4. White owned 100 shares of International Nickel, purchased at \$44.50, plus brokerage, which paid him \$2.00 per share annually. He decided to dispose of this stock at \$45.75 less brokerage, and purchase an even number of shares of Hudson Bay at \$31.25, plus brokerage. This stock will pay him \$1.75 per share annually. How many shares of Hudson Bay did he buy? What amount of money was unexpended and what change in his income resulted?

5. Miller purchased \$10,000.00 worth of Dominion of Canada 4$\frac{1}{2}$%'s, 1958, at \$109$\frac{1}{4}$, plus brokerage. What was his outlay; his annual income; and what per cent. did his investment pay him?

293. Illustration.—What is the rate per cent. dividend when 40 shares of stock yield an income of $240?

SOLUTION 1

40 shares = $4,000 par
(1 share = $100 par)
On $4,000 the dividend is $240
On $1 the dividend is $\frac{\$240}{4000}$
On $100 the dividend is $\frac{100 \times 240}{4000}$
= $6
The rate is 6%

SOLUTION 2

40 shares yield $240
1 share yields $\frac{240}{40}$
= $6
The rate is 6%

EXERCISE 91

What is the rate per cent. of dividend when

	SHARES.		INCOME.		STOCK.		INCOME.
1.	50	yield	$275	6.	$4,500	yield	$135
2.	60	"	$300	7.	$7,550	"	$453
3.	90	"	$390	8.	$8,600	"	$301
4.	75	"	$450	9.	$3,275	"	$131
5.	34	"	$170	10.	$4,125	"	$330

EXERCISE 92

294. Illustration.—What amount of stock par value $100 must be held to obtain $200 income from a 4% dividend?

SOLUTION

$4 income is derived from 1 share.
∴ $200 income is derived from 200 ÷ 4 = 50 shares.
50 shares = 50 × 100 = $5,000 stock.

1. What is the par value of stock if the dividend is $170.50, and the rate of dividend 5½%?

295. Illustration.—What sum must be invested in 5% bonds, at 105, to yield an annual income of $1,250?

SOLUTION

The dividend from 1 share = $5.
$1,250 ÷ $5 = 250 = number of shares.
The cost of 1 share = $105.
The cost of 250 shares = $105 × 250, or $26,250.

2. What sum must be invested at 93 to produce an income of $1,600, the rate of dividend being 8%?

3. How much must be invested in 8% stock, at 162, to secure an annual income of $1,280?

4. If 10% mining stock is at 87½, what sum must be invested in it to realize an annual income of $1,500?

5. If Ontario Express Co. stock pays 4% semi-annual dividends, and is quoted at 150¼, how much money must be invested in it to produce an annual income of $3,200?

296. Illustration.—How many shares Bank of Commerce stock selling at 21% premium can be bought for $7,275, brokerage 25c. a share?

SOLUTION

1 share at par	= $100
Premium	= 21
Market value	= 121
Brokerage	= ·25
Cost to purchaser	= 121·25

$7,275 ÷ $121¼ = 60

There are 60 shares, or $6,000 par of stock.

EXERCISE 93

How many shares may be bought for

	COST.	MAR. VAL.	BROK.		COST.	MAR. VAL.	BROK.
1.	$13,155	225	25c.	5.	$1,923	80	12½c.
2.	$9,760	121¾	25c.	6.	$3,850	96	25c.
3.	$5,610	140	25c.	7.	$12,025	240	50c.
4.	$13,620	85	12½c.	8.	$4,134	86	12½c.

297. Illustration.—How many shares Bank of Commerce stock at 121 must I sell to realize $7,245, brokerage 25c. a share?

SOLUTION

Market value	= $121
Brokerage	= ·25
Amount received from 1 share	= $120·75

$7,245 ÷ $120¾ = 60

60 shares are sold.

How many shares must be sold to realize

	S. P.	MAR. VAL.	BROK.		S. P.	MAR. VAL.	BROK.
9.	$8,505	121¾	25c.	13.	$19,755	220	50c.
10.	$10,245	85½	12½c.	14.	$2,400	96¼	25c.
11.	$4,314	90	12½c.	15.	$8,336	130½	25c.
12.	$4,350	87⅛	12½c.	16.	$10,548	110	12½c

EXERCISE 94

298. Illustration.—At what price must 5% stock be purchased to realize an income of 6% on the investment?

SOLUTION

\$6 is the income on an investment of \$100,
\$1 is the income on an investment of $\frac{1}{6}$ of \$100,
\$5 is the income on an investment of $\frac{5}{6}$ of \$100 = \$83$\frac{1}{3}$.

Or

Income per share (\$5) ÷ rate of income on investment (6%) = investment per share (\$83$\frac{1}{3}$).

At what price must:

1. 8% stock be bought to yield 6% on the investment?
2. 6% stock be bought to secure a yield of 7% on the investment?
3. 7% stock be bought to yield 8% on the investment?
4. 6% stock be bought to realize an income of 5%, if the brokerage for buying is 20c. a share?
5. 7% stock be bought to realize 8%, if the brokerage is 25c. a share?

EXERCISE 95

299. Illustration.—What % on the investment will be realized, if a preferred stock, paying a dividend of 8%, is bought at \$120 a share, brokerage for buying being 30c. a share?

SOLUTION

Market value = \$120.00
Brokerage = .30

Total cost of 1 share = \$120.30

An investment of \$120.30 yields an income of \$8.00

An investment of \$1.00 yields an income of $\frac{8.00}{120.30}$

An investment of \$100.00 yields an income of $\frac{100.00 \times 8.00}{120.30} = 6.65\ \%$.

What % on the investment will be realized:

1. By purchasing 4% stock at \$79$\frac{3}{4}$, brokerage 35c. a share?
2. By purchasing 6% stock at \$89, brokerage 35c. a share?

3. By purchasing 8% stock at $120, brokerage 50c. a share?

4. By purchasing 7% stock at $88, brokerage 35c. a share?

5. By purchasing 2% stock at $26.15, brokerage 25c. a share?

6. An investor bought sufficient 5% government bonds at $96.75, including brokerage, to secure an annual income of $375 a year. What was the amount of his investment?

7. I bought 60 shares of C.P.R. stock at 14\frac{3}{8}$; 75 shares of Dominion Stores at 16\frac{3}{4}$; and 20 shares of Photo Engravers at 15\frac{1}{8}$. Find the total amount invested, if the brokerage for buying was 20c. a share.

8. A man drew a quarterly dividend of $68.75 on a stock paying $5 a share annually, and afterwards sold the stock at $75.25 a share. What were the proceeds of the sale, if the broker's commission was 30c. a share for selling?

9. I sold 175 shares of Noranda at $16 a share, and with the proceeds bought Teck-Hughes at $3.85. How many shares of Teck-Hughes did I receive, brokerage for selling being 20c. a share, and for buying 4c. a share?

10. Which is the better investment, and how much: a 7% stock at $110 a share, or a 5% stock at $75 a share, not considering brokerage in either case?

11. A man bought 82 shares of Bank of Montreal stock at $205 and sold it at a net profit of $266.50. At what quotation were they sold, brokerage each way being 50c. a share?

12. A capitalist drew a quarterly dividend of 3% on his bank stock, and received $120. He afterwards sold 25 shares of that stock at $176, and the remainder at $174.50, paying a broker 50c. a share for selling. What were the net proceeds of the sale?

13. A broker, on his own account, bought 75 shares of a 6% stock at $105 a share. He afterwards sold this stock at $108 a share, and re-invested the proceeds in a 5% stock at $81. How much was his annual income increased? (No brokerage charges considered.)

14. A merchant retired from business with a capital of $21,417, and with it purchased $8,000 par of 5% bonds at $104.75, $5,000 par of 6% bonds at $96.50, and invested the remainder in manufacturing stock at 55\frac{1}{4}$, paying dividends of 4%. What was his

annual income, the broker's commission being 30c. for each $100 bond, and 30c. for each share of stock bought?

15. A man owns a house which he has rented at $55 a month. His annual expenses for taxes, insurance, upkeep, etc., are $275 a year. He sells the house for $6,600, and invests the proceeds in Goodyear Tire stock at $59.75 a share, the broker's charge being 25c. a share for buying. If the stock pays a dividend of $4 a share annually, what will be the difference in his yearly income?

ACCOUNTS WITH STOCK BROKERS

300. The adjustment of accounts with stock brokers gives us a familiar example of the application of the principles of cash balance.

301. Stock is either bought outright or on margin.

302. Where stock is bought outright, through a broker, it is expected that the buyer shall furnish the broker with sufficient money to buy the stock required at the market price. Thus, if it is desired to buy outright 50 shares of C.P.R., selling at $5.00, the buyer must put into the hands of his broker 50 times $5.00, or $250.00, plus the broker's charge for his services.

303. Buying on margin is simply a form of speculation. The buyer orders his broker to buy, but does not intend to put up the purchase price. Instead, he puts up a margin or a percentage of the par value of the stock bought. This varies from 20% to 50% of the par value. The broker then buys the stock with his own funds.

If the stock, after purchase, starts to decline in value, the broker has the margin amount to protect himself. If the speculator does not keep his margin good, the broker will sell the stock to protect himself. This he will always do before the decline has become sufficient to wipe out the margin.

If the stock starts to go up in value, the broker awaits the speculator's order in the matter of selling, and when the sale is ordered, the speculator expects to get back his margin plus his profits.

304. Illustration 1.—On Jan. 2nd J. Walters deposited with his broker $500 as margin for the purchase of 50 shares of Halifax R.R. stock at 92¼. The stock was sold Jan. 26th at 96¾. Allowing 6% on deposit, and charging 6% on purchase price, and 12½c. a share brokerage each way, what does the broker owe Walters?

SOLUTION

Broker's Account with J. WALTERS.

Dr.			*Cr.*	
Jan. 2—To 50 shares Halifax R.R., at 92¼ . .		$4,612.50	Jan. 2—By Cash . .	$ 500.00
" 2—To Brokerage, 12½c. a share .		6.25	" 26—By 50 shares Halifax R.R., at 96¾ . .	4,837.50
" 26—To Interest on $4,618.75 for 24 days at 6%		18.22	" 26—By Interest on $500 for 24 days at 6% .	1.97
" 26—To Brokerage, 12½c. a share .		6.25		
" 26—Balance due J. Walters .		696.25		
		$5,339.47		$5,339.47

EXPLANATION

In this account, J. Walters is charged with purchase price of the stock, the interest on this money plus the brokerage forbuying for the 24 dys. from date of purchase to date of sale, 12½c. a share brokerage for buying, and 12½c. a share for selling. Brokerage is always reckoned on market value.

Walters is credited with $500 margin, the interest on it for 24 dys., and the selling price of the stock.

The difference of $696.25 shows the cash balance due Walters. If he wants to figure his profit, he compares the $500 that he invested with the $696.25 he received from the broker. The profit is $196.25.

NOTE.—In charging interest on purchase price plus brokerage, we are following the custom of brokers.

305. Illustration 2.—Suppose in the previous question that on Jan. 26th the stock had been quoted at 88, how much would Walters need to deposit to make good his margin?

SOLUTION

Dr.	J. WALTERS.		*Cr.*	
Jan. 2—To 50 shares Halifax R.R., at 92¼ . .		$4,612.50	Jan. 2—By Cash . .	$ 500.00
" 2—To Brokerage, 12½c. a share		6.25	" 26—By 50 shares Halifax R.R., at 88 . .	4,400.00
" 6—To Interest on $4,618.75 for 24 days at 6%		18.22	" 26—By Interest on $500 for 24 days at 6% .	1.97
" 26—Balance . .		265.00		
		$4,901.97		$4,901.97

Walters would need to deposit $500–$265, or $235, to make his margin good.

306. Illustration 3.—On July 1st a broker bought for W. C. McCarter 200 shares Dominion Coal at $43\frac{5}{8}$. Thirty days later he sold 100 shares at $41\frac{1}{8}$, and on Aug. 18th he sold 100 shares at $39\frac{3}{4}$. Allowing brokerage of $12\frac{1}{2}$c. a share each way and 6% interest, how does the account stand?

SOLUTION

W. C. MCCARTER.

Dr.		*Cr.*	
July 1—200 shares Dom. Coal at $43\frac{5}{8}$	$8,725.00	July 1—Margin	$2,000.00
" 1—Brokerage, $12\frac{1}{2}$c. a share	25.00	" 31—100 shares Dom. Coal at $41\frac{1}{8}$	4,112.50
" 31—Brokerage, $12\frac{1}{2}$c. a share on 100 shares	12.50	Aug. 18—Int., 48 days, on $2,000	15.78
Aug. 18—Brokerage, $12\frac{1}{2}$c. a share on 100 shares	12.50	" 18—Int., 18 days, on $4,100	12.13
" 18—Int., 48 days, on $8,750	69.04	" 18—100 shares Dom. Coal at $39\frac{3}{4}$	3,975.00
" 18—Balance due W. C. McCarter	1,271.37		
	$10,115.41		$10,115.41

McCarter put up $2,000 margin. He receives from the broker only $1,271.37. He is therefore a loser by $2,000 – $1271.37, or $728.63.

EXERCISE 96

1. On May 10th W. G. Harwood deposited with his broker $1,000 for the purchase of 100 shares of Bell Telephone stock at $132\frac{3}{4}$. The stock was sold June 12th at $136\frac{1}{2}$. What were the profits of the sale, if money was worth 5% and there was brokerage each way of $12\frac{1}{2}$c. a share?

2. On Aug. 3rd I purchased through a broker 25 shares of Sao Paulo at $126\frac{1}{2}$, and deposited $250 as margin. On Aug. 30th the stock was sold at $123\frac{3}{4}$. What was my loss, allowing $12\frac{1}{2}$c. a share brokerage each way and 6% interest on purchase and margin?

3. On June 6th a speculator deposited with his broker $300 as margin, with instructions to buy Richelieu Navigation stock at $75\frac{1}{2}$. On June 20th the stock was quoted at 70. How much should he have deposited with his broker to make his margin of 10% good? Brokerage $12\frac{1}{2}$c. a share, interest 5%.

4. On Sept. 20th I bought through my broker 200 shares of Canadian General Electric at $87\frac{1}{4}$, and deposited with him $2,000 as margin. On Oct. 15th he sold 125 shares at $85\frac{1}{2}$, and on Oct. 23rd the balance at $83\frac{3}{4}$. Find my loss, if the broker's charges were $12\frac{1}{2}$c. a share for buying and the same for selling, and money was worth 8%.

5. On May 2nd I deposited with my broker $800, with instructions to buy 10 shares C.P.R. at $156\frac{3}{4}$, 30 shares Consumers' Gas at $192\frac{1}{4}$, and 40 shares Winnipeg Ry, at $157\frac{1}{2}$. On May 29th the C.P.R. was sold at $160\frac{1}{2}$, the Consumers' Gas at $194\frac{1}{2}$, and the Winnipeg Ry. at $158\frac{1}{4}$. What was my profit if my broker charged me $12\frac{1}{2}$c. a share for buying and same for selling, and money was worth 5%?

6. On March 3rd I deposited with my broker $8,500 as margin, and he purchased for me 150 shares Niagara Nav. at $123\frac{1}{4}$, 200 shares Twin City at $93\frac{3}{4}$, and 500 shares Toronto Ry. at 108. The stocks on Mar. 28th were quoted as follows : Niagara Nav., $120\frac{3}{4}$; Twin City, $88\frac{1}{2}$; and Toronto Ry., $102\frac{3}{4}$. How much should I have deposited with my broker to make my margin of 10% good, and to cover commission of $12\frac{1}{2}$c. a share for buying and interest at 6%? If I had been unable to have made an additional deposit, and the broker had "sold me out," what would have been my loss?

COMMERCIAL ARITHMETIC

TEST PAPER—No. 9

STOCKS AND BONDS

Your work for this paper is to be submitted to the instructor for correction and criticism.

1. A broker receives $25.00 brokerage for investing in stock at 86. The amount of the investment, including brokerage, is $4,325.00. What did the broker charge per share?

2. A man invests $10,800.00 in 3 per cent. stock at 75. He sells out at 88 and re-invests one-third of the proceeds in $3\frac{1}{2}$ per cent. stock at 96 and the remainder in 5 per cent. at 132. Find the change in his income. (Disregard brokerage in this question.)

3. A gentleman, having $25,000.00 of Dominion Bank Stock (which he purchased at par), paying dividends of 13% per annum, sells out at 209 and reinvests in Bank of Montreal Stock which has a market value of 266. The Bank of Montreal Stock pays dividends of 14%. Find the alteration in his income.

4. If 82 shares were bought at $106\frac{3}{8}$ and sold so as to make a net profit of $266.50, at what quotation were they sold? Brokerage for buying 30 cents per share and for selling 50 cents per share.

5. The present income of a railway company would justify a dividend of $3\frac{3}{4}$% if there were no preference shares. As, however, $1,200,000 of the stock consists of preference shares, which are guaranteed 5% per annum, the ordinary shareholders receive only 3%. What is the whole amount of stock?

6. An investor has $25,000.00 of Standard Bank Stock which he purchased at par and which pays a dividend of 12%. When his

stock is worth 196 he sells out and invests in Bank of Toronto Stock at 219, paying a dividend of 13%. What is the difference in his income after allowing his agent $37\frac{1}{2}$ cents a share for selling and 50 cents a share for buying?

7. A man, desiring to invest some of his surplus funds in dividend bearing stocks, requests you to advise him which will be the more profitable: to purchase 5% stock at $79\frac{3}{4}$, or 7% stock at $174\frac{3}{4}$. Brokerage in the first case is 30 cents per share and in the second case $37\frac{1}{2}$ cents per share.

(*a*) Which should be the more profitable as an investment?

(*b*) What rate on the investment does each earn?

(*c*) What difference in his income would be made by investing in one rather than the other if he invests $14,018.00?

8. On July 1, a broker bought for A. F. Sprott, who deposited $2,181.25 as margin, 200 shares Dominion Coal at $43\frac{5}{8}$. On July 31, he sold 100 shares at $41\frac{1}{8}$, and on August 18, he sold the remainder at $39\frac{3}{4}$. Allowing 30 cents per share brokerage each way and 6% interest, does A. F. Sprott lose or gain in the speculation?

9. On January 2, 1936, A. McPherson deposited with his broker $33\frac{1}{3}$% (of the market value) as margin for the purchase of 50 shares of Michigan Central Stock at $92\frac{1}{4}$. The stock was sold January 26, 1936, at $96\frac{3}{4}$. Allowing 6% interest on deposit and charging 6% on purchase price, and 30 cents per share brokerage each way, what does the broker owe A. McPherson?

Sticking to It

Diamonds are only chunks of coal
That stuck to their jobs, you see.
If they'd petered out like most of us do,
Where would the diamonds be ?

It isn't the fact of making a start,
It's the sticking that counts, I'll say.
It's the fellow who knows not the meaning of fail,
But hammers and hammers away.

Whenever you think you have come to the end
And you're beaten as bad as can be ;
Remember that diamonds are chunks of coal
That stuck to their jobs, you see.

TAXES

307. A **Tax** is a sum of money levied on the person, property, business, or income of an individual for the support of the government or for any public purpose.

308. A **Property Tax** is a tax assessed upon property at a given rate per cent. of the valuation.

309. Property is of two kinds : personal and real.

310. Personal Property is any movable property, such as merchandise, ships, cattle, money, stocks, mortgages.

311. Real Property, or **Real Estate**, is any fixed or immovable property, such as houses, lands, mines, quarries.

312. An **Assessor** is a public or government officer, appointed to estimate the value of property to be taxed, and to apportion the taxes in proportion to the value of each man's property.

313. A **Collector** is a public officer appointed to receive and collect taxes.

314. The **Rate of Taxation** is the sum charged on each dollar of the assessed valuation to raise the required amount of taxes, and is usually quoted at so many mills on the dollar. A tax of 30 mills on the dollar of assessed value is equal to $30.00 on $1000 assessed value.

EXERCISE 97

315. Illustration.—The taxable value of the property of a town is $675,000, and the rate of taxation is 7 mills on the dollar. What is the tax?

$675,000 \times .007 = \$4,725$

SOLUTION.—If the tax on $1 is 7 mills, on $675,000 it will be 675,000 times $.007, which is $4,725.

1. The rate of taxation in a certain city was $11\frac{1}{4}$ mills on the dollar. What tax was paid by a person whose property was assessed for $12,000?

2. My property is assessed at $6,400. At the rate of $3\frac{1}{2}$ mills on the dollar, how much tax will I be required to pay?

3. What amount of tax must a man pay who is assessed $12,000 for real estate and $4,500 for personal property, if he pays a rate of 25 mills on the dollar?

4. I have property worth $19,600, which was assessed at $\frac{3}{4}$ of its value. What was my entire tax at $7\frac{1}{2}$ mills on the dollar?

5. My real estate was assessed at $7,500, and my personal property $12,500. What amount of tax did I pay thereon at $11.80 per $1000.00, having been allowed a discount of 3% for prompt payment?

6. What is my entire tax for the present year upon real estate worth $12,500, and personal property worth $5,000, the real estate having been assessed at $\frac{4}{5}$ of its value, the rate of taxation being $14.00 on the $1,000.00.

316. Illustration.—A tax of $3,750 is to be levied upon a valuation of $1,250,000. Find the rate of taxation.

SOLUTION

$3,750 ÷ $1,250,000 = .003

Therefore, the rate of taxation is $.003 on $1, or 3 mills on the dollar, or $3.00 on $1,000.00.

EXERCISE 98

1. A tax of $7,380 was levied upon the taxable property of a county, valued at $2,460,000. What was the rate, and what was the tax on a farm assessed at $4,000?

Find the rate of taxation in mills on $1 if the assessed valuation is

2. $514,000, and the gross tax $6,233, including 130 polls at 50c.

3. $15,000, and the gross tax $137.25, including 3 polls at 75c.

4. $15,387,200, and the total property tax $169,259.20.

Find the rate of taxation in cents on $100 if the assessed valuation is

5. $34,000, and the total property tax is $469.20.

6. $1,275,000, and the total property tax is $12,495.

7. The assessed valuation of the property of a village is $832,000. The estimate of corporate expenses includes $1,500 for schools, $1,600 for streets, $1,200 for salaries and commissions, and $692 for sundry other expenses. What will be the rate of taxation expressed as mills on the dollar?

317. **Illustration.**—The tax on a certain property was $96.10, and the rate of taxation $7\frac{3}{4}$ mills on the dollar ($7.75 on each $1,000.00 of assessment). For how much was the property assessed?

SOLUTION

$0.00775 is the tax on $1

$1 is the tax on $\frac{\$1}{.00775}$

$96.10 is the tax on $\frac{96.10}{.00775} = \$12{,}400$

8. What is the assessed value of a property that pays a tax of $182, at the rate of $3\frac{1}{4}$ mills on the dollar?

9. A town hall, costing $12,250, was built by a tax assessed upon the property of the town. The tax rate was 5 mills on the dollar, and the cost of collection 2%. What was the valuation?

FIRE INSURANCE

318. **Insurance** is a contract in which one party to the contract agrees for a certain consideration to make up loss which another party may sustain. It is distinguished as Property Insurance, Life Insurance, Accident Insurance, and Health Insurance.

319. The **Policy** is the name applied to the written form of the contract made between the insurance company and the party insured.

320. A **Valued** or **Closed Policy** is one in which the amount insured is definitely determined when the insurance is effected. Houses and furniture are insured in this way.

321. An **Open Policy** is one upon which additional insurance may be entered at any time.

322. The **Premium** is the amount paid for the insurance.

323. Adjustment of Fire Insurance Losses.—Under the ordinary form of policy, the insurance company undertakes to pay the full amount of the loss or damage, provided such loss does not exceed the sum mentioned in the policy.

The policy, however, may contain an *average* clause, in which case the payment made is such proportion of the loss as the amount of the insurance bears to the total value of the property.

Again, the policy may contain a "co-insurance clause." As an example, we quote the following 80% co-insurance clause—

"It is a part of the consideration of this policy, and the basis upon which the rate of premium is fixed, that the insured shall maintain insurance concurrent in form with this policy, on each and every item of the property hereby insured, to the extent of at least eighty per cent. of the actual cash value thereof, and that, failing so to do, the insured shall be a co-insurer to the extent of an amount sufficient to make the aggregate insurance equal to eighty per cent. of the actual cash value of each and every item of the property hereby insured, and, in that capacity, shall bear his, her, or their proportion of any loss that may occur."

324. **Short Rates** are rates of premium charged when the term of insurance is less than a year.

In case a policy is terminated at the request of the insured, he is charged the short rate premium: If the policy is terminated by the company, the

lower long rate will be charged, and the company will refund the premium for the unexpired time of the policy.

325. Marine Insurance refers to insurance of vessels and their cargoes against the dangers of navigation.

326. Adjustment of Marine Insurance Losses.—Such losses are usually settled by the insurance company paying only such proportion of the losses as the sum insured is to the entire value of the vessel.

327. Salvage is an allowance made to those rendering aid in saving vessels or cargoes from marine disasters.

328. Explanation of the Application of the Co-insurance Clause, where 80% is the basis for co-insurance.

The 80% Co-insurance Clause is an agreement between the insurance company and the assured that insurance will be maintained to the amount of 80% of the value of the insured property, or any other percentage agreed upon. By agreeing to insure up to 80% of the value of the property, or any other percentage agreed upon, the policy-holder is given the advantage of a lower rate.

If insurance to 80% or more of the value is carried, the adjustment of the loss would not be affected under the following conditions:

1. When the property is destroyed to the extent of 80% or more of its value.

2. When the property is insured for 80% or more of its actual cash value, whether the loss is total or partial.

329. The co-insurance clause does affect the settlement of loss in the event of the property being partially destroyed, when the insurance carried is less than 80% of the actual cash value.

Illustration 1.—A stock of goods valued at $8,000.00 was insured for $7,500.00 under a policy containing a 90% co-insurance clause. Find the amount paid to the insured (1) if the goods are totally destroyed; (2) if the goods are damaged by fire to the extent of $2,400.00.

SOLUTION

(1) In the case of a total loss, the 90% co-insurance clause would not affect the settlement, and the insured would collect the full amount of insurance carried, $7,500.00.

(2) In the above example, the value of the goods is $8,000.00. Under a 90% co-insurance clause, the insured would be required to carry 90% of $8,000.00, or $7,200.00. Since the insurance carried exceeds this amount, the company would pay the full amount of the loss, $2,400.00.

Illustration 2.—Property worth $8,000.00 is insured for $5,000.00 under a policy containing an 80% co-insurance clause. What would be the amount paid by the insurance company if (1) the property is totally destroyed? (2) the property is damaged by fire to the extent of $3,000.00?

SOLUTION

(1) Since the co-insurance clause does not affect the settlement in case of a loss of more than 80% of the actual cash value, the amount paid would be the full amount of the policy, $5,000.00.

(2) Value of the property $8,000.00
Amount required under 80% co-insurance clause,
80% of $8,000.00, or 6,400.00
Amount actually carried 5,000.00

Amount paid by insurance company—

$\frac{5,000}{6,400}$ of $3000.00. or $2,343.75.

The insured is a co-insurer to the extent of $1,400.00, and thus bears

$\frac{1,400}{6,400}$ of $3,000.00, or $656.25 of the loss.

SUMMARY OF RESULTS USING THE 80% CO-INSURANCE CLAUSE

If the valuation of the property is $10,000.00, and the insurance carried is subject to the 80% co-insurance clause, then insurance amounting to at least $8,000.00 should be carried. If this amount is carried, the insured will receive from the insurance company the amount of a fire loss up to $8,000.00, or if $9,000.00, or $10,000.00 insurance is carried, these amounts can be recovered.

Suppose the insured *is not* carrying $8,000.00 (the amount he ought to carry) and he has a loss of $7,200.00, then his recovery would be cut down as follows:

If he Carries	*He Collects as*	*Or*	*Equals*
$1,000	$1,000 is to $8,000	$\frac{1}{8}$ of $7,200	$ 900
2,000	2,000 is to 8,000	$\frac{2}{8}$ of 7,200	1,800
3,000	3,000 is to 8,000	$\frac{3}{8}$ of 7,200	2,700
4,000	4,000 is to 8,000	$\frac{4}{8}$ of 7,200	3,600
5,000	5,000 is to 8,000	$\frac{5}{8}$ of 7,200	4,500
6,000	6,000 is to 8,000	$\frac{6}{8}$ of 7,200	5,400
7,000	7,000 is to 8,000	$\frac{7}{8}$ of 7,200	6,300
8,000	8,000 is to 8,000	$\frac{8}{8}$ of 7,200	7,200

NOTE.—Other percentages may be used, and results would be obtained in a similar way.

330. Co-insurance Clause Waiver.—Waiver of same in case of small losses. It is not permissible to insert any wording which shall in any way interfere with the full operation of the Co-insurance Clause, except one or the other (but not both) of the following waivers—

(*a*) "In case any claim for loss shall not exceed 5% of the total sum insured on the invoiced item or items of this schedule no special inventory or appraisement of the undamaged property shall be required."

(*b*) "In case of loss the Co-insurance Clause shall not be held to apply where the total loss does neither exceed Twenty-five Hundred Dollars nor 2% of the sum insured on the involved item or items of the schedule."

If the schedule consists of only one item the wording of the waiver will require to be changed to conform thereto.

331. Illustration.—How much will it cost to insure a store and contents for $42,000 at $1\frac{1}{2}\%$?

SOLUTION

$42,000 = the amount insured.

$1\frac{1}{2}\%$ of $42,000 = $630, the premium charged.

EXERCISE 99

1. The stock of a wholesale boot and shoe house is insured for $45,000 under an 80% co-insurance clause. At a time when their inventory amounts to $68,000, they suffer a partial fire loss of $10,500. What amount should they receive from the insurance company?

2. An insurance company received $117 premium on property insured for $18,000. If a building be insured for $35,000 in another

company at the same rate, what will be the premium on the latter?

3. A property valued at $60,000 is insured for $36,000. Fire does damage to the extent of $20,000. What difference will it make in the amount received from the insurance company if the insurance policy contains, or does not contain, an 80% co-insurance clause?

4. A merchant has property insured as follows: Stock-in-trade worth $16,000, for $\frac{7}{8}$ of its value, premium $1.20 per $100 for a one-year period; store fixtures worth $600 for their full value, premium 75c. per $100 for a period of 3 years; building worth $9,000, for 90% of its value, premium 90c. per $100 for a three-year period. Find the total average monthly cost of premiums on the three policies.

5. A manufacturer pays 95c. per $100 for insurance on the building, and $1.15 per $100 for insurance on the stock. He decides to install a sprinkler system at a cost of $4,500, which will reduce the cost of his insurance by 60%. If the building was insured for $30,000, and the stock for $80,000, how much did he save each year by installing the sprinkler system, after allowing interest at 6% per annum on the cost of the system, and 5% for depreciation?

6. Property worth $36,000 is insured under policies containing a 90% co-insurance clause. If the property is insured for $30,000, how much will the owner receive if fire does damage to the extent of $9,600?

7. I pay an insurance premium of $70.20 on a policy on property worth $12,000. The rate is 90c. per $100. If the policy contains an 80% co-insurance clause and a fire loss of $3,500 occurs, how much should I collect from the insurance company?

8. A ship's cargo valued at $240,504.21 was insured for such an amount that in case of total loss, the owner would receive the value of the cargo and the premium paid. Find the face of the policy, if the rate was $1\frac{3}{4}$%.

EXERCISE 100

332. Illustration.—I paid $30 for insuring a house, worth $6,400, at $\frac{3}{4}$ valuation. What was the rate?

SOLUTION

$\frac{3}{4}$ of $6,400 = $4,800, the face of the policy.

$30 ÷ 4,800 = $.00625, or 62½c. on the $100 = the rate of insurance.

1. The premium for insuring a house worth $12,000 at its full value was $80. What was the rate per $100?

2. If $156 are paid annually for insuring $24,000, what is the rate per $100?

3. A fire insurance company charged $129.60 for insuring a house for $13,500. What was the rate of insurance per $100?

4. I paid $175 premium on a schooner worth $25,000, which was insured at $\frac{4}{5}$ of its value. What was the rate per cent. of insurance? What was the rate of insurance in cents on the $100?

5. A marine insurance company received $484.50 for insuring a vessel worth $80,000 at $\frac{2}{5}$ of its value. What was the rate per $100 of insurance if $4.50 was charged for the policy and survey?

333. Illustration.—For what sum is a house insured if the premium paid is $75, and the rate of insurance is 62½c. per $100?

SOLUTION

62½c. is the premium on $100.

$75 is the premium on $\frac{75.00}{.625} \times 100 = \$12{,}000.$

6. A house is insured at 60 cents on $100, and the premium is $93.60. For how much is it insured?

7. The cargo of a steamer is insured at ½%, 50 cents on $100. For what sum is it insured, the premium being $1,500?

8. Paid $350 on a shipment of goods to insure $\frac{3}{4}$ the value, at $3.62½ on $100. What was the whole value?

9. A manufacturing company paid $214.80 premium for insurance on $\frac{3}{4}$ of the cost of its building and machinery, at 60c. per $100. What was their cost?

CUSTOM HOUSE BUSINESS

334. The **Customs Tariff** is a book issued by the Department of Customs at Ottawa, giving a list of dutiable goods, together with rates of duty thereon, and also giving the statutes which regulate the matter of levying duties.

335. The **General Tariff** is the rate of duties to which goods are

subject, unless specially provided for under the British preferential tariff, the intermediate tariff, or a surtax.

336. The **British Preferential Tariff** applies to goods which are the produce or manufacture of British countries when imported direct from any British country. This duty is always less than the general tariff.

337. The **Intermediate Tariff** is, as the name implies, a tariff between the general tariff and the British preferential tariff. It may by order in council be extended, in consideration of benefits, to any country the produce or manufactures of which have previously been subject to the general tariff.

338. A **Surtax** is a duty above the general tariff, and is levied on the produce of manufacturers of any foreign country which treats imports from Canada less favourably than those from any other country.

339. An **Ad Valorem Duty** is reckoned as a certain percentage of the " Value for duty in dollars."

340. A **Specific Duty** is reckoned according to quantity, as, for instance, a certain number of cents per pound, or per bushel, or per barrel, and so on.

Some articles imported are subject to both ad valorem and specific duties.

Dutiable goods arrive either by vessel or by rail or by express. For the purpose of handling these imports, the work is divided according to the amount of goods imported between the Long Room and the Short Room. All goods coming by rail or vessel are passed through the Long Room, no matter what the amount of the invoice may be. Goods arriving by express are passed through the Short Room if the amount is less than $25.00. If the amount is $25.00 or exceeds $25.00, the goods must be passed through the Long Room. (Toronto Custom House practice.)

341. Illustration.—Find the duty on an importation of 50 gals. of varnish invoiced at 97c. a gallon, the rate of duty being 20c. a gallon specific and 22½% ad valorem.

(*a*) Specific Duty:

20c.
50

$10.00

(*b*) Ad valorem Duty:

50 gals. at 97c.= $48.50, or for duty purposes this is called $49. 22½% of $49 = $11.03.

(*c*) $10 + $11.03 = $21.03 total duty.

SOLUTION.—The specific duty is simply 20c. on each gallon, or $10 on 50 gal. The ad valorem duty is a percentage of the cost. The cost of 50 gal. at 97c. is $48.50. As the value for duty is always given to the nearest dollar, we add another dollar if the cents are 50 or more, and drop the cents if they are less than 50. $48.50 thus becomes $49. 22½% of $49 is $11.03.

The whole duty is $10 + $11.03 or $21.03.

EXERCISE 101

What is the specific duty upon an importation of

1. 7,835 sq. yds. of dress goods, rate of duty 25c. per square yard?

2. 300 tons of medicinal roots, rate of duty 1c. per pound?

3. 1,200 tons guano, rate of duty 75c. per ton?

4. 30 doz. bottles wine, duty $2.50 per dozen?

5. 650 gal. brandy, rate of duty $1.50 per gallon?

6. 160 gross bottles, rate of duty 10c. per dozen?

7. What is the duty upon an importation of 1,500 yds. flannel weighing 350 lbs. net, and valued at 60c. per yard, the rate of duty thereon being 24c. per pound and 35% ad valorem?

8. I imported from the United States 7,240 bus. of corn and 17½ tons of hay, invoiced at $9.50 per ton. What amount of duties had I to pay at 15c. per bushel on the corn and 20% on the hay?

9. What is the duty on an invoice of linens amounting to £3,256 sterling at 27%?

10. What is the duty on 1,000 yds. of Brussels carpet, 27 in. wide, invoiced at 6s. 9d. per yard; duty 44c. per square yard specific, and 35% ad valorem?

11. An invoice of woollen cloth, imported from England, was valued at £956 6s. If its weight was 684 lbs., how much was the duty at 50c. per pound specific, and 35% ad valorem?

EXERCISE 102

1. Gowans, Kent & Co., Toronto, Ont., import from France

1 crate China dishes, containing 50 sets, at 10 frs. a set, on which the duty is 30%. The crate is invoiced at 45 frs. What is the amount of duty payable?

NOTE.—Cases where invoiced are dutiable. The general tariff is 20%. The preferential is 15%.

2. George Hamilton, of Medicine Hat, imports from England: 1 Wool Suit at £2, 1 doz. Silk Ties at 1s. 8d. each, 1 Silk Hat at 15s., 1 pair of Shoes at 12s. The duty on the hat is 22½%, on the other 30%. The package is invoiced at 4s. What is the amount of duty payable?

3. McLeod Bros., of Glasgow, sell to Brown, Morgan & Co., Montreal, a line of ready-made Shirts as follows—

8 @ 17/11, 8 @ 21/–, 6 @ 27/3, 6 @ 35/6, Case 13/5.
6 @ 19/6, 6 @ 25/3, 6 @ 27–, 6 @ 34/6, 4 @ 40/–, Case 19/9.
6 @ 46/6, 4 @ 52/6, 6 @ 39/6, 4 @ 40/6, 4 @ 41/–, Case 17/2.
14 @ 45/–, 8 @ 56/–.

The duty on shirts is 30%. What is the amount of duty to be paid?

4. John McDonald & Co., Toronto, buy from Wm. Anderson & Co., Glasgow, as follows—

57 pcs. Madras Shirting at 4⅜d. per yard, 32½, 46½, 47, 48½, 50, 54, 57, 2/58½, 5/59, 2/60, 60½, 4/61, 4/61½, 6/62, 9/62½, 10/63, 3/63½, 64, 64½, 71, 76½. Case 13/–.

34 pcs. Oxford Shirting at 5¾d. per yard, 55, 55½, 56, 4/57, 3/57½, 2/58, 2/58½, 2/59, 4/59½, 3/60½, 3/61, 2/61½, 2/62, 3/63, 60. Case 11/4.

9 pcs. Madras Shirting at 6¾d. per yard, 54½, 55, 55½, 2/57, 2/58½, 60, 61½. Case 12/9.

If the duty on shirting is 25%, what amount is payable?

5. Thomas Miller & Co., Winnipeg, Man., import from M. Myers & Son, of Birmingham, Eng., the following

882	gross Pens	@ 1/–
100½	" "	@ 1/3
25	gross Pens 340	@ 1/–
30	" " 42203	@ 1/–
50	Binders M. 1	@ 1/1
1	" M. 5	@ 2/–

1 Binder M. 7 . . . . . @ 4/6
Case 5/2

If the duty on pens is 15%, and on binders 20%, what is the total amount of duty to be paid?

6. Davies & Henderson, of Toronto, import from Andrew Whyte & Son, Ltd., Edinburgh, Scotland, the following—

96 rms. Printing "Water Mill" Cream Wove, 17 by 28, 2,304 lbs., at 2¾d. per lb.; 9 rms. do. do., 2nds, 230 lbs., at 2¾d. per lb., less 10%; 10 rms. do. do., 3rds, 246 lbs., at 2¾d. per lb., less 20%; 20 rms. Printing "Water Mill" Cream Wove, 16½ by 21, 420 lbs., at 2¾d. per lb.

Deduct trade discount of 5% from whole bill. 7 cases cost £2 0s. 7d.; duty 15%. What is the amount of the duty?

7. F. Tiffin & Co., Vancouver, B.C., import from the American Lead Pencil Co., New York, the following—

6	gross	557 Pencils	@ $3.50
3	"	450R "	@ 3.00
6	"	295Bx Penholders . . .	@ 3.00
6	"	658 " . . .	@ 1.40
12	"	6/379 6/383 Penholders . .	@ 3.60
1	"	410 Eras. Pencils . . .	@ 3.00
½	"	428 " " . . .	@ 3.00
6	"	3/F, 3/H.V.D. Pencils . .	@ 6.50
1	"	F.W./Prot. V.D. " . .	@ 8.10
6	"	Steno " . .	@ 3.00

Duty 27½%. What is the total duty to be paid?

COMMERCIAL ARITHMETIC

TEST PAPER—No. 10

TAXES—INSURANCE—CUSTOMS DUTIES

Your work for this paper is to be submitted to the instructor for correction and criticism.

1. Allowing 5% for taxes uncollectible and 3% for collection, what sum must be levied that $22,250 may be realized for the building of a town hall?

2. A city hall costing $36,750 was built by a special tax upon the property of the city. The rate of taxation being 5 mills on the dollar, and the cost of collection 2%, find the total assessment.

3. A man's salary is $2,900, and his net income is $2,852.50 after paying income tax on all over $1,000. What is the rate (in mills) of taxation?

4. A block of stores and contents was insured for $220,000, and became damaged by fire and water to the amount of $150,000. Of the risk, $40,000 was taken by the Quebec Company, $65,000 by the British American, $35,000 by the Western, and the remainder was divided equally between the Royal Canadian and the London Mutual. What was the net loss of each company, if the premium paid was $1\frac{7}{8}\%$?

5. Property valued at $10,000 and insured for $6,000 is damaged by fire to the extent of $7,000.

Find the amount payable by the Insurance Company under a policy containing the 80% co-insurance clause, which provided that the insured shall be a co-insurer to the extent of an amount sufficient to make the aggregate insurance equal to 80% of the property insured, and in that capacity shall bear his proportion of any loss that may occur.

6. Messrs. Upper & Lobb, St. Catharines, Ont., import from Rodrigo De La Guardia & Co., Seville, Spain, as follows: 1 butt Sherry Wine, 108 gal., at £85 per butt, duty 30% and 43c. per gallon; 4 hhds. (2 butts) Sherry Wine, 216 gal., at £96 per butt, duty 30% and 46c. per gallon; 10 qr. (2½ butts) Sherry Wine, 270 gal., at £102 per butt, duty 30% and 40c. per gallon. What is the amount of duty paid. (£1 = $4.86⅔.)

7. Wentzell & Eaton, Halifax, import from Arnett & Co., London, Eng., 80 yd. Black Sateen, at 4s. 6d.; 2½ doz. Silk Shawls, at 15s. 8d. each; 75 yd. Muslin, at 1s. 2d.

(*a*) Show extensions of invoice.

(*b*) Show duty on each in Canadian currency, rates being 35%, 30%, and 25% respectively, less 33⅓% for British preferential tariff.

(*c*) Show value of invoice in Canadian currency at $4.86⅔ per £1.

8. What is the duty on 1,000 yd. of Brussels carpet, 27 in. wide, invoiced at 6s. 9d. per yard; duty 28c. per square yard and 40%. (£1 = $4.86⅔.)

The Value of Education

Increases Power for Accomplishment and Enjoyment of Life

By Major C. K. Newcombe

When it is demonstrated that education makes for higher salary levels and higher standards of living, that it greatly enhances our prospects for what is known as success in life, the half has not been told.

Education liberalizes our thinking. It frees us from the bondage of space and time. Great men of all ages and of all lands are our daily companions. Their thought becomes our thought. The petty annoyances of the day cease to bulk so largely in our minds. We insensibly come to distinguish the things which matter, for above all education gives a sense of perspective.

Education notably increases the range of our rational enjoyments, music, art, the worlds of nature, microscopic and telescopic, philosophy and poetry, all make us free of their various realms with all their wealth.

To live more serviceably, to enjoy more intensely, and, shall we say, to die more contentedly, these are three worthy results of true education.

AVERAGE AND ITS APPLICATIONS

342. An Average is a medium sum or quantity made out of unequal sums or quantities.

To illustrate: A grocer sells a certain tea at 50c. a pound and another grade at 70c. a pound. He is getting an average of 60c. a pound for his tea. One pound sold at 50c. and one at 70c. returns him exactly $1.20. Two pounds at 60c. a pound gives the same return. Note that in selling a 50c. tea at 60c. a pound there is a gain of 10c. on a pound, while in selling a 70c. tea at 60c. a pound there is a loss of 10c. a pound. A true average should be such that the gains just counterbalance the losses.

Again, a partner in a business invests $5,000 for one month, and for the next month he invests $7,000. On an average he is giving the business the use of $6,000 a month. Note, again, that while $6,000 is $1,000 more than he invests the first month, it is $1,000 less than he invests the second month. The average is greater than the investment for one month by exactly the figure that it is less than the investment for the other month.

Again, a man stores 500 bus. of grain in a storehouse for one month, and for the next month he stores 700 bus. On an average he has in store 600 bus. a month. This is 100 bus. more than he had in the first month, but 100 bus. less than he had in the second month. The surplus in one case exactly balances the deficiency in the other.

QUESTIONS OF THE FIRST ASPECT

343. Given the items, to find the average.

Illustration 1.—A grocer mixed 25 lbs. tea at 30c. a pound, 20 lbs. at 50c. a pound, 40 lbs. at 75c. a pound, and 15 lbs. at a dollar a pound. What was the average value of the mixture?

SOLUTION

25 lbs. at 30c.	$7.50
20 lbs. at 50c.	10.00
40 lbs. at 75c.	30.00
15 lbs. at $1.00	15.00
100	$62.50

$62.50 ÷ 100 = $.62½

The tea is worth on an average 62½c. a pound.

344. Illustration 2.—On Jan. 1, 1944, J. Watson commences business, investing $2,000. On April 1st he invests $5,000 more, and on Sept. 1st $2,000 more. His investment then remains unaltered for the balance of the year. What is his average investment per month throughout the year?

SOLUTION

From Jan. 1st to April 1st = 3 mos.
$2,000 invested for 3 mos. = $6,000 for 1 mo.
April 1st to Sept. 1st = 5 mos.
$7,000 invested for 5 mos. = $35,000 for 1 mo.
Sept. 1st to end of year = 4 mos.
$9,000 invested for 4 mos. = $36,000 for 1 mo.

Total investment . $77,000 for 1 mo.
$77,000 ÷ 12 = $6,416⅔—average investment per month.

345. Illustration 3.—A commission merchant places in store the following items—

Sept. 9th, 200 barrels of pork.
Oct. 17th, 500 " of beef.
Nov. 10th, 250 " of pork.
Dec. 1st, 150 " of beef.

On Dec. 8th he desires to make a settlement. Suppose he is charged at the rate of 4c. a barrel for a period of 30 dys., what does he owe?

SOLUTION

The storage of 200 bbls. for 90 dys. = the storage of 18,000 bbls. for 1 dy.
The storage of 500 bbls. for 52 dys. = the storage of 26,000 bbls. for 1 dy.
The storage of 250 bbls. for 28 dys. = the storage of 7,000 bbls. for 1 dy.
The storage of 150 bbls. for 7 dys. = the storage of 1,050 bbls. for 1 dy.

All = the storage of 52,050 bbls. for 1 dy.
The storage of 52,050 bbls. for 1 dy. = the storage of

$$\frac{52050}{30} \text{ or } 1{,}735 \text{ bbls. for 30 dys.}$$

The storage bill will be 4c. × 1735 = $69.40

EXERCISE 103

1. Eight men weigh respectively 145 lbs., 187 lbs., 164 lbs., 138 lbs., 197 lbs., 205 lbs., 179 lbs., and 169 lbs. What is their average weight?

2. A grocer mixed 40 lbs. of tea costing 45c. a pound, 48 lbs. costing 47c. a pound, and 64 lbs. costing 53c. a pound. What was the value of the mixture per pound?

3. Find the average value of a mixture of 17 lbs. coffee at 32c., 45 lbs. at 26c., 37 lbs. at 29c., 42 lbs. at 37c., and 24 lbs. at 30c.

4. The following summary is taken from a book of cash sales: Aug. 7, sold 310 at $1.09; Aug. 8, sold 470 at $1.25; Aug. 9, sold 640 at $.95; Aug. 10, sold 430 at $1.07; Aug. 11, sold 580 at $.99; Aug. 12, sold 360 at $1.16. What was (*a*) the average number sold daily, (*b*) the average daily cash business, and (*c*) the average selling price?

5. Alex. Brown invested in business on Jan. 1, 1944, $4,000; on Mar. 1, $1,000 more; July 1, $2,000 more; and Nov. 1, $1,000 more. From Nov. 1 to the end of the year his investment remained unchanged. What was his average investment for the year? (Calculate time in months.)

6. Weir & Thayer formed a partnership. On April 1 Weir invested $3,500, and on Aug. 17 $1,200 more. On April 1 Thayer invested $3,200, and on Jan. 10 $2,000 more. Find the average investment of each for the year.

7. Liddle & Tufford became partners in a business, agreeing to share gains and losses according to average investments. On May 10 Liddle invested $1,600; on June 17, $500 more; and on Aug. 9, $700 more. On May 10 Tufford invested $1,300; on July 3, $600 more; and on Sept. 20, $400 more. The partnership was dissolved on Oct. 17. Find the average investment of each.

8. A warehouseman received the following quantities of wheat at the rate of 1c. per bushel for a period of 30 dys. of average storage: June 18, 500 bus.; June 28, 400 bus.; July 9, 300 bus.; and Aug. 2, 800 bus. What was the amount of storage due Aug. 15?

9. At 3c. a barrel per 30 dys. of average storage, what is the storage due May 18 on the following receipts: Jan. 20, 125 bbls.; Jan. 31, 75 bbls.; Feb. 12, 140 bbls., Feb. 29, 80 bbls.; Mar. 15, 150 bbls.; Mar. 24, 95 bbls.; April 4, 475 bbls.; April 9, 38 bbls.; April 15, 100 bbls.; April 28, 50 bbls.; May 10, 150 bbls?

10. A farmer received for pasture: April 30, 12 head of cattle; May 15, 14 head of cattle; May 23, 27 head of cattle; June 9, 5 head of cattle; June 30, 8 head of cattle; July 16, 40 head of cattle. If the charges were 75c. per head for each week of 7 dys. of average pasture, what would be owing for the pasture on July 25?

346. Illustration 4.—A warehouseman received and delivered the following—

Received.		Delivered.	
June 11,	150 bbls.	June 26,	120 bbls.
" 30,	200 "	July 15,	150 "
July 18,	90 "	" 25,	160 "
" 28,	180 "	Aug. 4,	190 "

What was paid for storage at 3c. per barrel for a period of 30 dys. average storage, a settlement having been made Aug. 4?

Solution

Dates.	Days.	Barrels.	
June 11 to June 26	= 15 ×	150 (received)	= 2,250 bbls. stored for 1 dy.
" "		120 (delivered)	
" 26 to " 30	= 4 ×	30 (in store)	= 120 bbls. stored for 1 dy.
" "		200 (received)	
" 30 to July 15	= 15 ×	230 (in store)	= 3,450 bbls. stored for 1 dy.
		150 (delivered)	
July 15 to " 18	= 3 ×	80 (in store)	= 240 bbls. stored for 1 dy.
" "		90 (received)	
" 18 to " 25	= 7 ×	170 (in store)	= 1,190 bbls. stored for 1 dy.
" "		160 (delivered)	
" 25 to " 28	= 3 ×	10 (in store)	= 30 bbls. stored for 1 dy.
" "		180 (received)	
" 28 to Aug. 4	= 7 ×	190 (in store)	= 1,330 bbls. stored for 1 dy.
			Total storage = 8,610 bbls. stored for 1 dy.

Total storage = $\left(\frac{8610}{30} = 287\right)$ bbls. stored for 30 dys.

$.03 × 287 = $8.61, required amount of storage.

347. Illustration 5.—The following accounts show the investments and withdrawals of two partners during a year. Find the average investment of each.

Dr.	FRANK SMITH.		*Cr.*
1944.	WITHDRAWALS.	1944.	INVESTMENTS.
Apr. 23	$1,000	Jan. 1	$16,000
Aug. 17	900	Oct. 20	3,000

Dr.	CHAS. ROBERTSON		*Cr.*
1944.	WITHDRAWALS.	1944.	INVESTMENTS.
July 28	$600	Jan. 1	$12,000
Dec. 4	800	May 17	600

SOLUTION

SMITH'S ACCOUNT

From Jan. 1 to Apr. 23 = 113 dys. Hence $16,000 for 113 dys. = $1,808,000 for 1 dy.
Withdrawal 1,000

From Apr. 23 to Aug. 17 = 116 dys. Hence $15,000 for 116 dys. = $1,740,000 for 1 dy.
Withdrawal 900

From Aug. 17 to Oct. 20 = 64 dys. Hence $14,100 for 64 dys. = $902,400 for 1 dy.
Investment 3,000

From Oct. 20 to Jan. 1 = 73 dys. Hence $17,100 for 73 dys. = $1,248,300 for 1 dy.

Smith's total investment $5,698,700 for 1 dy.

Dividing $5,698,700 by 365, we get $15,612$\frac{4}{73}$ as the average investment for the year.

ROBERTSON'S ACCOUNT

From Jan. 1 to May 17 = 137 dys. Hence $12,000 for 137 dys = $1,644,000 for 1 dy.
Investment 600

From May 17 to July 28 = 72 dys. Hence $12,600 for 72 dys. = $907,200 for 1 dy.
Withdrawal 600

From July 28 to Dec. 4 = 129 dys. Hence $12,000 for 129 dys. = $1,548,000 for 1 dy.
Withdrawal 800

From Dec. 4 to Jan. 1 = 28 dys. Hence $11,200 for 28 dys. = $313,600 for 1 dy.

Robertson's total investment $4,412,800 for 1 dy.

Dividing $4,412,800 by 365, we get $12,089$\frac{83}{73}$ as the average investment for the year.

Question.—Suppose the gain for the year were $3,000, and the partners shared this gain in proportion to their average net investments, how much is each entitled to?

SOLUTION

Smith's average net investment is $15,612\frac{44}{73}$
Robertson's average net investment is $12,089\frac{33}{73}$

Total average net investment is $27,702\frac{54}{73}$

Smith's share of the gain is $\frac{15612\frac{44}{73}}{27702\frac{54}{73}}$ of $3,000 = $1,690.76

Robertson's share of the gain is $\frac{12089\frac{33}{73}}{27702\frac{54}{73}}$ of $3,000 = $1,309.24

EXERCISE 104

1. Compute the storage on the following account at 1c. per bushel for a period of 30 dys. of average storage—

RECEIVED.		DELIVERED.	
Sept. 7,	130 bus.	Oct. 1,	200 bus.
" 23,	90 "	" 10,	145 "
Oct. 3,	180 "	" 30,	75 "
" 29,	160 "	Nov. 11,	140 "

2. At 4c. per barrel for a period of 30 dys. of average storage, what is the storage due Aug. 15 on the following produce, received and delivered as stated?

RECEIVED.			DELIVERED.		
June 9,	160 bbls.	apples	June 21,	85 bbls.	apples
" 22,	140 "	potatoes	" 26,	110 "	potatoes
July 14,	70 "	turnips	" 29,	75 "	apples
Aug. 5,	100 "	onions	July 16,	30 "	turnips
			" 30,	30 "	potatoes
			Aug. 10,	75 "	onions
			" 12,	40 "	turnips
			" 15,	25 "	onions

3. Find the pasturage, at 65c. per head, for a period of 30 dys. of average pasturage on the following account—

CATTLE RECEIVED.		CATTLE WITHDRAWN.	
June 3,	16 cattle	July 2,	18 cattle
" 16,	27 "	" 18,	20 "
July 7,	11 "	Aug. 4,	9 "
" 15,	26 "	" 19,	13 "
" 27,	7 "	Sept. 5,	27 "

4. A and B formed a partnership Mar. 1, and agreed to divide gains and losses according to their average net investments. At first A put in $3,000 and B $2,500. On June 1 A put in $1,000 more and B $2,000. On Sept. 1 A withdrew $1,500, and on Nov. 1 B withdrew $1,000. For the remainder of the year their investments remained unchanged. The profit for the year being $1,200, how much should each receive?

5. Jan. 1, 1890, Hess, Dunn, and Norris engaged in business, Hess investing $8,000, Dunn $7,000, and Norris $6,000. June 1 Hess invested $2,000 additional, but on Sept. 1 he withdrew $1,500. May 1 Dunn withdrew $2,000, but July 1 invested $1,500 additional. April 1 Norris invested $2,000 additional, but on Aug. 1 withdrew $1,000. Divide their gain, $5,320, in proportion to their average investments.

6. A, B, and C began business Jan. 1, when A put in $7,500, and July 1 he put in $2,500 more; B put in Jan. 1 $12,000, and May 1 withdrew $4,000; C put in Jan. 1 $10,000, Aug. 1 he added $3,000, and Oct. 1 he withdrew $7,000. At the close of the year the profit was $8,500. How much ought each to have, the gains being divided according to their average investment? (Calculate time in months.)

348. Given the average, to find the items.

Illustration.—A grocer wishes to mix four kinds of coffee, worth respectively 17, 18, 24, and 25c. a pound, in such a way that the mixture will be worth 20c. a pound. How many pounds of each will be used?

SOLUTION

17c.		plus 3c.
18c.	Selling Price average 20c.	plus 2c.
24c.		minus 4c.
25c.		minus 5c.

Put down the prices of the four kinds of coffee to be mixed and the selling price, 20c. If 1 lb. of each were taken, there would be gains of 3c. and 2c., with losses of 4c. and 5c., as shown above, or 5c. total gain, with 9c. total loss.

The loss must equal the gain to secure the correct average, and to obtain this let us take as follows—

3 lb. at 25c. =	75c.	plus	15	plus 23
2 lb. at 24c. =	48c.	plus	08	
4 lb. at 18c. =	72c.	minus	08	minus 23
5 lb. at 17c. =	85c.	minus	15	
14 lb. at	2.80			

1 lb. at $\frac{2.80}{14}$ = 20c.

You will note that we took the gain and loss figures in the order in which they came and reversed the prices per pound, thus averaging our gains with the losses, and securing the desired price of 20c. per lb.

349. Illustration.—A merchant has 15 pounds of tea worth 40c. a pound and 60 pounds worth 50c. a pound; how much tea at 60c. a pound must he add to have a mixture worth 55c. a pound?

SOLUTION

15 lb. at 40c.		plus	15	plus $2.25
60 lb. at 50c.	.55	plus	05	plus 3.00
? lb. at 60c.		minus	05	

By using 15 lb. at 40c. he has a gain of 15c. a pound, or $2.25; 60 lb. at 50c., he has a gain of 5c. a pound, or $3.00, a total of $5.25 gain. On each pound of 60c. tea he loses 5c.; therefore to equalize his gain of $5.25 he requires $\frac{\$5.25}{5}$ = 105 lb. of 60c. tea, which would give a loss of $5.25.

EXERCISE 105

1. In what proportion must two kinds of coffee, which cost 50c. and 65c. per pound respectively, be mixed to give a mixture worth 54c. per pound?

2. In what proportion must teas worth 25c., 35c., and 40c. per pound respectively be mixed so that a mixture may be formed which will be worth 34c. per pound?

3. A liquor dealer has wines worth 30c., 40c., 80c., and 83⅓c. per quart. He wishes to make a mixture of 80 qts. so that he may sell at 70c. per quart, and make 20% profit. How much of each kind must he use?

4. A mixture of 60 lbs. of two kinds of tea cost $24.60; the

cheaper is worth 35c. per pound and the dearer 45c. Find the number of pounds of each in the mixture.

5. A mixture of 7 lbs. of black tea and 8 lbs. of green is worth $5.28, while a mixture of 12 lbs. of black and 3 lbs. of green is worth $5.73. Find the value per pound of each.

6. Six geese and 5 turkeys are worth $5.95, and 7 geese and 8 turkeys are worth $8.35. Find the price of each.

7. The mean height of 6 mountains is 10,357 ft. What must be the height of a seventh mountain to make the mean height of the seven 10,643 ft.?

8. A and B enter into partnership for a year, and A receives $\frac{3}{5}$ of the profits and B the remainder. A invests $2,000 additional the second year, and at the end of the year receives $\frac{4}{9}$ of the profits. How much did each invest the first year?

EXERCISE 106

1. The average of seven numbers is 26.98. The average of the first two is 34.5, and of the next three 19.3. Find the average of the remaining two.

2. A dishonest milk-dealer buys 135 gals. of pure milk at 23c. a gallon, and, after mixing it with water, sells the mixture at 4½c. a quart, thereby gaining $2.43. How much water has he used?

3. A, B, and C engage in manufacturing shoes. A puts in $1,920 for 6 mos., B a sum not specified for 12 mos., and C $1,280 for a time not specified. A received $2,400 for his stock and profits, B $4,800 for his, and C $2,080 for his. Required, B's stock and C's time.

4. Dodd and Brown became partners June 1, 1943, Dodd investing $10,000 and Brown $8,000. On Oct. 1 Dodd invested $2,000 additional, and Brown withdrew $2,000. How much should Brown invest on Jan. 1, 1944, to entitle him to half the year's profits?

5. How much tea worth respectively 55c. and 75c. per pound must be mixed with 30 lbs., worth 90c. per pound, in order that the mixture may be sold for 70c. per pound?

6. How much water will it require to dilute 60 gal. of alcohol, worth $1.50 per gallon, so that the mixture may be worth only $1.20 per gallon?

7. How many gallons of kerosene oil, worth 60c. per gallon, must be mixed with 12 gal. of coal oil, worth 36c., and 8 gal. of Aurora oil, worth 56c., so that the mixture may be sold for 50c. per gallon?

8. A farmer has 16 bus. of corn, worth 48c. per bushel, and 12 bus. of oats, worth 34c. per bushel, which he wishes to mix with rye, at 60c., and barley, at 80c., in order to sell the mixture at 56c. per bushel. How many bushels of rye and barley will be required?

EXERCISE 107

1. A farmer has three different qualities of wool worth 33c., 37c., and 45c. per pound respectively. He wishes to make up a package amounting to 120 lbs., which he can afford to sell at 39c. per pound. How many pounds of each kind must he take?

2. A confectioner mixes three different qualities of candy, worth 14c., 18c., and 30c. per pound respectively, so as to make a box of 84 lbs. How many pounds of each sort must he take so as to sell the mixture at an average price of 24c. per pound?

3. How much sugar at 10c., 13c., 15c., 17c., and 18c. per pound must be taken to make a mixture worth 16c.?

4. A dealer mixed two kinds of wines, worth respectively $2.40 and $3.20 per gallon, in such proportion that by selling the mixture at $2.80 per gallon he made a profit of 10%. Find the proportion in which the wines were mixed.

5. When wheat is worth 90c. per bushel, 17 bus. of a mixture of wheat and oats are worth $12.55; but if the proportions in the mixture were interchanged, its value would be $8.70. Find the price of oats per bushel.

6. A cask contains 7 parts of brandy and 5 parts of water; $\frac{1}{7}$ of the mixture is drawn off, and the cask filled with water. What is the strength of the mixture then?

7. A mixture of 50 gal. of alcohol and water contains 80% alcohol. (*a*) How much water must be added to reduce the strength to $62\frac{1}{2}\%$? (*b*) How much alcohol must be added to increase its strength to $87\frac{1}{2}\%$?

8. How much water must be added to a mixture of 16 qts. of vinegar at 13c. and 10 qts. at 10c. that the whole mixture may be worth 11c. the quart?

COMMERCIAL ARITHMETIC

TEST PAPER—No. 11

AVERAGE AND ITS APPLICATION

Your work for this paper is to be submitted to the instructor for correction and criticism.

1. A grocer wishes to mix 20 lb. of sugar, worth 9c. per pound, and 10 lb. worth 12c. per pound, with clarified sugar, worth 15c. per pound, so that the compound may sell for 13c. How much of the clarified must he take ?

2. Two men and five boys can do a piece of work in 20 days, and one man and eight boys can do it in 18 days. In what time can three men and three boys do the work ?

3. On June 1, 1924, A and B formed a co-partnership for 1 yr., each investing $5,000. On September 1, each invested $1,000. On November 1, A withdrew $1,500 and B invested $500, and on April 1, 1925, A withdrew $2,000 and B $3,000. At the end of the year the partnership was dissolved with a net gain of $6,000. What proportion of the gain should each partner then receive ?

NOTE.—Gain shared according to average net investment. Calculate the time in months.

4. Wade, Harmon and Ferris engaged in business on January 1, 1925, Wade investing $8,000. Harmon $5,000, and Ferris $6,000. On May 1, Wade put in $2,000 additional ; on July 1, he withdrew $3,000 ; and on October 1, he put in $1,500. On February 1, Harmon withdrew $1,000 ; on April 1, he put in $4,000 ; on August 1, he withdrew $500 ; and on September 1, he put in $3,000 On July 1, Ferris put in $2,500 additional. At the end of the year the net gain was found to be $7,000. What was each partner's

share of the gain if it was divided in proportion to their average investments ? (Calculate time in months.)

5. Of $150,000.00 loaned by a bank in one locality, $50,000 is earning 5½%, $40,000.00 earns 6%, $25,000.00 earns 7%, and the balance earns 8%. What is the average rate earned on the whole ?

6. The average height of six men is 5 ft. 4 in. What must be the height of a seventh man in order that the average height of the seven may be 5 ft. 5 in. ?

7. On four consecutive days a train arrived at a certain station 15 min., 10 min. 30 sec., 11 min. 47 sec., and 13 min. 23 sec., respectively, past 10 a.m. If the train was on an average 3¼ min. late on these four days, at what time was it due at that station ?

8. I bought 50 yd. of calico, part at 13c. a yard, and the remainder at 18c. a yard, and paid $7.62 for the whole. How many yards did I buy at each price ?

Keeping Going

A railroad recently elected a new president. I read attentively the newspaper stories of his career, and nowhere did I discover any intimation that he is a genius, or endowed with brilliancy beyond the measure of his fellows. Rather, the reports laid emphasis on the fact that he began work in the lowest ranks, and for thirty-five years had plugged steadily ahead.

It reminded me of the conversation which took place between a friend of mine and the conductor of that railroad's crack train.

"We do not seem to be going very much faster than an ordinary train," said my friend as he puffed his cigar in the comfortable club-car. "I suppose that is a tribute to the smoothness of the road-bed."

"Not entirely," replied the conductor. "The truth is, we don't go very much faster than an ordinary train."

"No?" exclaimed my friend in surprise.

"Not very much faster," the conductor repeated; "but we keep on going all the time."

Every single year in New York I am more impressed with the certain triumph of steady-going mediocrity. The meek—which is to say, the pluggers—do verily inherit the earth.

"Do you know who it is that loses money in Wall Street?" a veteran broker asked once. "It is the temperamental men who are easily enthused and easily depressed. They get discouraged when things don't go right, and sell out at the bottom. The stolid man takes another hitch in his belt, hangs on and rides up again."

Said the president of one of the biggest corporations in the world: "There are only two brilliant minds in this whole organization." The brilliant minds occupy places of importance and profit, but not the top places. The top places are held by men who have arrived there through steady application and staying power.

I remember the first high-salaried man on whom my youthful eyes ever rested. I was a high-school kid, and an older friend pointed him out to dazzle me. "There," he whispered in awed tones, "is a ten-thousand-dollar man."

That was many years ago. The other day I met that glittering phenomenon. He is middle-aged and not at all dazzling; hundreds of pluggers, who were hardly making ends meet when he was in his glory, are far ahead of him to-day.

I do not covet for my sons any brilliance, any magnetism, any striking personal attributes that will open all doors to them at a touch. I covet for them hard jaws and firm muscles, the capacity to stay with their work an hour longer than the other fellow, and to hold on when more brilliant men have decided that the thing is too slow, and have quit.

I propose that they add a thirteenth exercise to their daily dozen—this to consist in reading this brief autobiographical note:

"Five times received I forty stripes save one; thrice was I beaten with rods, once was I stoned, thrice I suffered shipwreck; a night and a day have I been in the deep. In journeyings often, in perils of water, in perils of robbers . . . in weariness and painfulness . . . in hunger and thirst . . . in cold and nakedness . . ."

Some life, wasn't it? Yet the hand which penned those words also wrote: "I have fought a good fight, I have finished my course, I have kept the faith."

He kept the faith and kept going; and his soul still marches on.

—Bruce Barton.

COMPOUND INTEREST

350. In computing simple interest the principal is the same throughout the whole time. In reckoning compound interest, it increases with each interest period—the amount at the end of the first period becoming the principal for the second, that at the end of the second becoming the principal for the third, and so on.

351. Unless otherwise stated, the interest is compounded annually, thus making the interest period one year. But it may be compounded half-yearly, quarterly, or at any other time agreed upon.

Suppose A borrows from B $100 for 3 yrs., and agrees to pay him compound interest at the rate of 5%. At the end of the first year A would owe B $105. For the second year he would pay interest on $105. At the end of the second year he would owe $110.25. For the third year he would pay interest on $110.25, and at the end of that year would owe $115.76. The compound interest, therefore, on $100 for 3 yrs. at 5% is $115.76 − $100 = $15.76.

352. QUESTIONS OF THE FIRST ASPECT

(Without the use of interest tables)

Illustration 1.—Find the compound interest on $640 for 3 years at 4% per annum, compounded yearly.

SOLUTION 1

Principal	$640
Interest for 1st year ($640 × .04)	25.60
Amount for 1st year or principal for 2nd year	$665.60
Interest for 2nd year ($665.60 × .04)	26.624
Amount for 2nd year or principal for 3rd year	$692.224
Interest for 3rd year ($692.224 × .04)	27.68896
Amount for 3rd year	$719.91296
Original principal to be subtracted	640
Compound interest for 3 yrs. to nearest cent	$ 79.91

NOTE 1.—If interest is compounded semi-annually, work in exactly the same manner as above, but take $\frac{1}{2}$ the rate for twice the number of years. If quarterly, take $\frac{1}{4}$ the rate for 4 times the number of years, etc.

NOTE 2.—If the time is not an exact number of interest periods, calculate the interest for the integral number of periods; then, for the remaining time, take the interest for the full period, multiply it by the fraction which this remaining time is of a full period, and add to the amount for the exact periods.

SOLUTION 2

The interest for 1 year $= \frac{4}{100}$ of principal

The amount at end of 1 year $= \frac{104}{100}$ of principal

The interest for 2nd year $= \frac{4}{100}$ of this amount

The amount at end of 2nd year $= \frac{104}{100}$ of this amount

$= \frac{104}{100}$ of $\frac{104}{100}$ of principal

$= \left(\frac{104}{100}\right)^2$ of principal

Similarly, the amount at end of 3rd year $= \left(\frac{104}{100}\right)^3$ of principal

$= (1.04)^3$ of principal

$= \$640 \times (1.04)^3$

Hence the interest for the 3 yrs. $= 640 \times (1.04)^3 - 640$

$= 640\{(1.04)^3 - 1\}$

The above solution will be found very valuable in finding the compound interest or the amount when the number of years is large and no interest table is available. This solution really depends on finding 1.04^3, which in this case is easily found. If the time had been 16 yrs., however, we would have to find 1.04^{16}, which by multiplying by individual factors would be a laborious process, but may be shortened by squaring 1.04, then squaring the result, which would give 1.04^4, then squaring this result, which would give 1.04^8, and again squaring this result, which would give 1.04^{16}, in each case using the contracted method of multiplication of decimals.

A little thought will show how this plan may be adapted to find any power for any rate. Thus, 1.05^{20} would be got by finding 1.05^{16} as above, and multiplying this result by 1.05^4, and so on.

Compound Interest Table

Showing the amount of $1 at compound interest for any number of years from 1 year to 55 years inclusive

Yrs.	1 per ct.	1½ per ct.	2 per ct.	2½ per ct.	3 per ct.	3½ per ct.	4 per ct.	Yrs.
1	1.0100 000	1.0150 000	1.0200 0000	1.0250 0000	1.0300 0000	1.0350 0000	1.0400 0000	1
2	1.0201 000	1.0302 250	1.0404 0000	1.0506 2500	1.0609 0000	1.0712 2500	1.0816 0000	2
3	1.0303 010	1.0456 784	1.0612 0800	1.0768 9062	1.0927 2700	1.1087 1787	1.1248 6400	3
4	1.0406 040	1.0613 636	1.0824 3216	1.1038 1289	1.1255 0881	1.1475 2300	1.1698 5856	4
5	1.0510 101	1.0772 840	1.1040 8080	1.1314 0821	1.1592 7407	1.1876 8631	1.2166 5290	5
6	1.0615 202	1.0934 433	1.1261 6242	1.1596 9342	1.1940 5230	1.2292 5533	1.2653 1902	6
7	1.0721 354	1.1098 450	1.1486 8567	1.1886 8575	1.2298 7387	1.2722 7926	1.3159 3178	7
8	1.0828 567	1.1264 926	1.1716 5938	1.2184 0290	1.2667 7008	1.3168 0904	1.3685 6905	8
9	1.0936 853	1.1433 900	1.1950 9257	1.2488 6297	1.3047 7318	1.3628 9735	1.4233 1181	9
10	1.1046 221	1.1605 408	1.2189 9442	1.2800 8454	1.3439 1638	1.4105 9876	1.4802 4428	10
11	1.1156 683	1.1779 489	1.2433 7431	1.3120 8666	1.3842 3387	1.4599 6972	1.5394 5406	11
12	1.1268 250	1.1956 182	1.2682 4179	1.3448 8382	1.4257 6089	1.5110 6686	1.6010 3222	12
13	1.1380 933	1.2135 524	1.2936 0663	1.3785 1104	1.4685 3371	1.5639 5606	1.6650 7351	13
14	1.1494 742	1.2317 557	1.3194 7876	1.4129 7382	1.5125 8972	1.6186 9452	1.7316 7645	14
15	1.1609 690	1.2502 321	1.3458 6834	1.4482 9817	1.5579 6742	1.6753 4883	1.8009 4351	15
16	1.1725 786	1.2689 855	1.3727 8570	1.4845 0562	1.6047 0644	1.7339 8601	1.8729 8125	16
17	1.1843 044	1.2880 203	1.4002 4142	1.5216 1826	1.6528 4763	1.7946 7555	1.9479 0050	17
18	1.1961 475	1.3073 406	1.4282 4625	1.5596 5872	1.7024 3306	1.8574 8920	2.0258 1652	18
19	1.2081 090	1.3269 507	1.4568 1117	1.5986 5019	1.7535 0605	1.9225 0132	2.1068 4918	19
20	1.2201 900	1.3468 550	1.4859 4740	1.6386 1644	1.8061 1123	1.9897 8886	2.1911 2514	20
21	1.2323 919	1.3670 578	1.5156 6634	1.6795 8185	1.8602 9457	2.0594 3147	2.2787 6807	21
22	1.2447 159	1.3875 637	1.5459 7967	1.7215 7140	1.9161 0341	2.1315 1152	2.3699 1879	22
23	1.2571 630	1.4083 772	1.5768 9926	1.7646 1068	1.9735 8651	2.2061 1448	2.4647 1555	23
24	1.2697 346	1.4295 028	1.6084 3725	1.8087 2595	2.0327 9411	2.2833 2849	2.5633 0417	24
25	1.2824 320	1.4509 454	1.6406 0599	1.8539 4410	2.0937 7793	2.3632 4498	2.6658 3633	25
26	1.2952 563	1.4727 095	1.6734 1811	1.9002 9270	2.1565 9127	2.4459 5856	2.7724 6979	26
27	1.3082 089	1.4948 002	1.7068 8648	1.9478 0002	2.2212 8901	2.5315 6711	2.8833 6858	27
28	1.3212 910	1.5172 222	1.7410 2421	1.9964 9502	2.2879 2768	2.6201 7196	2.9987 0332	28
29	1.3345 039	1.5399 805	1.7758 4469	2.0464 0739	2.3565 6551	2.7118 7798	3.1186 5145	29
30	1.3478 490	1.5630 802	1.8113 6158	2.0975 6758	2.4272 6247	2.8067 9370	3.2433 9751	30
31	1.3613 274	1.5865 264	1.8475 8882	2.1500 0677	2.5000 8035	2.9050 3148	3.3731 3341	31
32	1.3749 407	1.6103 243	1.8845 4059	2.2037 5694	2.5750 8276	3.0067 0759	3.5080 5875	32
33	1.3886 901	1.6344 792	1.9222 3140	2.2588 5086	2.6523 3524	3.1119 4235	3.6483 8110	33
34	1.4025 770	1.6589 964	1.9606 7603	2.3153 2213	2.7319 0530	3.2208 6033	3.7943 1634	34
35	1.4166 028	1.6838 813	1.9998 8955	2.3732 0519	2.8138 6245	3.3335 9045	3.9460 8899	35
36	1.4307 688	1.7091 395	2.0398 8734	2.4325 3532	2.8982 7833	3.4502 6611	4.1039 3255	36
37	1.4450 765	1.7347 766	2.0806 8509	2.4933 4870	2.9852 2668	3.5710 2543	4.2680 8985	37
38	1.4595 272	1.7607 983	2.1222 9879	2.5556 8242	3.0747 8348	3.6959 1132	4.4388 1345	38
39	1.4741 225	1.7872 103	2.1647 4477	2.6195 7448	3.1670 2698	3.8253 7171	4.6163 6599	39
40	1.4888 637	1.8140 184	2.2080 3966	2.6850 6384	3.2620 3779	3.9592 5972	4.8010 2063	40
41	1.5037 524	1.8412 287	2.2522 0046	2.7521 9043	3.3598 9893	4.0978 3381	4.9930 6145	41
42	1.5187 899	1.8688 471	2.2972 4447	2.8209 9520	3.4606 9589	4.2412 5799	5.1927 8391	42
43	1.5339 778	1.8968 798	2.3431 8936	2.8915 2008	3.5645 1677	4.3897 0202	5.4005 9527	43
44	1.5493 176	1.9253 330	2.3900 5314	2.9638 0808	3.6714 5227	4.5433 4160	5.6165 1508	44
45	1.5648 107	1.9542 130	2.4378 5421	3.0379 0328	3.7815 9584	4.7023 5855	5.8411 7568	45
46	1.5804 589	1.9835 262	2.4866 1129	3.1138 5086	3.8950 4372	4.8669 4110	6.0748 2271	46
47	1.5962 634	2.0132 791	2.5363 4351	3.1916 9713	4.0118 9503	5.0372 8404	6.3178 1562	47
48	1.6122 261	2.0434 783	2.5870 7039	3.2714 8956	4.1322 5188	5.2135 8398	6.5705 2824	48
49	1.6283 483	2.0741 305	2.6388 1179	3.3532 7680	4.2562 1944	5.3960 6459	6.8333 4937	49
50	1.6446 318	2.1052 424	2.6915 8803	3.4371 0872	4.3839 0602	5.5849 2686	7.1066 8335	50
51	1.6610 781	2.1368 211	2.7454 1979	3.5230 3644	4.5154 2320	5.7803 9930	7.3909 5068	51
52	1.6776 889	2.1688 734	2.8003 2819	3.6111 1235	4.6508 8590	5.9827 1327	7.6865 8871	52
53	1.6944 658	2.2014 065	2.8563 3475	3.7013 9016	4.7904 1247	6.1921 0824	7.9940 5226	53
54	1.7114 105	2.2344 276	2.9134 6144	3.7939 2491	4.9341 2485	6.4088 3202	8.3138 1435	54
55	1.7285 246	2.2679 439	2.9717 3067	3.8887 7303	5.0821 4859	6.6331 4114	8.6463 6692	55

NOTE 1.—To find the amount to which any given principal, at compound interest, will increase at any rate per annum, and for any number of years given in the preceding tables:

Multiply the given principal by the amount of $1 of principal for the given time and rate per annum, as shown in the table.

NOTE 2.—To find the compound interest which any given principal will produce at any rate per annum, and for any number of years given in the preceding tables:

Subtract $1 from the amount of $1 for the given time and rate per annum, as shown in the table; the result will be the compound interest of $1 of principal. Multiply the compound interest of $1 of principal by the given principal.

Compound Interest Table

Showing the amount of $1 at compound interest for any number of years from 1 year to 55 years inclusive.

Yrs.	4½ per ct.	5 per ct.	6 per ct.	7 per ct.	8 per ct.	9 per ct.	10 per ct.	Yrs.
1	1.0450 0000	1.0500 000	1.0600 000	1.0700 000	1.0800 000	1.0900 000	1.1000 000	1
2	1.0920 2500	1.1025 000	1.1236 000	1.1449 000	1.1664 000	1.1881 000	1.2100 000	2
3	1.1411 6612	1.1576 250	1.1910 160	1.2250 430	1.2597 120	1.2950 290	1.3310 000	3
4	1.1925 1860	1.2155 063	1.2624 770	1.3107 960	1.3604 890	1.4115 816	1.4641 000	4
5	1.2461 8194	1.2762 816	1.3382 256	1.4025 517	1.4693 281	1.5386 240	1.6105 100	5
6	1.3022 6012	1.3400 956	1.4185 191	1.5007 304	1.5868 743	1.6771 001	1.7715 610	6
7	1.3608 6183	1.4071 004	1.5036 303	1.6057 815	1.7138 243	1.8280 391	1.9487 171	7
8	1.4221 0061	1.4774 554	1.5938 481	1.7181 862	1.8509 302	1.9925 626	2.1435 888	8
9	1.4860 9514	1.5513 282	1.6894 790	1.8384 592	1.9990 046	2.1718 933	2.3579 477	9
10	1.5529 6942	1.6288 946	1.7908 477	1.9671 514	2.1589 250	2.3673 637	2.5937 425	10
11	1.6228 5505	1.7103 394	1.8982 986	2.1048 520	2.3316 390	2.5804 264	2.8531 167	11
12	1.6958 8143	1.7958 563	2.0121 965	2.2521 916	2.5181 701	2.8126 648	3.1384 284	12
13	1.7721 9610	1.8856 491	2.1329 283	2.4098 450	2.7196 237	3.0658 046	3.4522 712	13
14	1.8519 4492	1.9799 316	2.2609 040	2.5785 342	2.9371 936	3.3417 270	3.7974 983	14
15	1.9352 8244	2.0789 282	2.3965 582	2.7590 315	3.1721 691	3.6424 825	4.1772 482	15
16	2.0223 7015	2.1828 746	2.5403 517	2.9521 638	3.4259 426	3.9703 059	4.5949 730	16
17	2.1133 7681	2.2920 183	2.6927 728	3.1588 152	3.7000 181	4.3276 334	5.0544 703	17
18	2.2084 7877	2.4066 192	2.8543 392	3.3799 323	3.9960 195	4.7171 204	5.5599 173	18
19	2.3078 6031	2.5269 502	3.0255 995	3.6165 275	4.3157 011	5.1416 613	6.1159 390	19
20	2.4117 1402	2.6532 977	3.2071 305	3.8696 845	4.6609 571	5.6044 108	6.7275 000	20
21	2.5202 4116	2.7859 626	3.3995 636	4.1405 624	5.0338 337	6.1088 077	7.4002 499	21
22	2.6336 5201	2.9252 607	3.6035 374	4.4304 017	5.4365 404	6.6586 004	8.1402 749	22
23	2.7521 6635	3.0715 238	3.8197 497	4.7405 299	5.8714 637	7.2578 745	8.9543 024	23
24	2.8760 1383	3.2250 999	4.0489 346	5.0723 670	6.3411 807	7.9110 832	9.8497 327	24
25	3.0054 3446	3.3863 549	4.2918 707	5.4274 326	6.8484 752	8.6230 807	10.8347 059	25
26	3.1406 7901	3.5556 727	4.5493 830	5.8073 529	7.3963 532	9.3991 579	11.9181 765	26
27	3.2820 0956	3.7334 563	4.8223 459	6.2138 676	7.9880 615	10.2450 821	13.1099 942	27
28	3.4296 9999	3.9201 291	5.1116 867	6.6488 384	8.6271 064	11.1671 395	14.4209 936	28
29	3.5840 3649	4.1161 356	5.4183 879	7.1142 571	9.3172 749	12.1721 821	15.8630 930	29
30	3.7453 1813	4.3219 424	5.7434 912	7.6122 550	10.0626 569	13.2676 785	17.4494 023	30
31	3.9138 5745	4.5380 395	6.0881 006	8.1451 129	10.8676 694	14.4617 695	19.1943 425	31
32	4.0899 8104	4.7649 415	6.4533 867	8.7152 708	11.7370 830	15.7633 288	21.1137 768	32
33	4.2740 3018	5.0031 885	6.8405 899	9.3253 398	12.6760 496	17.1820 284	23.2251 544	33
34	4.4663 6154	5.2533 480	7.2510 253	9.9781 135	13.6901 336	18.7284 109	25.5476 699	34
35	4.6673 4781	5.5160 154	7.6860 868	10.6765 815	14.7853 443	20.4139 679	28.1024 369	35
36	4.8773 7846	5.7918 161	8.1472 520	11.4239 422	15.9681 718	22.2512 250	30.9126 805	36
37	5.0968 6049	6.0814 069	8.6360 871	12.2236 181	17.2456 256	24.2538 353	34.0039 486	37
38	5.3262 1921	6.3854 773	9.1542 524	13.0792 714	18.6252 756	26.4366 805	37 4043 434	38
39	5.5658 9908	6.7047 512	9.7035 075	13.9948 204	20.1152 977	28.8159 817	41.1447 778	39
40	5.8163 6454	7.0399 887	10.2857 179	14.9744 578	21.7245 215	31.4094 200	45.2592 556	40
41	6.0781 0094	7.3919 882	10.9028 610	16.0226 699	23.4624 832	34.2562 679	49.7851 811	41
42	6.3516 1548	7.7615 876	11.5570 327	17.1442 568	25.3394 810	37.3175 320	54.7636 992	42
43	6.6374 5818	8.1496 669	12.2504 546	18.3443 548	27.3666 404	40.6761 098	60.2400 692	43
44	6.9361 2290	8.5571 503	12.9854 819	19.6284 596	29.5559 717	44.3369 597	66.2640 761	44
45	7.2482 4843	8.9850 078	13.7646 108	21.0024 518	31.9204 494	48.3272 861	72.8904 837	45
46	7.5744 1961	9.4342 582	14.5904 875	22.4726 234	34.4740 853	52.6767 419	80.1795 321	46
47	7.9152 6849	9.9059 711	15.4659 167	24.0457 070	37.2320 122	57.4176 486	88.1974 853	47
48	8.2714 5557	10.4012 697	16.3938 717	25.7289 065	40.2105 731	62.5852 370	97.0172 338	48
49	8.6436 7107	10.9213 351	17.3775 040	27.5299 300	43.4274 190	68.2179 083	106.7189 572	49
50	9.0326 3627	11.4673 998	18.4201 543	29.4570 251	46.9016 125	74 3575 201	117.3908 529	50
51	9.4391 0490	12.0407 698	19.5253 635	31.5190 168	50.6537 415	81.0496 969	129.1299 382	51
52	9.8638 6463	12.6428 083	20.6968 853	33.7253 480	54.7060 408	88.3441 696	142.0429 320	52
53	10.3077 3853	13.2749 487	21.9386 985	36.0861 224	59.0825 241	96.2951 449	156.2472 252	53
54	10.7715 8677	13.9386 961	23.2550 204	38.6121 509	63.8091 260	104.9617 079	171.8719 477	54
55	11.2563 0817	14.6356 309	24.6503 216	41.3150 015	68.9138 561	114.4083 616	189.0591 425	55

NOTE 3.—The amount of $1 for more than 55 years is equal to the product of the amount of $1 for any two or more numbers of years in the table whose sum is equal to the given time. Thus, the amount of $1 at compound interest for 98 years at 8 per cent. per annum, is equal to the product of the amount of $1, at 8 per cent., for fifty years ($46.9016125) multiplied by the amount of $1, at 8 per cent., for 48 years (40.2105731), that is, $46.9016125 × 40.2105731, or $1885.94071794+.

NOTE 4.—When the interest is to be compounded semi-annually, find, from the table, the amount or interest at *one-half* the rate per annum for twice the number of years. Thus, the compound interest of $1 for 10 years at 5 per cent. per annum, *payable semi-annually*, is the same as the compound interest of $1 for 20 years at 2½ per cent. per annum, *payable annually*; and the compound interest of any sum for 8 years at 4 per cent. per annum, *compounded quarterly* is the same as the compound interest of the same sum for 32 years, at 1 per cent. per annum, *compounded annually*.

EXERCISE 108

1. Find the compound interest on $7,500 for 3 yrs. at 5%, compounded annually.

2. Find the compound interest on $3,700 for 4 yrs. at 7%, compounded annually.

3. Find the compound interest on $3,250 for 3 yrs. at 6%, compounded semi-annually.

4. Find the amount of $1,850 for 1 yr. 6 mos. at 8%, compounded quarterly.

5. What is the compound amount of $1,520 for 5 yrs. at 5% ?

6. Find the compound interest on $1,415 from July 16, 1942, to Dec. 27 1947, at 3%.

7. What is the amount due Oct. 17, 1947, upon a note of $468.75, dated Nov. 12, 1943, and bearing interest at 5% per annum, compounded annually, no payment having previously been made ?

8. Find the compound interest on $250 for 2 yrs. at 5% per annum, compounded semi-annually.

9. By what decimal must a sum be multiplied to give the amount at compound interest for 4 yrs. at 4%, compounded yearly ?

10. Find 1.06^{12} correct to 6 places of decimals.

11. Find 1.04^{24} correct to 6 decimal places.

12. By the process suggested in the previous example find the compound interest on $362.75 for 15 yrs. at 7%, compounded yearly.

13. In the same way find the compound interest on $745.80 for 11 yrs. at 4% per annum, compounded semi-annually.

353. QUESTIONS OF THE SECOND ASPECT

(Interest table not to be used)

Illustration.—What sum will amount to $4,024.55 in 3 yrs. at 7% compound interest ?

SOLUTION

$1 will amount to $1.225043 in 3 yrs. at 7%.

$1.225043 is amount of $1.

$4,024.55 is amount of $4,024.55 ÷ $1.225043 = $3,285.23.

EXERCISE 109

1. What sum will amount to \$182.33 in 4 yrs. at 5% per annum, calculated yearly?

2. What is the present worth of \$984.54 for 3 yrs. at 4% compounded yearly?

3. What sum of money will give \$150 interest in 4 yrs. at $3\frac{1}{2}\%$ per annum, compounded yearly?

4. The difference between the simple and compound interest on a certain sum for 4 yrs. at 6%, compounded yearly, is \$100. Find the sum.

5. Find the present worth of \$367.25 for 3 yrs. at 5% per annum, compounded half-yearly.

6. What sum put out at interest now at $4\frac{1}{2}\%$ will in 3 yrs. at compound interest, compounded yearly, amount to \$1,200?

7. Find the true discount on \$1,450 for 11 yrs. at 3%, compounded yearly.

8. The compound interest, compounded yearly at 5%, on a certain sum from Jan. 10, 1941, to Sept. 17, 1947, was \$216.15. Find the sum.

354. QUESTIONS OF THE THIRD ASPECT

(Interest table to be used)

Illustration.—At what rate per annum will \$750 amount to \$2,549.6747 in 21 yrs.?

SOLUTION

\$750 amounts to \$2,549.6747

\$1 amounts to \$3.3995636

Then run the eye down the column of years until the given number of years (21) is reached, and thence to the right until the amount is reached finding it in the 6% column. Hence the required rate is 6%.

EXERCISE 110

1. At what rate will the compound interest on \$200 for 3 yrs. be \$38.81, if the interest is compounded half-yearly?

2. At what rate, compounded yearly, will \$9,600 in 17 yrs. amount to \$30,324.63?

3. The compound interest on \$1,250 for 16 yrs., compounded half-yearly, is $\$1,105.67\frac{1}{2}$. What is the rate?

4. The amount of $850 for 23 yrs., interest compounded yearly, is $1,677.5515. What is the rate?

355. QUESTIONS OF THE FOURTH ASPECT

(Interest tables to be used)

Illustration.—In what time will $5,000 amount to $6,100.95 if compounded quarterly at 4% per annum.

SOLUTION

$5,000 amounts to $6,100.95

$1 amounts to $1.22019

Since the given rate is 4% per annum, or 1% per quarter, run the eye down the 1% column until the amount $1.22019 is reached, which will be found to be the amount for 20 yrs., or, in this instance, 20 qrs. Hence, the required time is 20 qrs., or 5 yrs.

EXERCISE 111

1. In what time will $1,500 amount to $2,984.68½ at 3½% compounded yearly?

2. In what time will the compound interest on $800 at 5% per annum, compounded half-yearly, be $720.232?

3. In what time will $750 amount to $3,322.80 at 7% per annum, compounded yearly?

4. In what time will the compound interest on $625 be $463.13¾ if compounded quarterly at 8%?

EXERCISE 112

Work the following questions with the use of the compound interest tables—

1. Find the interest on $3,600 for 18 yrs. at 7%, compounded annually.

2. Find the amount of $787.30 for 36 yrs. if compounded annually at 5%.

3. Find the interest on $1,960 for 20 yrs. at 5%, compounded half-yearly.

4. Find the amount of $4,500 for 8 yrs. at 6%, compounded quarterly.

5. Find the compound interest on $1,600 for 28 yrs. at 7%, compounded yearly.

6. If $300 be deposited in a savings bank for 16 yrs., and the interest be compounded semi-annually at 7%, how much would the amount be at the end of the time ?

7. What sum will amount to $2,912.71 in 15 yrs. at 6%, compounded semi-annually ?

8. On what sum will the compound interest for 24 yrs. at 5% per annum, compounded yearly, be $4,105.31 ?

9. At what rate will $3,500 amount to $8,007.74688 if compounded semi-annually for 14 yrs. ?

10. At what rate will $750 produce $1,355.09528 interest if compounded semi-annually for 15 yrs. ?

EXERCISE 113

1. In what time will $750 amount to $1,578.639 if compounded annually at 7% ?

2. In what time will $100 yield $75.350605 interest if compounded half-yearly at 6% ?

3. What rate per cent. per annum, compounded yearly, is equivalent to 6% per annum compounded half-yearly ?

4. What rate per cent. per annum, compounded half-yearly. is equivalent to 6% per annum compounded yearly ?

5. The difference between the simple and compound interest on a certain sum for 4 yrs. at 6%, compounded yearly, is $100. Find the sum.

EXAMINATION No. 5

(PERCENTAGE)

80% *required.* *Time* 2½ *hrs.*

1. A grocer gained 20% by selling 20 lbs. sugar for $1. Afterward he increased his price, giving only 18 lbs. for $1. How much per cent. did he make at the increased price ?

2. What per cent. is made by investing in 4½% stock at 75 ?

3. A merchant bought goods at 25% and 20% off, and sold at 20% and 10% off. What per cent. profit did he make ?

4. Find the per cent. of commission on a purchase, if the gross cost is $2,048.51, the commission $87.30, the cartage $20, and other charges $1.21.

5. A merchant failed with liabilities amounting to $50,000; his assets in merchandise are $20,000; in real estate, $8,000; in accounts receivable, $12,000. If the assignee received 10% of the assets for making the settlement, how many cents on the dollar will the creditors receive?

6. A merchant has a stock of goods valued at $1,200, and a store building valued at $9,000. At $\frac{3}{4}$%, what amount of premium does he pay on a policy if the insurance is on a $\frac{2}{3}$ valuation?

7. At what price must I mark cloth which cost me $2.40 per yard, so that after throwing off 20% I may still make a gain of 20%?

8. At 50% ad valorem, the duty on 1,580 yds. of silk was $1,975. What was the invoice price per yard, and for what price per yard must the importer sell it to gain 20%?

9. A city hall costing $36,750 was built by a special tax upon the property of the city. The rate of taxation being 5 mills on the dollar and the cost of collection 2%, find the total assessment.

10. A business man reckons his percentages on his turnover. If the turnover is $36,729.28, wages $15,387.20, salaries $8,756.80, gross profit $26,239.50, and net profit $13,729.20, calculate his percentage of wages, salaries, gross profit, and net profit.

EXAMINATION No. 6

80% required. Time 2½ hours.

1. Find the income derived from $22,831.50 invested in Bank stock, which sells at 184 and pays a dividend of 8% per annum, brokerage being 12½c. per share.

2. Bought a bill of hardware amounting to $1,560: of this $125 less 10% and 10%; $268 less 40% and 10%; $600 less 25% and 20%; the balance of the bill 33⅓% and 10%. What is the net price?

3. The property of a town is assessed at $2,600,000; the rate of taxation is 17 mills on the dollar; it costs 2% of the tax for collection. Find the net amount received by the town.

4. A man sold 2 lots for $445, gaining 12½% on one and losing 12½% on the other. Find the cost of each, if he gained $5 on the whole transaction.

5. A merchant had 500 bbls. of flour insured for $\frac{3}{4}$ of its value

at 2½%, paying $75 premium. At what price per barrel must he sell to gain 25% of cost, as well as premium paid?

6. What is the difference between a discount of 40% and three discounts of 20, 10, and 10%?

7. At what price must I mark goods which cost $16.20 so that I can allow a discount of 10% on marked price, and still make a profit of 10% on cost, allowing 10% of sales for bad debts?

8. Find the total duty on the following at the rate of $7 per ton—

45 Steel angles, 3″ × 3″ × ¼″, at 4.9 lbs. per foot, 32 ft. long.

29 Steel angles, 2″ × 2″ × ¼″, at 3.2 lbs. per foot, 32 ft. long.

30 Steel beams, 4 in., at 7½ lbs. per foot, 38 ft. long.

9. A sells goods to B at a gain of 12%, and B sells the same goods to C at a gain of 7½%; C paid $3,762.50 for the goods. How much did A pay for them?

10. A corporation, having net earnings of $5,665, wishes to declare a dividend after reserving a working fund of $2,000. The corporation organized with a capital stock of 250 shares, but has since issued 90 shares of preferred stock, secured at 6%. What rate of dividend can they declare?

EXAMINATION No. 7

80% *required. Time* 2½ *hours.*

1. The profits of a loan company for a year were sufficient to enable the directors to add $20,000 to their reserve fund, to pay $5,965 for cost of management, to pay two half-yearly dividends of 3½% on a paid-up capital stock of $309,056, and to have still on hand $4,236. Find the profits for the year.

2. I purchased 12 shares of C.P.R. stock at 112½, and, after receiving a semi-annual dividend thereon of 3½%, sold again at 135, brokerage in each case 25c. per share. What rate per cent. per annum did I realize on my investment if the shares were in my possession just six months?

3. The assignee of a bankrupt firm sold its realty for $9,000, and its merchandise for $7,000. He collected $2,000 on bills receivable, and $500 on personal accounts. The statement showed the liabilities to be $27,000. If A's claim is $7,200, how much would he receive, the expenses of the assignment being $500?

4. A farmer took the following insurance in the Perth Mutual: House, valued at \$6,000, for $\frac{3}{4}$ valuation at $1\frac{1}{4}\%$; barn, valued at \$4,500, for $\frac{2}{3}$ valuation at $1\frac{1}{2}\%$; live stock for \$5,500 at $\frac{3}{4}\%$; grain for \$3,000 at $\frac{1}{2}\%$. What was his total premium?

5. I bought a bicycle for \$96. I wish to make 25% profit and yet offer $33\frac{1}{3}\%$ and 10% off. How much must I ask for it?

6. A tailor buys cloth at \$1.75 a yard, which, in sponging, shrinks 5%. At what price per yard must he sell it to gain 20% on his outlay?

7. A house and lot were offered for sale at an advance of 50% above their cost; but the agent sold the house at 20% below the asking price, receiving \$500 more than the cost. What is the owner's per cent. gain, after allowing the agent 2% for his services?

8. An insurance agent takes a risk of \$20,000 on a vessel at $2\frac{1}{4}\%$, and immediately re-insures $\frac{1}{2}$ of the risk at $2\frac{3}{4}\%$. What net premium does he receive?

9. A merchant invested \$42,500 in dry goods. The first year he gained 20%, the second year he gained $12\frac{1}{2}\%$, the third year he lost $16\frac{2}{3}\%$, and the fourth year he gained 5%. Find the amount invested in business at the end of the fourth year, the gain being added or the loss subtracted each year.

10. A house that cost \$15,500 rents for \$155 a month. It is insured for \$10,850 at $\frac{4}{5}\%$ yearly; the taxes are 15 mills on an assessment of \$12,450, and \$346.45 is spent each year on repairs. What rate of interest does the investment pay?

COMMERCIAL ARITHMETIC

TEST PAPER—No. 12

COMPOUND INTEREST

Your work for this paper is to be submitted to the instructor for correction and criticism.

1. (*a*) Find the Compound Interest on \$1,800 for 3 yr. at 4% compounded annually. (Do not use tables.)

(*b*) What sum will amount to \$27,783 in 3 yr. at 5% ? (Do not use tables.)

2. Find the Compound Interest on \$478.25 for 16 yr. and 175 days at 5% per annum.

3. Give the interest on \$1,250 for 4 yr. and 6 months at 6% per annum as follows—

(*a*) At simple interest.

(*b*) Due and compounded half-yearly.

(*c*) Due yearly with interest on interest.

4. The difference between the simple and compound interest on a sum of money for 2 yr. at 5% is \$12.00. Find the sum.

5. The compound interest, compounded yearly at 5%, on a certain sum from January 10, 1919, to September 17, 1925, was \$432.30. Find the sum.

6. Which is the most profitable investment ?

(*a*) Deposit in Savings Bank at $3\frac{1}{2}$% per annum, compounded half-yearly.

(*b*) Imperial Bank Stock at 204. Dividend 6% half-yearly.

(*c*) Bank of Montreal Stock at 266. Dividend 7% half-yearly.

SECTION 3

FOREIGN TRADE

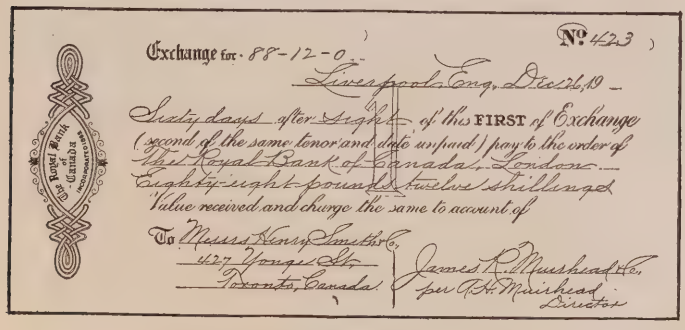
BILLS OF EXCHANGE

356. Importing goods into Canada makes it necessary that remittances of money should be made in payment of these goods to the persons from whom the goods are bought.

As in the case of cancelling domestic debts, where the creditor lives in a different city in Canada, we find that the usual custom in making a foreign remittance is to purchase a bill of exchange, which may in turn be remitted in cancellation of the debt.

357. Foreign Bills of Exchange are usually drawn in sets of two to provide against loss in transmitting. This custom is a long-established one and still obtains, although the reason for the custom in these days of safe and rapid ocean transit is not so apparent. The wording of each of the bills in a set of exchange will make it clear that the honoring of one practically cancels the other. The following form will illustrate the first of a set of foreign exchange—

358. The First of a Set of Exchange


Exchange for £88-12-0

No. 423

Liverpool, Eng., Dec. 26, 19—

Sixty days after sight of this FIRST of Exchange (second of the same tenor and date unpaid) pay to the order of The Royal Bank of Canada, London

Eighty-eight pounds twelve shillings

Value received and charge the same to account of

To Messrs Henry Smith & Co.
427 Yonge St.
Toronto, Canada

James K. L. Muirhead & Co.
per A. H. Muirhead
Director

The Royal Bank of Canada

Foreign Bill of Exchange (Foreign Draft)

359. The **Par of Exchange** is the established value of the monetary unit of one country expressed in the currency of the other.

Thus, we read in our Canadian Currency Act that "The currency of Canada shall be such that the British sovereign of the weight and fineness now prescribed by the laws of the United Kingdom shall be equal to and pass current for $4.86\frac{2}{3}$ of the currency of Canada." This means that the par of exchange between Canada and Great Britain is $4.86\frac{2}{3}$ for a sovereign or one pound sterling.

In the same way the value of the monetary units of all foreign countries has been established for us, and the table we append herewith will give these values.

360. The **Rate of Exchange** is the market value of the monetary unit of one country estimated in the currency of another. Sterling rates are quoted in dollars and cents per pound sterling. Other rates are quoted in cents per foreign unit.

Thus, we find that, while one pound is equal to $4.86\frac{2}{3}$ of our money, it does not always follow that in purchasing a sterling bill of exchange that we can get it at the par value. The bank is entitled to a commission for issuing the bill of exchange. This always has a tendency to increase the cost of the exchange above the par value. Then, again, foreign bills may be drawn at any length of time. The customary bills, however, are drawn either at sight or at sixty days' sight. It is natural that a quotation on a sight bill is going to be higher than a quotation on a sixty-day bill.

361. Peculiar Methods of Quoting the Rate of Exchange

Ordinarily we expect that the rate of exchange would be quoted by giving the value of the monetary unit of any country in terms of our own currency.

Thus, the obvious way of quoting the rate for sterling exchange would be to give the value of £1 in our currency. Accordingly, we may find that sterling exchange is quoted at $4.88, $4.89, and so on.

There is another method, however, formerly in use in Canada, of quoting sterling exchange, which the student will do well to recognize. This method of quoting is based on a reference to the old par of exchange, which used to be $4\frac{4}{9}$ for £1 sterling. As this was below the intrinsic value, the par was raised to its present figure of $4.86\frac{2}{3}$. The old par had been in use so long, however, that the custom grew up, even after the new par had been established, of quoting sterling exchange at a certain increase on the old par. Thus, if we find that sterling exchange is quoted at a premium of

Values of Foreign Money in Canadian Currency

Country.	Unit.		Grammes (fine gold).	Value in Cents.
Argentine	Peso	(100 centavos)	1.45161	96.49
" (paper)	Peso	(44 centavos gold)		42.45
Austria	Schilling	(100 groschen)	.21172086	14.069
Belgium	Belga	(100 centimes)	.2092109	13.9063
Bolivia	Boliviano	(100 centavos)	.5491786	36.50
Brazil	Milreis	(1000 reis)	.8217783	54.634
"	Cruzerio	(100 centavos)	.720	11.96306
Chili	Peso	(100 centavos)	.183057	12.1666
Colombia	Peso	(100 centavos)	1.464466	97.331
Cuba	Peso	(100 centavos)	1.5046316	100.
Denmark	Krone	(100 ore)	.4032258	27.799
Ecuador	Sucre	(100 centavos)	.300993	20.
Egypt	Pound E.	(100 piastres)	7.4375	494.307
France	Franc	(100 centimes)	.05895	3.9179
Great Britain	Pound	(20 shillings)	7.3223848	486.656
Greece	Drachma	(100 lepta)		1.30
Germany	Reichsmark	(100 pfennig)	.3584229	23.828
India	Rupee	(16 annas)	.5491786	36.50
Italy	Lira	(100 centesimi)	.079191	5.26316
Japan	Yen	(100 sen)	.75	49.8461
Liberia	Dollar	(100 cents)	1.5046316	100.
Mexico	Peso	(100 centavos)	.75	49.8461
Netherlands	Guilder	(100 cents)	.6048	40.196
Norway	Krone	(100 ore)	.4032258	26.799
Peru	Pound	(100 soles)	7.3223848	486.656
Portugal	Escudo	(100 centavos)	1.62585	108.056
Russia	Chervonetz	(1000 kopecks)	7.74234	514.55
Spain	Peseta	(100 centimos)	.2903225	19.2947
Sweden	Krona	(100 ore)	.4032258	26.799
Turkey	Piastre	(40 paras)	.066147	4.3965
United States	Dollar	(100 cents)	1.5046316	100.0
Uruguay	Peso	(100 centesimos)	1.556149	103.424
Venezuela	Bolivar	(100 centimos)	.2903225	19.2947

$9\frac{1}{2}\%$, or, which is the same thing, at $109\frac{1}{2}$, we are to understand this as being a premium of $9\frac{1}{2}\%$ on the old par of $\$4\frac{4}{5}$. This increase of $9\frac{1}{2}\%$, by the way, just brings us to the present par of $\$4.86\frac{2}{3}$, and explains the somewhat queer expression that sterling exchange is at par when it is at a premium of $9\frac{1}{2}\%$. This means that it is at the present par when it is at $9\frac{1}{2}\%$ premium over the old par. If sterling exchange is quoted at 10% premium, it means 10% premium on the old par of $\$4\frac{4}{5}$. In this way any percentage may be quoted on this old par.

Exchange on France, in which country it will be noted in the table the franc is the monetary unit, would naturally be expected to be quoted by giving the value of 1 fr. in our money. Thus, a franc at par is worth 3.9179c.

Exchange on Germany, in which country the monetary unit is the reichsmark, we would naturally expect to be stated by giving the value of 1 reichsmark in our money. One mark at par is worth 23.828c. Therefore, if we find German exchange quoted at 23.828 or $23\frac{1}{4}$ or 24, the meaning will be quite clear. These figures will represent the value in cents of 1 reichsmark under different conditions.

DIRECT EXCHANGE

362. Questions of the First Aspect

Illustration 1.—Find the cost in Montreal of a bill of exchange on London, England, for £900 when exchange is quoted at $\$4.87\frac{1}{3}$.

SOLUTION

Cost of bill for £1 = $\$4.87\frac{1}{3}$

Cost of bill for £900 = $\$4.87\frac{1}{3} \times 900 = \$4,386$

Illustration 2.—How much must be paid in Toronto for a draft on Liverpool for £458 13s. 4d., exchange being quoted at $109\frac{3}{4}$?

SOLUTION

£458 13s. 4d. = $£458\frac{2}{3}$

Cost of draft for £1 when exchange is at $109\frac{3}{4} = \$4\frac{4}{5} \times 1.09\frac{3}{4}$

Cost of draft for $£458\frac{2}{3}$ when exchange is at $109\frac{3}{4} = \$4\frac{4}{5} \times 1.09\frac{3}{4} \times 458\frac{2}{3}$

$= \$2,237.27\frac{1}{27}$ or $\$2,237.27$

Illustration 3.—Find the cost of a bill of exchange on Berlin for 1,600 mks, when exchange is quoted at $23\frac{7}{16}$c.

SOLUTION

Cost of bill for 1 mark = $\$.23\frac{7}{16}$

Cost of bill for 1600 marks = $\$.23\frac{7}{16} \times 1600$

$= \frac{\$3.75}{16} \times 1600$

$= \$375.00$

EXERCISE 114

Find the cost of the following London drafts—

1. For £374 7s. 6d., exchange $4.80.
2. For £193 6s. 8d., exchange $4.85\frac{5}{9}$.
3. For £836 17s. 6d., exchange $4.88\frac{8}{9}$.
4. For £527 15s., exchange $108\frac{3}{4}$.
5. For £1,638 14s. 7d., exchange $9\frac{3}{4}\%$ premium.
6. For £947 11s. 9d., exchange $9\frac{1}{2}\%$ premium.
7. For £385 19s. 5d., exchange $8\frac{3}{4}\%$ premium.
8. For £593 16s. 4d., exchange $4.86\frac{3}{4}$.
9. For £267 5s. 9d., exchange par.
10. How much must be paid in Winnipeg for a bill of exchange on Paris for 3,048 fr. when exchange is quoted at $3.91\frac{3}{4}$c. ?
11. What is the cost in Toronto for a draft on New York for $3,479.37 when exchange is at $\frac{3}{8}\%$ premium ?
12. Find the cost of a draft on Berlin for 7,329 mks. when exchange is quoted at $25\frac{3}{8}$c.
13. What will it cost to cancel a debt in Yokohama for 850 yen when exchange is at par (49.8c.) ?
14. Find the cost of a bill of exchange on Amsterdam for 2,367 glds. when exchange is quoted at $40\frac{3}{8}$.
15. Sold through a broker a bill of exchange on Geneva for 3,650 frs. at 60 dys. sight. What were the proceeds of the draft, exchange being 19.3c., brokerage $\frac{1}{8}\%$?

363. Questions of the Second Aspect

Illustration 1.—The cost of a draft on London for £383 12s. 6d. was $1,854.18\frac{3}{4}$. Find the rate of exchange.

SOLUTION

£383 12s. 6d. = £383$\frac{5}{8}$
Draft for £383$\frac{5}{8}$ cost $1,854.18\frac{3}{4}$
Draft for £1 cost $1,854.18\frac{3}{4} \div 383\frac{5}{8} = \$4.83\frac{1}{3}$
$\$4.83\frac{1}{3} \div \$4\frac{1}{3} = 108\frac{3}{4}$
The rate of exchange may be given as $\$4.83\frac{1}{3}$, $108\frac{3}{4}$, or $8\frac{1}{4}\%$ premium.

Illustration 2.—The cost of a draft on Paris for 12,500 frs. was $\$500.62\frac{1}{2}$. Find the rate of exchange, brokerage $\frac{1}{8}\%$.

SOLUTION

$100\% + \frac{1}{8}\% = 100\frac{1}{8}\%$

If there had been no brokerage, the cost of the draft would have been

$\frac{100}{100\frac{1}{8}}$ of $\$500.62\frac{1}{2}$, or $\frac{800}{801}$ of $\$500.62\frac{1}{2} = \$500$

12500 frs. = \$500

$1 \text{ fr.} = \frac{50000\text{c.}}{12500}$

$= 4\text{c.}$

EXERCISE 115

Find the rate of exchange

1. When a draft for £900 cost \$4,370.
2. When a draft for £306 13s. 6d. cost $\$1,492.44\frac{4}{9}$.
3. When a draft for £563 12s. 9d. cost \$2,724.25.
4. When a draft for 4,800 mks. cost $\$1,147.43\frac{1}{4}$, brokerage $\frac{1}{8}\%$.
5. When a draft for 3,576 mks. cost \$840.36.
6. When a draft for 3,575 glds. cost \$1,443.41.
7. When a draft for 33,250 frs. cost \$1313.38.
8. When a draft for 2,400 frs. cost \$72.09, brokerage $\frac{1}{8}\%$.
9. The cost of a bill on Bordeaux for 2,746 frs. was \$107·64, including $\frac{1}{8}\%$ brokerage. What was the rate of exchange?

364. Questions of the Third Aspect

Illustration 1.—A bill of exchange on London, England, cost $\$2,104.22\frac{1}{2}$ when exchange was at $109\frac{1}{2}$. What was the face of the bill?

SOLUTION

Cost of bill for £1 when exchange is $109\frac{1}{2} = \$4.44\frac{4}{9} \times 1.09\frac{1}{2} = \$4.86\frac{2}{3}$

$\$4.86\frac{2}{3}$ will buy a bill for £1

$\$2104.22\frac{1}{2}$ will buy a bill for $£1 \times 2104.22\frac{1}{2} \div 4.86\frac{2}{3}$

$= £432.375$

$= £432\ 7\text{s.}\ 6\text{d.}$

Illustration 2.—The cost of a bill of exchange on Paris was \$600, and exchange was at $3.91\frac{7}{16}$c. Find the face of the bill.

SOLUTION

\$$.0391\frac{7}{16}$ will buy a bill for 1 fr.

\$600 will buy a bill for $\frac{1 \times 600}{.0391\frac{7}{16}}$

$= \frac{600}{.03914375}$

$= 15{,}328.12$ fr.

EXERCISE 116

1. A draft on London, England, cost \$1,194.94 when exchange was at \$4.88. Find the face of the draft.

2. Find the face of a bill of exchange on Manchester, England, which cost \$2,730 when exchange was at $9\frac{11}{16}\%$ premium.

3. An exporter sold through a broker a bill of exchange on Hamburg at $23\frac{3}{8}\frac{7}{2}$c., and received \$5,953.49 as net proceeds. What was the face of the bill, brokerage $\frac{1}{8}\%$?

4. I bought a bill of exchange on Paris and paid \$2,156 for it. What was the face of the bill, exchange being quoted at $3.91\frac{3}{4}$c.?

5. What is the face of a bill of exchange on London which can be bought for \$5,807.25 if exchange is at \$4.85, brokerage $\frac{1}{8}\%$?

6. An importer purchased a 60-day bill of exchange on Bremen at par for \$446.20. What was the face of the bill?

7. The cost of a bill of exchange on Bordeaux, bought at $3.90\frac{7}{8}$c., was \$941.17$\frac{1}{2}$, including $\frac{1}{8}\%$ brokerage. What was the face of the bill?

8. I bought a draft on Amsterdam when exchange was at $41\frac{3}{8}$c. to the guilder, and paid \$1,489.50 for it. What was the face of the draft?

365. INDIRECT OR CIRCUITOUS EXCHANGE

Illustration 1.—A Toronto merchant paid a debt of 27,940 florins in Amsterdam by remitting to London when exchange was \$4.885; thence to Paris when exchange was 25.4 frs. for £1; and thence to Amsterdam when exchange was 212 frs. for 100 florins. What did he pay in Canadian money?

SOLUTION

$$100 \text{ fls.} = 212 \text{ frs.}$$

$$\therefore 27940 \text{ fls.} = \frac{27940 \times 212}{100} \text{ frs.}$$

$$25.4 \text{ frs.} = £1$$

$$\therefore \frac{27940 \times 212}{100} \text{ frs.} = £ \frac{27940 \times 212}{100 \times 25.4}$$

$$£1 = \$4.885$$

$$£\frac{27940 \times 212}{100 \times 25.4} = \$\frac{4.885 \times 27940 \times 212}{100 \times 25.4} = \$11{,}391.82$$

EXERCISE 117

1. If 11.65 Dutch fls. are given for 24.42 frs., 352 fls. for 407 mks., and 58½ mks. for 32 silver roubles, how many francs should be given for 932 silver roubles?

2. A montreal merchant wishes to transmit 2,400 mks. to Hamburg. He remits to London when exchange is $4.83; thence to Paris when 26 frs. are worth £1; and thence to Hamburg when 47 frs. are worth 25 mks. What does he pay in Canadian money?

3. If £1 is worth 12 fls. or 25.56 frs., how many francs is 1 fl. worth?

4. A merchant in Montreal drew on Amsterdam for 10,000 glds. at $.415. How much more would he have received if he had ordered remittance through London to Montreal, exchange at Amsterdam on London being 11¼ glds. for £1, and at London on Montreal 4.85⅝?

5. If the exchange of London on Hamburg is 14 mks. per pound sterling, that of Hamburg on Amsterdam is 20 mks. for 18 fls., that of Amsterdam on Paris is 28 fls. for 60 frs., and that of Paris on Toronto is 4 frs. for 72c., what is the value of £1 sterling in Canadian money?

6. Exchange at Paris upon London is at the rate of 25.7 frs. for £1 sterling, and exchange at Vienna upon Paris is at the rate of 40½ Austrian schillings for 20 frs. Find how many Austrian schillings should be paid at Vienna for a £50 note.

CASH STORAGE BILLS

366. **Storage** is a charge made for storing goods in a warehouse.

367. **Cash Storage** is that paid or estimated at the time of each withdrawal of goods from store.

368. **Credit** or **Average Storage** is that paid or estimated at the time of last withdrawal from store.

369. The **Term of Storage** is the period of time for which storage charges are made.

Rates of storage may be arranged between the parties to the contract, but are frequently regulated by boards of trade or by associations of warehousemen.

Only cash storage bills will be considered here. Average storage will be dealt with under its appropriate heading, as an application of the average principle.

370. **Illustration.**—At a warehouse there was received and delivered merchandise as follows—

Received.	Delivered.
May 6, 300 bbls. flour	May 23, 200 bbls. flour
,, 27, 250 " "	June 3, 80 " "
June 12, 180 " "	" 7, 220 " "
	" 25, 230 " "

How much must be paid for storage on the above at the rate of 5c. per barrel for the first 10 dys. or part thereof, and 3c. per barrel for each subsequent 10 dys. or part thereof?

Solution

Date.	*Time of Storage.*	*Receipts and Deliveries.*		*Rate.*	*Storage*
May 6,		received	300 bbl.		
" 23	(17 dys., or 2 terms),	delivered	200 " ×	8c. =	$16.00
		Bal. of 1st receipt =	100 "		
June 3	(28 days., or 3 terms),	delivered	80 " ×	11c. =	8.80
		Bal. of 1st receipt =	20 "		
" 7	(32 dys., or 4 terms),	Of 220, delivered	20 " ×	14c. =	2.80

May 27		received	250 bbl.		
June 7 (11 dys., or 2 terms),		Of 220, delivered	200 "	× 8c. =	16.00
		Bal. of 2nd receipt	50 "		
" 25 (29 dys., or 3 terms),		Of 230, delivered	50 "	× 11c. =	5.50
" 12,		received	180 "		
" 25 (13 dys., or 2 terms),		Of 230, delivered	180 "	× 8c. =	14.40
				Storage due,	$63.50

371. EXPLANATION.—The first delivery of 200 bbl. was made May 23, or 17 dys. after the date of the first receipt, constituting one term of 10 dys. and part of another, or practically 2 storage terms, which, at the given rate of storage (5c. for first term + 3c. for second) produces 200 times 8c., or $16 storage. Of the remainder of the first receipt (100 bbls.), 80 bbls. were delivered June 3, or 28 dys. after the date of their receipt, constituting 2 terms of 10 dys. and part of another, or 3 storage terms; hence the storage thereon must be 80 times (5 + 3 + 3 =) 11c., or $8.80. The remainder of the first receipt (20 bbls.) was included in the delivery of 220 bbls. on June 7, and hence was stored 32 dys., or 4 terms, producing 20 times (5 + 3 + 3 + 3 = 14c.) or $2.80 storage.

The second receipt (250 bbls.) was on May 27, and the next delivery (of 220 bbls.) on June 7, on 20 bbls. of which storage has already been computed, leaving 200 bbls. which were in store 11 dys., or 2 terms, producing 200 times 8c., or $16 storage. The remaining 50 bbls. of the second receipt were included in the delivery of 230 bbls. on June 25, after being stored 29 dys., or 3 terms, producing 50 times 11c., or $5.50 storage.

The third receipt (180 bbls.) was on June 12, all of which were delivered on June 25 (being included in the delivery of 230 bbl.), after being stored 13 dys., or 2 terms, producing 180 times 8c., or $14.40 storage. Hence, the total storage is the sum of the storage on all the deliveries, or $63.50.

372. RULE

1. *Multiply each delivery of the first receipt by the rate of storage for the number of terms which such delivery has been in store, considering any fraction of a term as an entire term.*

2. *Similarly compute the storage on each delivery of the second receipt, and so continue in regular order with subsequent receipts until the storage on all the deliveries has been found.*

3. *The sum of the storage on all the deliveries will be the required cash storage.*

EXERCISE 118

1. The receipts and deliveries at a certain warehouse were—

Received.		Delivered.	
Sept. 25,	350 bbls.	Oct. 11,	85 bbls.
Oct. 14,	250 "	" 19,	140 "
" 29,	80 "	" 28,	160 "
Nov. 10,	50 "	Nov. 8,	210 "
		" 16,	40 "
		" 30,	95 "

What was the entire cash storage due Nov. 30th, the rate being 5c. per barrel for the first 10 dys. or part of 10 dys., and 3c. per barrel for each subsequent 10 dys. or part of 10 dys. ?

2. Find the cash storage on the following account, the rate of storage being 6c. per barrel for the first 10 dys. or part thereof, and 3c. per barrel for each subsequent 10 dys. or part thereof—

Received.		Delivered.	
May 15,	350 bbls.	May 20,	200 bbls.
" 22,	70 "	" 25,	30 "
June 6,	30 "	June 16,	60 "
" 14,	50 "	" 28,	210 "

3. Compute the storage on the following account, the rate being 1c. per bushel for the first 20 dys. or part thereof, and ½c. per bushel for each subsequent 20 dys. or part thereof—

Received.		Delivered.	
July 1,	1,600 bus.	July 10,	1,000 bus.
" 15,	3,000 "	" 24,	2,500 "
" 28,	2,500 "	Aug. 7,	1,700 "
Aug. 12,	6,000 "	" 11,	1,300 "
" 27,	1,200 "	" 30,	4,500 "
Sept. 14,	3,400 "	Sept. 25,	6,700 "

COMMERCIAL ARITHMETIC

TEST PAPER—No. 13

FOREIGN TRADE

Your work for this paper is to be submitted to the instructor for correction and criticism.

NOTE.—At the present time Sterling rates are quoted in Canada and the United States at so many dollars and cents to the pound sterling, \$4.85, 4.86½, and so on. There are two methods of progression; namely, advancing by eighths of a cent \$4.85, \$4.85⅛, \$4.85¼, and advancing by five one-hundredths of a cent per pound (00.05c.), \$4.8510, \$4.8515, \$4.8520, and so on. The latter is, of course, the closer quotation, and is used in market transactions.

Franc rates are quoted in cents per franc.

German marks are quoted in cents per mark.

1. (*a*) Convert £130 10s. 9d. sterling into currency when rate is \$4.8550.

(*b*) Convert \$1,000 currency into sterling at the same rate. Answer to be in £ s. d.

2. A person in Montreal owes another in Copenhagen a debt of 500 Crowns, which must be remitted through London. He pays the requisite sum to his banker at a time when the exchange between Montreal and London is \$4.84½ per £1, and between Copenhagen and London 14 Crowns per £1. The remittance is delayed until the rates of exchange are \$4.86 per £1 and 16 Crowns per £1. What does the banker gain or lose by the transaction?

3. A Toronto merchant paid a debt of 27,940 florins in Amsterdam by remitting to London (Eng.), when exchange was \$4.8850; thence to Paris, when exchange was 25.4 francs for £1; and thence to

Amsterdam when exchange was 212 francs for 100 florins. What did he pay in Canadian money ?

4. A person in Barrie received 1,000 pounds sterling from England, when sterling was quoted at $4.86. He put it out at interest for 288 days, at 6% per annum. To how much did it amount ?

5. A. P. Wilson is purchasing agent in London, England, for two Canadian houses. For A he buys in France goods valued at 1168.11 francs ; for B he purchases in Germany goods worth 7,650 marks. His commission is 5%. What is the amount of the sterling drafts A and B must remit and what is the amount of the cheque they will each issue ? Sterling exchange is $\$4.88\frac{7}{8}$; 21 marks equal £1 ; and 25 francs equal £1.

6. A merchant in Toronto bought of Jones & Co., Liverpool, Eng., 840 yd. silk at 20s. 8d. per yard, 500 yd. tweed at 12s. 6d. per yard, and paid for same by bill of exchange bought at $4.8555. What was the cost of the bill of exchange ?

7. The cost, including $\frac{1}{8}\%$ brokerage, for a draft on Antwerp for 833 francs was $27. What is the exchange rate ? (Give rate in cents per franc.)

8. The receipts and deliveries at a certain warehouse on the following account were as follows—

	Received.	Delivered.
1926.	June 20. 350 bbl. pork.	July 10, 90 bbl. pork.
,,	Aug. 1, 250 bbl. pork.	Aug. 15, 100 bbl. pork.
,,	Aug. 25, 100 bbl. pork.	Aug. 25, 250 bbl. pork.
,,	Sept. 12, 90 bbl. pork.	Sept. 10, 50 bbl. pork.
		Sept. 20, 300 bbl. pork.

What was the total storage paid, the rate being 5c. per bbl. for the first 10 days, and 3c. for each subsequent 10 days or part thereof.

Winners

By Berton Braley

The folks we call " Successes " ; what's the secret that they
know ?
What sort of magic food is it that makes them flourish so ?
There isn't any secret which you cannot figure out,
Nor any strange enchantment which is ringing them about ;
For when you come to study them the answer's quickly guessed
They worked a little harder and they planned a little better,
And they dreamed a little truer
Than the rest.

They weren't afraid of drudgery when drudgery was wanted,
They faced the little grimy jobs with spirit quite undaunted ;
They didn't like that sort of task, they didn't find it fun,
But they kept grinning at it till they knew that it was done ;
And thus their strength and courage grew to meet each larger
test,
They plugged a little harder, and they toiled a little longer,
And they thought a little clearer
Than the rest.

A little more of effort and a little more of stress
Would often alter failure into glorious success,
But the failure blames misfortune for his lack of fame and pelf ;
He blames the times, he blames his job—but never blames
himself ;
Yet the secret of the folks who win is easily expressed
They fight a little harder, and they act a little quicker,
And they stick a little longer
Than the rest.

PARTNERSHIP

373. A **Partnership** or **Co-Partnership** is the contract relation subsisting between persons who have combined their property, labor, or skill in an enterprise or business as principals for the purpose of joint profit.

The contracting parties are called partners or co-partners. Collectively they are called a firm, a house, or a company.

There are two kinds of partnership, general and limited.

374. A **General Partnership** is composed of two or more general partners. That is, partners who are liable not only as partners for the debts of the firm, but also personally liable if the resources of the firm are not sufficient to pay its debts in full.

375. A **Limited Partnership** is composed of one or more general partners, who are liable as general partners always are, and one or more limited partners, who are not liable for the debts of the firm beyond the amount of their investment. A limited partner must take no active interest in the business.

376. The **Liabilities** of a firm are its entire debts. These are of two kinds : first, its debts to the public ; second, its debts to the partners.

377. The **Resources** of a firm are the available means it has for the payment of its debts.

A perfectly solvent partnership should be able not only to pay its debts to the public, but to return to the partners the amount of their investment.

378. The **Investment** of a partner is the aggregate of the sums contributed by him for the purpose of carrying on the business. The contributions need not be in money, but may be in goods, real estate, trade marks, trade secrets, patent rights, or any other resources.

It will be understood that a partner in entering a business may invest resources, and he may also bring with him certain liabilities, which are assumed by the firm. Then, again, he may, during the course of the business, withdraw a part of his investment,

or, on the other hand, he may add additional amounts to his original investment. In other words, the investment is not necessarily a fixed figure.

379. The **Net Investment** or **Net Credit** of a partner is the difference between his investment on the one hand and the liabilities assumed for him and withdrawals on the other hand.

As the object of any business is to make a profit for those concerned in it, it is clear that the results of a year's trading or manufacturing will bring about a change in the way of adding a profit to the investment or subtracting a loss therefrom.

380. A **Balance Sheet** is a statement of the resources and liabilities of a business.

381. The **Net Capital** or **Present Worth** of a firm is the amount by which the resources exceed the liabilities to the public.

382. The **Net Gain** of a firm is the amount by which the net capital exceeds the net investment. In double-entry book-keeping this figure should be the same as the difference between the two sides of the loss and gain account, when the credit side is the larger, or, as it may be more broadly called, the revenue account, or income and expenditure account.

383. The **Net Loss** is the amount by which the net investment exceeds the net capital. In double-entry book-keeping this is the same as the amount by which the debit of the loss and gain account exceeds the credit.

384. The **Net Insolvency** of a firm is the amount by which the liabilities to the public exceed the resources.

PARTNERSHIP ADJUSTMENTS

385. **Partnership Adjustments** involve generally the adjusting between the partners of such matters as the distribution of the net profit or of the net loss, the payment of salaries according to agreement, or the payment of interest according to agreement.

It will be understood that, as a partnership is the outcome of a contract, the parties to that contract are at liberty to make

any agreement that they may see fit. No matter how absurd that agreement may appear, it must be followed in the adjustment of the affairs. There is, therefore, no set rule for the division of gains and losses, for instance, between partners, for the simple reason that partners may agree to divide gains and losses as they choose. The same thing is true concerning allowances for salary or allowances for interest. If, however, the agreement is silent on such points, it may be implied that gains and losses are to be divided proportionately to the number of partners. Unless there is a specific agreement, it could not be inferred that interest or salary was to be allowed.

386. Partners may also be brought into contact with the public through failure of their resources to cancel the debts to the public. In such cases general partners must expect to contribute from their private means. Here, again, partners must make adjustments between themselves.

387. We recommend a careful study of the following principles which must guide one in making partnership adjustments, and which will be understood to apply in all cases unless modified by agreement—

(*a*) Losses, including losses and deficiencies of capital, shall be paid, first, out of profits, next out of capital, and, lastly, if necessary, by the partners individually in the proportion in which they were entitled to share in the profits.

(*b*) The assets of the firm, including the sums, if any, contributed by the partners to make up losses or deficiencies of capital, shall be applied in the following manner and order—

1. In paying the debts and liabilities of the firm to persons who are not partners therein.

2. In paying to each partner rateably what is due from the firm to him for advances as distinguished from capital.

3. In paying to each partner rateably what is due from the firm to him in respect to capital.

4. The ultimate residue, if any, shall be divided among the partners in the proportion in which profits are divisible.

388. Illustration 1.—J. Coleman and R. Dawson formed a partnership on Jan. 1, 1943, agreeing to invest equally and to share equally in gains and losses. To provide for any possible inequality in investment, 8% interest was to be allowed each partner on investments, and interest at the same rate was to be charged on all withdrawals. It is further agreed that Coleman is to be paid a salary of $2,000 a year and Dawson a salary of $1,000 a year. The adjustment of both interest and salary to be made after the adjustment of the ordinary net profit or loss of the year has been made. The following is the statement of each partner's account—

Dr. J. COLEMAN. *Cr.*

1943.				1943.			
Apr.	3	Withdrawal	$3,000.00	Jan.	1	Investment	$6,000.00
June	7	"	1,500.00	Mar.	14	"	4,000.00
Oct.	15	"	800.00	July	3	"	3,000.00
1944.				Sept.	18	"	5,000.00
Jan.	1	⅓ loss from interest adjustment	601.75	1944.			
				Jan.	1	⅓ net gain of $5,800	2,900.00
"	1	⅓ loss from salary adjustment	1,500.00	"	1	Net credit int.	710.05
"	1	Present worth	16,208.30	"	1	Salary	2.000.00
			$23,610.05				$23,610.05

Dr. R. DAWSON. *Cr.*

1943.				1943.			
Mar.	8	Withdrawal	$2,000.00	Jan.	1	Investment	$4,000.00
Aug.	15	"	1,000.00	May	12	"	5,000.00
Sept.	30	"	1,200.00	July	16	"	2,000.00
1944.				Oct.	5	"	1,500.00
Jan.	1	⅓ loss from interest adjustment	601.76	1944.			
				Jan.	1	⅓ net gain of $5,800	2,900.00
"	1	⅓ loss from salary adjustment	1,500.00	"	1	Net credit int.	493.46
"	1	Present worth	10,591.70	"	1	Salary	1,000.00
			$16,893.46				$16,893.46

SOLUTION

NOTE.—It is presumed that the detail work of figuring interest on investments and withdrawals is already familiar to the student from that section of the text which deals with interest on partners' accounts.

It might not be out of place to mention here that a close acquaintance with the work on sharing, and also on average as it applies to partners' accounts, will help the student in work on partnership adjustments.

1st Step: The gain of $5,800, according to agreement, is to be divided equally. Therefore each partner is credited with $2,900 as his share of this gain.

2nd Step: Having found the net interest due to Coleman to be $710.05 and the net interest due to Dawson to be $493.46, these amounts are credited to their respective accounts. It is here that the principle of double-entry book-keeping must be kept in mind. We cannot credit the partners with these interest amounts without taking into consideration the fact that some other account would have to be debited with these amounts. If we followed the book-keeping for the transactions right through, we would find these amounts are debited to interest account. This, being a loss and gain account, would, in due course, be closed into loss and gain account. Loss and gain account, in turn, would be closed into the partners' accounts. Thus it is that the partners, who are credited with their net interests, are, in turn, debited, each for his share of the complete loss ($1,203.51) it would occasion to the business by paying these interests.

3rd Step: Each partner is credited for the amount of salary due to him as per agreement—namely, $2,000 to Coleman and $1,000 to Dawson. As in the case of the interest, the loss to the business occasioned by paying this salary ($3,000) will be divided equally between the partners, and each partner debited with his share, $1,500.

4th Step: With all the entries thus made in the accounts, a comparison of the debits with the credits will give us the present worth of each of the partners.

389. Illustration 2.—Adams, Wilson, and Green enter into partnership as grain dealers. Green is a special or limited partner, putting $10,000 into the business. Wilson contributes $15,000, and Adams nothing. Wilson and Adams are each allowed $1,500 a year salary. Profits are divided equally. Partnership deed duly drawn and registered. In the second year Green, being on a holiday trip, sees a chance of buying a quantity of grain, which he buys at a profitable figure, with the consent of his partners. He sells part of the lot on his way home, but, after his return, the

market takes a disastrous turn, and the firm is obliged to assign. Their statement of affairs is as follows—

Liabilities.			Assets.	
Bills Payable to Bank (Secured by warehouse receipts on 100,000 bushels of wheat at $1.10. Wheat realizes $80,000.)		$75,000	Office Furniture	$500
			Grain in Elevator, hypothecated to Bank	110,000
				$110,500
			Deficiency	14,500
Trade Creditors		25,000		
Capital Account—				
Green	$10,000			
Wilson	15,000			
		25,000		
		$125,000		$125,000

Partners have sufficient personal assets to pay all creditors. Furniture sells for $500. Adjust the loss and show the partners' accounts, including final adjustment with one another, it being understood that no payments have been made on account of salary.

Solution

Dr.	Green (Capital).		*Cr.*	
To ⅓ Loss		$15,833⅓	By Investment	$10,000
			" Cash to Settle	5,833⅓
		$15,833⅓		$15,83 ⅓

Dr.	Wilson (Capital).		*Cr.*	
To ⅓ Loss		$15,833⅓	By Investment	$15,000
" Cash to Settle		666⅔	" Salary	1,500
		$16,500		$16,500

Dr.	Adams (Capital).		*Cr.*	
To ⅓ Loss		$15,833⅓	By Salary	$1,500
			" Cash to Settle	14,333⅓
		$15,833⅓		$15,833⅓

Dr.	Cash.		*Cr.*
Surplus from Wheat	$5,000	Paid Trade Creditors	$25,000
Received for Furniture	500	" Wilson	666⅔
" from Green	5,833⅓		
" " Adams	14,333⅓		
	$25,666⅔		$25,666⅔

EXPLANATION.—Green, by purchasing and selling grain for the firm, violated one of the most important provisions of the Act governing limited partnerships, and so made himself liable as a general partner. This being the case, his loss is not limited by his investment, but he may be charged with one-third of the firm's losses, no matter how large they may be. The grain in the elevator has been sold for $80,000; $75,000 of this amount is retained by the bank to settle the bills payable due them. The $5,000 surplus from this source, together with the $500 received for the office furniture, makes a total of $5,500 cash, which represents the entire assets of the firm. Against this there are liabilities to trade creditors of $25,000, thus showing a shortage or deficiency of $19,500, which has to be made up by the partners. The loss of the partnership is the sum of the investments shown on the balance-sheet, together with the $19,500 needed to settle the accounts, and $3,000 for salary, making a total of $47,500, of which each has to bear one-third, or $15,833⅓. When these amounts have been charged, the capital accounts will show Green with a debit balance of $5,833⅓, Adams with a debit balance of $14,333⅓, and Wilson with a credit balance of $666⅔. Green and Adams must pay sufficient cash into the firm to square off their respective accounts. When this is done, the firm has $25,666⅔ available cash, of which $25,000 is paid over to the trade creditors and the remaining $666⅔ to Wilson. All of the accounts will then balance, as shown.

EXERCISE 119

1. A, B, and C form a partnership, investing respectively $9,600, $8,400, and $7,300. They agree to share gains and losses equally, and to allow interest on all investments and charge interest on all withdrawals at the rate of 6%. At the end of four months A invested $3,000 more, B $1,000, and C $1,200. At the end of the year no settlement of the interest is made until after the net gain, $12,800, has been divided and entered up. The interest is then adjusted. Show partners' accounts after such adjustment, properly closed, with balances brought down for next year's business. (Calculate time in months.)

2. H. Mann and J. Ross enter into partnership on Jan. 1, 1944, each investing $8,000, under agreement to share gains and losses according to average net investment. On April 1 Mann withdrew $3,200 and Ross $1,600. On Sept. 1 each took out $2,400, and on Nov. 1 Mann withdrew $900 and Ross $700. At the end of the year the capital is $6,000. Show the partners' accounts as they stand on Jan. 1, 1945. (Calculate time in months.)

3. A, B, C are partners. On Jan. 1, 1944, their respective capital accounts showed the following credit balances : A, $75,000 ; B, $25,000 ; C, $10,000. From that date each partner was to get 6% per annum on his capital. A was to share $\frac{5}{9}$ of the gains or losses ; B, $\frac{1}{3}$; and C the remainder. At the end of the year their assets and liabilities were as follows : Premises, $50,000 ; machinery, $21,000 ; tools, $4,000 ; sundry debtors, $40,000, less 5% allowance for bad debts ; bills receivable, $20,000, less 5% allowance for bad debts ; cash in bank, $10,000 ; cash on hand, $200 ; sundry creditors, $30,000 ; bills payable, $12,000. A was charged up with $4,000 withdrawn, B with $1,500, and C with $700. Write up the capital accounts of the three partners in the ledger.

4. A, B, and C form a partnership, investing respectively $9,700, $8,400, and $7,200. They agree to share gains and losses according to average net investment, and to allow interest on all investments and charge interest on all withdrawals at the rate of 6%. At the end of four months A invests $4,000 more, B $1,000, and C $1,200. At the end of the year no settlement of the interest is made until after the net gain, $12,800, has been divided and entered up. The interest is then adjusted. Show the partners' accounts after such adjustment, properly closed, with the balances brought down for next year's business. (Calculate time in months.)

5. A and B are partners in business, sharing gains and losses in the following manner : A, $\frac{3}{4}$; B, $\frac{1}{4}$. A is to receive a salary of $1,200 per annum for special services. How much money is he practically clearing in salary ? Why ?

6. A and B are partners, with respective investments of $18,000 and $30,000, and sharing equally losses and gains. At the end of the first year the interest on excess of capital at the

rate of 7% is adjusted, without entry in books, by private payments of A to B. How much is paid?

7. A, B, and C are partners, with equal investments, and share gains and losses equally. A is allowed a salary of $900 per year; B, $1,100; and C, $1,200. The net gain, irrespective of salaries, is $3,000. They have exactly this amount of cash on hand. As it is the intention of the firm to leave their original capital unchanged, this amount of money is divided among them. Prepare a statement showing clearly how much each partner is entitled to, no salaries having yet been paid.

8. Thompson and Wilson went into partnership Jan. 1, 1943, Thompson investing $22,000 and Wilson $4,000. During the year they lost $5,900, without reckoning interest on capital, which, by agreement, is to be taken at 8% per annum. During the year the partners have withdrawn the following amounts on which no interest is to be charged: Thompson, $2,800, and Wilson, $2,000. They share gains and losses as follows: Thompson, $\frac{5}{8}$; and Wilson, $\frac{3}{8}$. Draw up a statement showing the position of both partners at the beginning of 1944.

9. G, H, and N formed a co-partnership on Jan. 1, 1943, and agreed to invest equal sums, and share equally gains and losses, interest at 10% per annum being allowed on the excess and charged on the deficiency of each partner's required investment. G invested $8,000 on Jan. 1, $6,400 on Apr. 15, $5,600 on May 20; and withdrew $1,200 on May 5, $1,800 on Aug. 13, and $2,400 on Nov. 16. H invested $7,000 on Jan. 1, $9,000 on July 24, $1,600 on Oct. 12; and withdrew $6,000 on Mar. 26, $8,000 on May 5, and $6,700 on June 14. N invested $7,500 on Jan. 1, $6,000 on Jan. 25, $6,800 on Feb. 14; and withdrew $2,000 on Apr. 27, $3,600 on June 4, and $800 on Aug. 2. If the net gain during the year was $9,842.19, what was G's capital in the business on Jan. 1, 1944. What interest did H owe the remaining partners? How much interest did H owe N?

10. H and G entered into co-partnership, equally insolvent. The resources of the firm at closing are: Cash, $2,440.60; bills receivable, $1,644; personal accounts, $4,234.73. The liabilities are: Bills payable, $2,149; due S, $3,241.67; due J, $1,340.33.

The firm paid private debts of $260 for H, and G paid sundry expenses for the firm amounting to $375. There was a net gain of $2,136.72. What was each partner's insolvency at commencing? What was each partner's net capital at closing?

11. S, G, and H are partners in business, and on the date of settlement the books of the concern show that S drew out for private use $590, that the firm assumed a private debt of $930 for G, and that H's account was overdrawn $125. On the date of the last settlement each partner had an equal sum standing to his credit. The resources closing were: Cash, $8,294; personal accounts, $1,560; bills receivable, $4,400; merchandise, $18,220. The liabilities at closing were: Bills payable, $7,278; personal accounts due $13,651; interest on notes unpaid, $190. What was each partner's net capital at closing, there being a net gain of $2,100?

12. L and W are equal partners, but, having become financially involved, they arrange a compromise with their creditors at 50c. on the dollar, and wind up the business. Their balance-sheet at time of compromise stood as follows—

Liabilities.		Assets.	
L (Capital Acct.)	$190	Cash	$50
Sundry Creditors	3,910	Sundry Debtors	535
		Goods	1,800
		Furniture	380
		Machinery and Plant	1,000
		W (Capital Acct.)	335
	$4,100		$4,100

The assets realize as follows: Cash, as stated, $50; sundry debtors, $410; goods, $1,250; furniture, $150; machinery and plant, $500. W's private estate being insolvent, he is unable to contribute towards the debt. The creditors are paid as arranged. How should the partners settle with each other? Show ledger accounts dealt with in handling winding-up proceedings.

13. C, G, M, and F are partners, sharing profits or losses in the proportions of $\frac{5}{12}$, $\frac{1}{6}$, $\frac{1}{3}$, and $\frac{1}{12}$. The balance-sheet, after realizing on the estate and settling with the creditors, is as follows—

Liabilities.		Assets.	
C (Capital Acct.)	$280	M (Capital Acct.)	$400
G (Capital Acct.)	800	F (Capital Acct.)	223
		Cash	457
	$1,080		$1,080

As none of the partners are able to contribute anything further to the business, explain clearly the settlement, and show the disposition of the $457 cash.

14. H and J purchased a hotel for $40,000, the former investing $16,000, the latter investing the remainder. They agree to share profits and losses in these proportions. The business has two distinct branches, one of which is taken charge of by H and the other by J. Each keeps separate accounts of his receipts and disbursements for the term of partnership. H's total receipts were $40,910.15, and J's $37,201.19. H's disbursements were $50,912.24, and J's $26,870.15. At the end of the period the business was sold for $53,700.00, of which $30,000 was received in cash, and banked to their joint credit. A note of $23,700 was taken for the balance and then afterwards transferred to H at a discount of 10% on its face. A final settlement is then made, and each received a check for his share of the $30,000. Find the amount of the respective checks.

15. A and B are partners, sharing profits and losses equally. Their respective capital accounts are: A, $6,000; B, $1,500. Their liabilities amount to $15,000, which includes $3,000 due to A on loan account and $1,500 due to B on loan account. The assets of the firm were sold for $18,000 cash. How do the partners settle with each other? Show balance-sheet at start and accounts to explain your adjustment.

PARTIAL PAYMENTS

390. Partial Payments are simply part payments made from time to time on interest-bearing instruments, such as notes or mortgages.

Sometimes, as in the case of an instalment note, the paper itself specifies the time at which payments are to be made. A mortgage, also, is often drawn with the proviso that the principal is to be paid in definite instalments.

Instalment Note

$60.\frac{00}{100}$ *Stratford, July 2,* 1944.

On the first day of each month hereafter, for four months consecutively, I promise to pay to ..... *A. Kilgour* .....*or order Fifteen Dollars, the whole amounting to Sixty Dollars, the first of such payments to be made on the second day of August next, interest, both before and after maturity, and until paid, at the rate of six per cent. per annum.*

J. ROBERTS.

Unless thus specified that partial payments are to be made, it is quite at the option of the creditor as to whether partial payments shall be accepted or not.

The problem presented, where partial payments are made and accepted, is to find at any given date the amount remaining due.

391. Law of Application of Payments

It is a principle of law that, in making a payment on an interest-bearing paper, the payment is first to be applied to the reduction of the interest and then to the reduction of the principal. If the payment is not sufficient to at least cancel the interest, it might better be withheld, as the creditor is simply getting the use of the money without giving any return for it. All he can do is to hold such inadequate payments until such time as the sum of them will at least cancel the accrued interest. Then he may proceed to apply the payments to the reduction of the interest; and, if there is more than sufficient for the reduction of the interest, the balance may be applied to the principal. If this point be kept in mind, the work of partial payments presents a very easy application of simple interest.

392. Illustration 1.—Where every payment is large enough to at least cancel the accrued interest.

500\frac{00}{100}$ *Hamilton, Mar.* 24, 1944.

Six months after date I promise to pay.....

J. E. Emerson.....*or order*

Five Hundred.....$\frac{00}{100}$ *Dollars*

value received, with interest at six per cent. per annum.

ROBERT BROWNING.

On the back of this note there are the following endorsements of partial payments—

BACK OF NOTE

Received on the within note, May 20, 1944.

One Hundred and Fifty Dollars ($150).

J. E. EMERSON.

July 15, *One Hundred Dollars* ($100).

J. E. EMERSON.

What remains to be paid on the due date of the note, Sept. 27, 1944?

SOLUTION

Face of note Mar. 24, 1944	$500.00
Interest on $500 from Mar. 24, 1944, to date of first payment, May 20, 1944	4.68
Amount due May 20, 1944	$504.68
First payment	150.00
Balance	$354.68
Interest on $354.68 from May 20, 1944, to date of second payment, July 15	3.26
Amount due July 15	$357.94
Second payment	100.00
Balance	$257.94
Interest on $257.94 from July 15 to Sept. 27	3.14
Balance due Sept. 27, 1944	$261.08

393. Illustration 2.—Where payments are not in all cases large enough to cancel accrued interest.

$2,750$\frac{00}{100}$ *Hamilton, Aug.* 18, 1944.

Two years after date I promise to pay to.....
Roderick Bethune.....*or order*
Twenty-seven Hundred and Fifty.....$\frac{00}{100}$ *Dollars value received, with interest at seven per cent. per annum.*

GEORGE CAMPBELL.

On the above note are endorsed the following payments—

April 9, 1945, $75
July 3, 1945, 400
Dec. 5, 1945, 150

What remains to be paid on the due date of the note?

SOLUTION

Face of note	$2,750.00
Interest on $2,750 from Aug. 18 to Apr. 9, at 7%	123.41
Since the first payment is less than this interest, compute the interest on $2,750 from Apr. 9 to July 3	44.83
Amount due July 3	$2,918.24
First and second payments ($75 + $400)	475.00
Balance due	$2,443.24
Interest on $2,443.24 from July 3 to Dec. 5	72.63
Amount due Dec. 5	$2,515.87
Third payment	150.00
Balance due	$2,365.87
Interest on $2,365.87 from Dec. 5 to due date of note	117.5
Balance due Aug. 21, 1946	$2,483.39

NOTE.—Where notes are not paid at maturity, careful attention must be paid to the wording of the interest clause. A reference to the chapter giving a synopsis of the law concerning interest-bearing documents will aid the student here.

EXERCISE 120

1. $850.00. Belleville, Ont., April 24, 1944.

Six months after date I promise to pay to W. G. Harwood or Order.....Eight Hundred and Fifty.....$\frac{00}{100}$ Dollars, value received, with interest. W. W. GREEN.

The above note has the following payments endorsed upon it: June 17th, $125; Sept. 3rd, $200. Find the value of the note at maturity.

2. $1,200.00. Aylmer (West), Ont., June 6, 1944.

One year after date I promise to pay to Elmond Bowlby..... or Order.....Twelve Hundred.....$\frac{00}{100}$ Dollars, value received, with interest at seven per cent. per annum.

J. J. GOULD.

On the above note were endorsed these payments: Oct. 20th, $120; January 30th, $10; May 1st, $350. Find the maturity value of the note.

3. $900.00. Chatham, Ont., Mar. 3, 1944.

Nine months after date I promise to pay to George House or Order.....Nine Hundred.....$\frac{00}{100}$ Dollars, value received, with interest at four per cent. per annum.

GEORGE PARTLOW.

On the above note the following payments were made: June 12, 1944, $150; Nov. 9, 1944, $10. Find the amount necessary to pay the note on Mar. 3, 1945.

4. $2,500.00. Halifax, N.S., Jan. 1, 1942.

Two years after date I promise to pay to W. W. Ingram..... or Order.....Twenty-five Hundred.....$\frac{00}{100}$ Dollars, value received, with interest at six per cent. per annum.

J. D. BROWN.

The above note has endorsed upon it the following payments: Dec. 14, 1942, $100; Aug. 6, 1943, $500; Mar. 11, 1944, $1,000. Find the value of the note on Sept. 9, 1944.

5. $875.00. Winnipeg, Man., Oct. 8, 1943.

Nine months after date I promise to pay to John Gloin..... or Order.....Eight Hundred and Seventy-five.....$\frac{00}{100}$ Dollars, value received, with interest at six per cent. per annum until maturity, and thereafter at the same rate until paid.

J. SMALL.

The following payments were endorsed upon the above note: Jan. 3, 1944, $125; Mar. 24, 1944, $75; Sept. 12, 1944, $200. Find the amount necessary to pay the note on Jan. 1, 1945.

6. $1,500.00. Owen Sound, Ont., April 2, 1944.

One year after date I promise to pay to W. H. Weir..... or Order.....Fifteen Hundred.....$\frac{00}{100}$ Dollars, value received, with interest at four per cent. per annum until maturity, and thereafter at the rate of seven per cent. per annum until paid. J. G. DAVIDSON.

The following payments were made on the above note: Oct. 11, 1944, $25; Dec. 20, 1944, $300; July 9, 1945, $450. Find the amount necessary to pay the note on Sept. 10, 1945.

7. $1,000.00. Vancouver, B.C., Nov. 3, 1943.

Six months after date I promise to pay to Frank Kent..... or Order.....One Thousand.....$\frac{00}{100}$ Dollars, value received, with interest at four per cent. per annum until paid. J. R. GREEN.

On the above note the following payments were made: April 9, 1944, $250; July 30, 1944, $300. What was the balance due Sept. 15, 1944.

8. $750.00. Kingston, Ont., Mar. 8, 1944.

Fifteen months after date I promise to pay to Fred White .....or Order.....Seven Hundred and Fifty.....$\frac{00}{100}$ Dollars, value received, with interest at eight per cent. per annum until maturity, and thereafter at the rate of four per cent. per annum until paid. C. COWAN.

On the above note were endorsed the following payments: Oct. 10, 1944, $25; Jan. 14, 1945, $280; Sept. 20, 1945, $300. Find the balance due on the note Mar. 8, 1946.

9. $500.00. Peterborough, Ont., July 2, 1943.

On demand I promise to pay to C. S. Cutler.....or Order, Five Hundred.....$\frac{00}{100}$ Dollars, value received, with interest at nine per cent. per annum from maturity until paid. F. THAYER.

The following payments were made on the above note: Dec. 11, 1943, $10; Mar. 15, 1944, $150; Aug. 27, 1944, $200. What was the balance of the note on Dec. 31, 1944?

10. $1,450.00. Brantford, Ont., May 12, 1944.

Nine months after date I promise to pay to Albert Williams .....or Order.....Fourteen Hundred and Fifty.....$\frac{00}{100}$ Dollars, value received, with interest at $4\frac{1}{2}$ per cent. per annum until maturity, and thereafter at the same rate until paid. J. DOUGLAS.

The following payments were endorsed on the above note: Aug. 31, 1944, $400; Dec. 3, 1944, $15; April 11, 1945, $275. What remained due on the note on June 30, 1945?

11. $450.00. Woodstock, Ont., Aug. 20, 1943.

One year after date I promise to pay to John J. Doane..... or Order.....Four Hundred and Fifty.....$\frac{00}{100}$ Dollars, value received, with interest at four per cent. per annum until maturity, and thereafter at the rate of six per cent. per annum until paid. J. D. TODD.

The following payments were endorsed on the above note: Dec. 24, 1943, $50; April 10, 1944, $4; Sept. 11, 1944, $100. What was the balance due on the note on Dec. 15, 1944?

12. $1,800.00. London, Ont., July 2, 1944.

Six months after date I promise to pay to W. J. Long..... or Order.....Eighteen Hundred.....$\frac{00}{100}$ Dollars, value received, with interest at ten per cent. per annum until maturity, and thereafter at the same rate until paid.

C. BAILY.

The following payments were made on the above note: July 26, 1944, $300; Oct. 11, 1944, $400; Mar. 11, 1945, $500. Find the value of the note on July 9, 1945.

13. Required the balance due May 1, 1945, on a demand note for \$4,119.82, at 6% from maturity till paid, dated June 25, 1944, on which a payment of \$450.25 was made Aug. 1, 1944, and a payment of \$21.19 on the 15th of each subsequent month.

14. On Oct. 15, 1943, I borrowed a certain sum of money secured by mortgage at 6% interest. The following partial payments have been made: Nov. 24, 1943, \$200; April 15, 1944, \$300; Aug. 18, 1944, \$750. On Jan. 31, 1945, I still owe \$4,099.739. Find the sum borrowed.

15. Jones purchases a house from Smith, and gives a demand note bearing interest at 5% on Jan. 1, 1943. Subsequently the following payments were made and endorsed on the note: Mar. 15, \$59; July 28, \$10; Aug. 8, \$157; Jan. 1, \$214. On the last-mentioned date a renewal note is given for the balance then due, to run for one year, and bear interest at 7%. Smith immediately discounts this note at 5 per cent., and realizes \$508.25. Find the purchase price of the house. (In reckoning for last-mentioned note, omit days of grace.)

16. D bought a piano, catalogued at \$600, at a reduced price. In payment he gave his note dated Jan. 1, 1942, payable on demand, and bearing interest at 5% per annum. During the year the following payments were made on the note: Mar. 15, \$100; July 31, \$7.10; Aug. 8, \$206. No payments to be applied to reducing the debt until their sum is greater than the then due interest. On Jan. 1, 1943, D renewed the note for one year, the rate of interest being changed to 8%. The holder discounted the new note on Aug. 8, 1943, at the Bank of Toronto, at 7% per annum, the proceeds being \$214.15$\frac{13}{125}$. By what percentage was the catalogue price reduced? (Allow no days of grace.)

394. CASH BALANCE

The settlement of accounts between debtor and creditor often involves the calculation of simple interest.

A book account is different from a note, inasmuch as it does not bear interest when overdue, unless the merchant has it printed on his invoices and bills that interest will be charged after a certain date. Even then it can only be 5%, unless the debtor consents to pay more. Simply having

7 or 8 or 10%, as the case may be, printed on an invoice or statement does not make the charge binding, and the debtor, unless he has contracted to do so, may refuse to pay more than the legal rate of 5%.

The time when an account becomes subject to interest is a matter of the facts of the particular case. A retail dealer may establish a practice with his customer that goods may be bought on account at any time during the month, and are to be paid for when the statement of account is rendered at the end of the month. His bills naturally bear a statement such as this: " Accounts rendered monthly. Interest at the rate of 5 per cent. charged on overdue accounts."

Even wholesale dealers very often make the same arrangements with retail dealers, especially in the case of city sales. In the case of the ordinary sales by wholesaler to retailer, however, each sale is made on a definite term, and it is just as easy to find when each item of the account matures as it is to find when a note matures.

395. Questions of the First Aspect

Illustration 1

May 1st.—J. Brown & Co., of Toronto, sell to R. Young, of Woodstock, a bill of goods amounting to $500 on the following terms: 5/30, N/90.

Young's account, as it appears in Brown & Co.'s books, would show as follows—

Dr.		R. YOUNG.		*Cr.*
May 1st	5/30, N/90	$500		

According to the terms, Young can pay in 30 dys., and earn a 5% discount. In other words, he can settle the account for $475 in cash on any date up to May 31st.

If he does not settle in that time, he will have to pay the full amount of $500, and it must be paid in 90 dys. from May 1st or on July 30th.

If not paid on July 30th, and the interest notice has been duly given, the account will bear interest from July 30th until paid. If paid on August 31st, for instance, it will require $500 plus the interest on $500 for 32 dys., or, altogether, $502.19 in cash to settle the account.

396. The **Merchandise** or **Commercial Balance** is the difference between the debit and credit items—$500.

397. The **Cash Balance** is the sum required to settle the account on a given date—$475 on May 31st, or $502.19 on August 31st.

398. An Account Current is simply a detailed statement of an open account showing its cash balance on a given date.

The account current rendered by J. Brown & Co. to R. Young on August 31st will appear as follows—

R. YOUNG, Woodstock, Ont. TORONTO, Aug. 31, 1944.

In account with

J. BROWN & CO.

May 1	To Mdse. 5/30 N/90	$500.00
	" Interest from July 30 to Aug. 31	2.19
	Balance due	$502.19

NOTE.—Interest cannot be compounded in this case. If this account is not settled on August 31st, further interest is always reckoned on the $500, not on the $502.19.

Illustration 2

Find the cash balance of the following account on July 15, 1944, reckoning interest at 5%—

Dr. E. Jordan Cr.

19						19					
Mar.	10	Mdse	N/30	650	00	Apr.	20	Cash	500	00
Apr.	1	"	N.	1000	00	May	13	Draft 90 dys	900	00
May	26	"	N/60	1200	00	June	1	Cash	1000	00

SOLUTION

DUE DATE.	DAYS.	ITEMS.	INTEREST.	DUE DATE.	DAYS.	ITEMS.	INTEREST.
1944.				1944.			
Apr. 9	97	$650.00	$8.64	Apr. 20	86	$500.00	$5.89
" 1	105	1,000.00	14.38	Aug. 14	30	900.00	
July 25	10	1,200.00		June 1	44	1,000.00	6.03
			3.70				1.64
		$2,850.00	$26.72			$2,400.00	$13.56
			2,850.00				2,400.00
		Total Debit .	$2,876.72			Total Credit .	$2,413.56
			2,413.56				
		Cash Balance	$463.16				

EXPLANATION

The steps in the above solution may be set down as follows—

1. Find due date of each item. This is a general direction that applies to all work on accounts. No figuring is done with any dates but due dates.

2. Find the interest of each item from its due date to the date on which the cash balance is being found.

3. Add debit interests to debit items in order to get the total debit, and add credit interests to credit items in order to get the total credit.

4. Compare total credit with the total debit, and the difference gives cash balance.

NOTE.—There is only one possible point of exception to this general plan. It is illustrated in item 3 on the debit side and item 2 on the credit side. In both these cases the due date is *after* the date on which we are asked to find cash balance. The interest in each case is carried to the reverse side of the account. In thus subtracting the interest from the item, we do not get the absolutely correct present worth of the item, but the result is considered to be near enough for business purposes, and the plan is generally followed in business calculations.

399. Questions of the Second Aspect

The student who can find the cash balance of an account should have little difficulty in solving another question of prime importance in business. Let us illustrate.

The cash balance of the account just dealt with is \$463.16, calculated to July 15th. The merchandise balance of this account —that is, the difference between its debit and credit items—is \$450 (\$2,850 – \$2,400). Using these figures, our question may be stated thus—

400. Illustration

If \$463.16 will settle an account on July 15th, on what date will \$450 settle it?

Putting this in the more familiar form of one of our simple interest questions, it may be stated thus: In what time will \$450 amount to \$463.16 simple interest?

SOLUTION

\$450, to amount to \$463.16, must accumulate \$13.16 interest.
Interest on \$450 for 1 yr. at 5% = \$22.50.
\$22.50 is interest for 1 yr., or 365 dys.
\$13.16 is interest for $\frac{13.16}{22.50}$ of 365 dys. = 213 dys.

If $463.16 will settle an account on July 15th, $450 (a smaller sum of money) will settle it some time before July 15th, because we know that the longer an account runs, the more interest it will accumulate, and the greater will be the amount needed to settle it. This time we have just calculated at 213 dys. 213 dys. before July 15, 1944, gives us Dec. 15, 1943, Therefore, $450 would settle the account on Dec. 15, 1943.

NOTE.—It may be said that there is no sense in speaking of $450 settling this account on Dec. 15, 1943, as the first transaction recorded took place on March 10, 1944. It is quite true that the account cannot actually be settled on Dec. 15, 1943, but that fixes a date from which to reckon interest on the account, whenever the debtor comes forward and wishes to settle. We can illustrate this, and prove our previous work, by finding what would settle the account on July 15, 1944. It would require $450 plus the interest on $450 from Dec. 15, 1943, to July 15, 1944 (213 dys.), or, altogether, $463.16.

EXERCISE 121

1. (*a*) What is the cash balance of the following account on Mar. 15, 1944, reckoning interest at 6%?

   (*b*) On what date would the commercial balance settle it?

Dr. YOUELL & WRONG. *Cr.*

1944.								
Jan.	3	To Mdse., net 30 dys.	250					
Feb.	6	" " net 30 dys.	300					

2. (*a*) What is the cash balance of the following account on Dec. 3, 1944, reckoning at 5%?

   (*b*) On what date would the commercial balance settle it?

Dr. A. WHITE & SONS. *Cr.*

1943.				1943.			
Sept.	12	To Mdse., 30 dys.	450	Oct.	1	By Cash . .	400
Oct.	7	" " 30 "	310				

3. (*a*) What amount would settle the following account on May 10, 1944, interest 5%?

   (*b*) When would the commercial balance of the account be an equitable settlement?

Dr. INGRAM & DAVEY. *Cr.*

1944.				1944.			
Mar.	9	To Mdse., 60 dys.	275	Apr.	10	By Cash . .	240
Apr.	17	" " 30 "	340	May	1	" " . .	300

4. (*a*) What is due on settlement of following account, Dec. 20, 1944, interest 7%?

(*b*) When should interest start on the account?

Dr. J. F. Foster & Co. *Cr.*

1944.					1944.			
July	10	To Mdse., 2mos.	560		Aug.	11	By Cash . .	450
Aug.	1	" " 30 dys.	270		"	20	" Note, 30 dys.	300

5. (*a*) What sum would be required to pay the following account on Sept. 19, 1944, interest at 5%?

(*b*) Find when the balance of the account should be considered an equitable settlement.

Dr. C. Chute. *Cr.*

1944.					1944.			
May	30	To Sundries, 60dys	370		June	20	By Cash . .	450
June	15	" " 30 "	145		"	30	" Draft, 60 dys.	360
"	23	" " 90 "	420					

6. Of the following account find—

(*a*) The amount due in settlement Aug. 23, 1944, interest at 7%.

(*b*) When a three months' note given in payment of the account for the commercial balance should be dated so that the note would fall due on the equitable date.

Dr. Geo. Young. *Cr.*

1944.					1944.				
Jan.	10	To Mdse. . .	468	70	Mar.	28	By Cash . .	400	
Apr.	5	" " . .	392	60	June	7	" " . .	145	

7. (*a*) What amount would settle this account on June 20, 1944, interest at 5%?

(*b*) When would the commercial balance of the account pay it?

Dr. W. H. Hayslip. *Cr.*

1944.					1944.				
Feb.	19	To Mdse., 60 dys.	387	25	Mar.	22	By Cash . .	276	70
Mar.	25	" " . .	175		Apr.	11	" " . .	200	
Apr.	30	" " 60 "	297	30	June	7	" Note, 1 mo.	185	

8. (*a*) What amount would be necessary to pay the following account on Dec. 2, 1944, reckoning interest at 4%?

(*b*) When would the balance of the account be accepted in settlement?

Dr. F. L. SANDERS. *Cr.*

1943.						1943.				
July	7	To Mdse., 3 mos.	179	30	Aug.	17	By Cash . .	150		
Sept.	12	" " 60 dys.	210		Sept.	19	" Draft . .	162	75	
Nov.	30	" " 60 "	87	20						

9. (*a*) What is the cash balance of the following account on July 3, 1944, interest at 5%?

(*b*) When should a note, at 60 dys., for the commercial balance of the account be dated so that by the prompt payment of same Mr. Boyle would avoid paying interest on the account?

Dr. H. J. BOYLE. *Cr.*

1944.					1944.				
Apr.	9	To Mdse., 90 dys.	327	50	May	3	By Cash . .	268	30
"	12	" " 30 "	169	20	"	15	" Note at 15 dy.	200	
May	15	" " 30 "	126	35	"	31	" Cash . .	150	

10. (*a*) What amount would settle the following account on Jan. 10, 1945, interest at 5%?

(*b*) On what date should a note, at 3 mos., for the balance of the account be dated so as to fall due on the equitable date of payment?

Dr. G. P. BROWN. *Cr.*

1944.					1944.			
June	10	To Mdse., 4 mos.	850		Aug.	10	By Cash . .	560
July	23	" " 3 "	600		Sept.	29	" " . .	470
Sept.	15	" " 4 "	730		Oct.	7	" Note, 10 dy.	350

EQUATION OF PAYMENTS AND AVERAGING OF ACCOUNTS

401. To **Average** an account is to find the date on which the account can be settled by the payment of a sum in cash equal to the amount of the merchandise or commercial balance. (*See* **work on cash balance.**)

402. The **Equated** or **Average Date** for the payment of an account is the date on which the merchandise balance may, with fairness to both debtor and creditor, be proffered in payment.

Sometimes an account has items on one side only. Sometimes there are both debit and credit items. In the first case the work of averaging is spoken of as Simple Average or Equation of Payments. In the second case it is spoken of as Compound Average. These distinctions, however, cause no particular difference in the plan of working.

403. **Working Plan**

Those who have carefully followed the work on cash balance know already how to average an account. The questions of the second aspect under the head of cash balance are all of them questions in averaging accounts. There is just one point to be noted by the student. In asking for the cash balance of an account, it is natural that the question should give the date on which it is required to find the cash balance. In finding the average date of an account, the student may select his own date up to which he will find the cash balance. He may also select his own rate of interest, as this work of finding the cash balance is merely a means to an end.

There are but two simple steps in the work.

Step 1.—Find the cash balance of the account to any date (preferably the latest due date) and at any rate of interest.

NOTE.—This date selected is known as the focal date.

Step 2.—Compare the cash balance with the merchandise balance, and find the date on which the merchandise balance will settle the account.

404. Illustration 1.—Find the average date for the payment of the following account—

Dr. ANDREW ADAMS. *Cr.*

1944.			
May 15	Mdse.	@ 30 dys.	$500.00
June 9	"	@ 60 "	350.00
July 18	"	@ net .	250.00
Aug. 5	"	@ 30 dys.	470.00
			$1,570.00

SOLUTION

1st Step: Cash Balance to Sept. 4, at 5%.

Dr. ANDREW ADAMS. *Cr.*

DUE DATES.	ITEMS.	INT.
June 14	$500.00	$5.62
Aug. 8	350.00	1.29
July 18	250.00	1.64
Sept. 4	470.00	0
	$1,570.00	$8.55
		1,570.00
Cash Balance		$1,578.55

2nd Step: Knowing that $1,578.55 will settle the account on Sept. 4, find when $1,570 will settle it.

In other words, find in what time $1,570 will amount to $1,578.55 at 5%.

$78.50 is interest on $1,570 for 1 yr., or 365 dys.

8.55 is interest on $1,570 for 365 dys. $\times \frac{8.55}{78.50}$, or 40 dys.

If $1,578.55 will settle the account Sept. 4, $1,570, a *smaller* amount, will settle it 40 dys. *before* Sept. 4, or July 26, 1944.

405. Illustration 2.—Find the average date of the following account—

Dr. CHARLES B. HEATH. *Cr.*

1944.			1944.		
May 21	Mdse., 3 mos.	$500.00	May 24	Cash	$300.00
" 28	" 3 "	250.00	June 5	Note, 60 dys.	400.00
June 9	" 30 dys.	160.00	July 21	Cash	100.00

SOLUTION

1st Step: Cash Balance to Aug. 28 at 5%.

Dr. CHARLES B. HEATH. *Cr.*

DUE DATES. 1944.	ITEMS.	INT.	DUE DATES. 1944.	ITEMS.	INT.
Aug. 21	$500	$0.48	May 24	$300	$3.95
" 28	250	.00	Aug. 7	400	1.15
July 9	160	1.10	July 21	100	.52
	$910	$1.58		$800	$5.62
		910.00			800.00
		$911.58			$805.62
			Cash Balance		105.96
					$911.58

2nd Step: If \$105.96 will settle the account on Aug. 28, find when \$110 (merchandise balance) will settle it. In other words, find in what time the interest on \$110 will be \$4.04 (\$110.00 – \$105.96).

\$5.50 is the interest on \$110 for 1 yr., or 365 dys.

4.04 is the interest on \$110 for $\frac{4.04}{5.50} \times 365$ dys., or 268 dys.

If \$105.96 will settle the account on Aug. 28, 1944, \$110, which is a *greater* amount, will settle it 268 dys. *after* Aug. 28, or May 23, 1945.

406. The Interest v. Product Method of Averaging Accounts

We have chosen to make the work in averaging accounts an outgrowth of finding cash balance. This we have done only after mature deliberation, as the result of many years' experience in presenting this subject of averaging accounts.

Naturally, the interest method of handling the work has been followed. Our first reason for using this method will be found in the fact that finding cash balance requires the finding of interests, and, if we use the cash balance work as a means of averaging accounts, we therefore naturally use the interest method. There are several other reasons, however. In the first place, it will be found that in many offices where there is a great deal of work along the line of averaging accounts to be done, the work is done with the assistance of interest tables. In the next place, we believe that the plan is more satisfactory to those to whom accounts are rendered, as they can appreciate the idea of interest calculations where they might not be able to see the point of the product method calculation.

From the teacher's standpoint, we are persuaded that much of the difficulty in presenting the work of averaging will be overcome by following the order we have suggested in the work. However, if it is required to use the product method, it again can be quite easily explained as an outgrowth of the interest method, and this will be especially clear, if, as a rate of interest, we use 3.65 or 36.5%, or even 365%. We submit herewith a second solution of the last illustrative example, in which the product method is explained, and, moreover, developed from the interest method.

407. Illustration.—Find the average date of the Charles B. Heath account, first using the cash balance method, and then developing the product method.

SOLUTION BY CASH BALANCE METHOD

1st Step: Cash Balance to Aug. 28 at 3.65% interest.

Dr. CHARLES B. HEATH. *Cr.*

DUE DATES. 1944.	ITEMS.	INT.	DUE DATES. 1944.	ITEMS.	INT.
Aug. 21	$500	$0.35	May 24	$300	$2.88
" 28	250	.00	Aug. 7	400	.84
July 9	160	.80	July 21	100	.38
	$910	$1.15		$800	$4.10
		910.00			800.00
		$911.15			$804.10
			Cash Balance		107.05
					$911.15

2nd Step: If $107.05 will settle the account on Aug. 28, on what date will $110 settle it? In other words, how long will it take $110 to accumulate $2.95 interest?

$4.015 is interest on $110 for 1 yr., or 365 dys.

2.95 is interest on $110 for $\frac{2.95}{4.015} \times 365$ dys., or 268 dys.

If $107.05 will settle the account on Aug. 28, 1944, $110, which is a *greater* amount, will settle it 268 dys. *after* Aug. 28, or on May 23, 1945.

SOLUTION BY PRODUCT METHOD

Focal date, Aug. 28, 1944

Dr. CHARLES B. HEATH. *Cr.*

DUE DATES. 1944.	ITEMS.	HEATH GAINS. Use of $1 for	DUE DATES. 1944.	ITEMS.	HEATH LOSSES. Use of $1 for
Aug. 21	$500	3,500 dys.	May 24	$300	28,800 dys.
" 28	250	0 "	Aug. 7	400	8,400 "
July 9	160	8,000 "	July 21	100	3,800 "
		11,500 dys.			41,000 dys.

Heath loses use of $1 for 41,000 dys.

Heath gains use of $1 for 11,500 dys.

Heath's net loss = use of $1 for 29,500 dys.

Expressed in terms of $110, which is the amount owing by Heath, this is equal to a loss of $110 for 268 dys.

Now, an average date should be such that payment on that date should not occasion loss to either person. Therefore, if Heath, who is the one owing the money, is a loser by our date being selected as Aug. 28, 1944, that date is wrong, in that it is too early. In other words, Heath should be given 268 dys. after Aug. 28, 1944, or until May 23, 1945, in which to pay.

Explanation

To understand why Heath gains in the one series of items and loses in the other, we have only to consider the actual state of affairs as shown by the accounts in the light of our supposition. By the actual account, Heath owes us $500, which is due on Aug. 21. By our supposition we consider that the proper date for settling the account is Aug. 28th. When a man, who owes $500 due on Aug. 21st, is given until Aug. 28th to pay his account, he is gaining use of his money for the intervening 7 dys. The use of $500 for 7 dys. is equal to the use of $1 for 3,500 dys. The same reasoning can be employed in all items on the debit side.

Coming to the credit side, we find that the account shows that Heath actually paid us $300 on May 24th. Where a man makes any payment on May 24th, and we suppose that Aug. 28th is the correct date for settlement, it is clear that, in paying us $300, Heath has deprived himself of the use of this money from May 24th to Aug. 28th. This means the use of $300 for 96 dys., which is equivalent to the use of $1 for 28,800 dys. The same line of reasoning may be employed with all items on the credit side.

EXERCISE 122

1. On Feb. 10, 1944, I bought a bill of goods amounting to $1,200 on the following terms: $300 cash, $500 on a credit of 30 dys., and $400 on a credit of 60 dys. On what day might I pay the whole bill of $1,200 without gain or loss?

2. Smith bought goods of Brown as follows: On May 30, $200; June 5, $400; June 15, $300. No payments having been made by Smith, on what date should he start to pay interest on the whole amount?

3. Bought goods as follows—

Mar. 3, 1944, $350 on 3 mos.' credit.
Apr. 10, 1944, $270 on 2 mos.' credit.
May 15, 1944, $125 on 30 dys' credit.

What is the average date of payment?

4. John Brown bought goods as follows—

On Jan. 15, 1945, $500 worth at 30 dys.' credit.
On Feb. 25, 1945, $300 worth at 40 dys.' credit.
On Mar. 20, 1945, $800 worth at 15 dys.' credit.

Find the equated date of payment.

5. A bill of goods was purchased on the following terms: $\frac{1}{4}$ cash, $\frac{1}{3}$ payable in 10 dys., $\frac{1}{4}$ in 30 dys., and the balance in 60 dys. Find the average term of credit.

6. A man bought a farm on Mar. 10, and was to pay $1,000 cash, $800 in 3 mos., $1,200 in 6 mos., and $1,500 in 9 mos. Find the average term of credit and the equated time.

7. Find the equated date of payment of the following account—

June 3,	Mdse. @	3 mos.	$1,275.00
" 15,	"	" 60 dys.	500.00
July 12,	"	" 3 mos.	450.50
Aug. 18,	"	" 90 dys.	320.87
Sept. 25,	"	" 3 mos.	145.63

8. A person owes a debt of $1,920, due in 9 mos. He pays $\frac{1}{3}$ in 4 mos., $\frac{1}{4}$ in 5 mos., $\frac{1}{6}$ in 6 mos., and $\frac{1}{8}$ in 7 mos. By how much should the term of credit on the balance be extended?

9. H. J. Boyle owes C. Crawford: $75, due to-day; $90, due in 30 dys.; $80, due in 40 dys.; and $120, due in 60 dys. If Mr. Boyle pays Mr. Crawford $60 to-day and $120 in ten days, how long a term of credit should he have for the payment of the balance?

10. On a debt of $3,000 due in 9 mos. from April 1, the following payments were made: July, 1 $600; Aug. 1, $500; Oct. 1, $700. What is the equitable date for the payment of the balance?

11. On Feb. 10, 1944, a wholesale fruit dealer made purchases as follows: $600 on 10 dys., $750 on 20 dys., $625 on 30 dys., $725 on 60 dys., and $850 on 90 dys. On what date must he give his note at 30 dys. to have it mature on the average date of payment?

12. When may the remainder of the following account be paid without loss to either debtor or creditor?

Dr. THOMAS BROWN. *Cr.*

1945.					1945.				
Aug.	1	To Mdse. . .	375		Sept.	1	By Cash .	200	
"	25	" " . .	425		"	20	" " . .	410	

13. Average the following account—

Dr. J. B. DUNBAR. *Cr.*

1944.					1944.				
Mar.	9	To Mdse., 1 mo.	250		Apr.	1	By Cash . .	175	
Apr.	11	" " 60 dys.	312	50	May	9	" " . .	280	
May	17	" " 3 mos.	275	40	June	21	" " . .	202	90

14. What is the equated time for the payment of the balance of the following account?

Dr. A. C. NEWCOMB. *Cr.*

1945.					1945.				
May	15	To Mdse., 30 dys.	500		June	20	By Cash . .	300	
June	8	" " 60 "	750		Aug.	15	" " . .	600	
July	12	" " 90 "	650		Sept.	1	" Note, 30 dys.	800	

15. Find the equated date of payment for the balance of the following account—

Dr. A. D. SINCLAIR. *Cr.*

1944.					1944.				
Jan.	3	To Mdse., 2 mos.	375		Mar.	9	By Cash . .	250	
Feb.	15	" " 3 "	450		Apr.	15	" " . .	400	
Apr.	20	" " Net .	775		May	24	" Draft, 60 dys.	750	

16. A ledger contained the following account: when should the balance be paid, that no loss may be incurred?

Dr. H. PRESSEY. *Cr.*

1943.					1944.				
Nov.	10	To Mdse., 3 mos.	325		Jan.	5	By Note, 90 dys.	300	
Dec.	12	" " 60 dys.	475		Mar.	1	" Sundries .	450	
"	27	" " 45 "	600		"	25	" Cash . .	350	

17. Find the equated date for paying the balance of the following—

Dr. C. D. BENSON. *Cr.*

1943.					1944.			
Dec.	10	To Mdse., 30dys	475		Jan.	2	By Cash . .	400
"	24	" " 30 "	515		"	31	" Note, 30 dys.	600
"	31	"Accept'ce,30 "	410					

18. Average the following account—

Dr. W. H. HAYSLIP. *Cr.*

1943.					1943.			
Aug.	10	To Mdse., 30 dys.	375		Sept.	3	By Draft, 30 dys	250
"	25	" Cash . .	450		"	27	" Cash . .	425
Sept.	10	" Mdse., 2 mos.	175		Oct.	1	" " . .	25
Oct.	1	" " Net .	200					

19. Find the equated time for settlement of the following account—

Dr. JAMES MUNRO. *Cr.*

1943.						1943.				
Mar.	1	To Balance .	325			Mar.	16	By Cash . .	250	
Apr.	16	" Mdse., 2 mos.	623	47		May	20	" " . .	300	
May	11	" " 2 "	1722	30		July	30	" Note, 30 dys.	1000	
June	16	" " 30 dys.	975	12		Aug.	31	" Cash . .	500	
July	30	" " 60 "	150							
Aug.	17	" " 30 "	1600							

AVERAGING ACCOUNT SALES

408. Accounts between principal and agent are kept as personal accounts are usually handled. For instance, the agent, in his books, credits the principal's account with all amounts received from sales, collections, or whatever work he is doing for the principal. He debits the account with all amounts paid on account of charges incurred in handling the principal's business, and also with all amounts representing the value of his services, such as commissions, and amounts representing the value of conveniences placed at principal's disposal, such as storage or cartage. The difference between the two sides of the account represents a balance due to principal.

Where it is a case of selling goods for the principal, the statement that is rendered to the principal is spoken of as an *Account Sales*.

409. Rendering an account sales, like rendering a statement of account, necessitates averaging the account in order that the equated date for the payment of the balance may be found.

Averaging an account sales presents no difficulties that have not been handled in averaging accounts.

There is only one feature to be noticed. Sometimes the commission is put in at the date of rendering the account sales, sometimes at the average date of the sales as made, and sometimes at the average due date of the sales. It is all a matter of the custom of the commission house or of agreement between the parties.

An inquiry among a number of commission houses reveals the fact that, in many of them averaging is a thing unknown. When the last sale is made a check is made out for the proceeds and along with the account sales is mailed to the consignor. The transaction is immediately closed.

410. Illustration

Toronto, Ont., March 4, 1945.

Account sales of 1,200 bbls. of apples received from J. LLOYD, Stratford, Ont., to be sold on their account and risk by

RUTHERFORD & MARSHALL.

1945.				
	SALES.			
Mar. 5	To A. Brown, at 30 dys., 200 bbls.	$1.50	$300.00	
" 12	" C. Dunn, note at 30 dys., 600 bbls.	1.25	750.00	
" 27	" E. Fall, note at 30 dys., 400 bbls.	1.25	500.00	$1,550.00
	CHARGES.			
Mar. 3	Freight in cash		110.00	
" 4	Drayage, note 30 dys.		25.00	
	Storage		31.75	
	Commission, 3% of sales		46.50	213.25
	Net proceeds			$1,336.75

Find the average date for the payment of the proceeds of the above account sales. The date of the storage and of the commission to be the same as the average due date of the sales.

SOLUTION

1st Step: Set down the sales at their due dates, and find average date as before directed.

April 4	$300
" 14	750
" 29	500
	$1,550

Average date, April 17

2nd Step: Arrange an account showing sales on Cr. side and charges on Dr. side, putting storage and commission in at date just found.

Dr. LLOYD'S CONSIGNMENT. *Cr.*

1945.	CHARGES.		1945.	SALES.	
Mar. 3		$110.00	Apr. 17		$1,550.00
Apr. 6		25.00			
" 17		31.75			
" "		46.50			

3rd Step: Average this account as before directed.
Average date for balance of account, April 21st.

Therefore net proceeds, $1,336.75 are due, at per average, April 21st.

NOTE.—In performing the second average, it will be found much shorter to take as the focal date the average due date of the sales.

EXERCISE 123

Find the net proceeds of the following account sales, and when due by equation. Consider the commission due as per average due date of sales.

1.

Montreal, Quebec, April 3, 1944.

Account Sales of Flour.
Sold for acct. of THE LAKE OF THE WOODS MILLING CO.
By C. PINEO & SON.

		SALES.			
Mar.	3	124 bbls. @ $6.10 Cash			
"	8	48 " @ 6.20 "			
"	28	120 " @ 5.95 "			
Apr.	2	60 " @ 6.20 "			
		CHARGES.			
Mar.	4	Freight	124	50	
"	20	Advertising	18	25	
Apr.	2	Storage	25	80	
		Commission, 3%			

2.

Account Sales of Apples.
By F. RUTHERFORD.

Winnipeg, Dec. 12, 1943.
For acct. of V. WEISBROD,
Aylmer, Ont.

		SALES.		
Nov.	10	85 bbls. @ $2.75, 30 dys.		
"	14	63 " @ 3.10, 20 "		
"	19	110 " @ 2.90, 2 mos.		
"	23	47 " @ 3.20, 10 dys.		
Dec.	12	115 " @ 2.95, cash		
		CHARGES.		
Nov.	8	Freight	65	20
"	15	Cooperage	13	
"	20	Cash, advanced	300	
Dec.	12	Storage	23	60
		Commission, 4%		

3.

Account Sales of Bacon.
Sold for acct. of THE WM. DAVIES CO.

Halifax, N.S., April 3, 1944.
By L. LANG & SONS.

		SALES.		
Feb.	28	800 lbs. @ 7½c., cash		
Mar.	20	900 " @ 8½c., 30 dys.		
"	27	700 " @ 7¾c., 10 "		
Apr.	3	600 " @ 8⅛c., 20 "		
		CHARGES.		
Feb.	28	Freight and cartage	23	90
"	28	Insurance	3	50
Mar.	9	Advertising	8	25
Apr.	3	Storage	4	70
		Commission, 3½%		

4.

Account Sales of Coffee.
Sold for acct. of JOSE MARQUEZ.

Toronto, Feb. 10, 1944.
By A. REGAN.

		SALES.		
Jan.	3	47 bgs., 5,380 lbs. @ 11½c., 30 dys.		
"	10	56 " 6,720 " @ 11¾c., 20 "		
Feb	10	38 " 4,320 " @ 11⅞c., 1 mo.		
		CHARGES.		
Jan.	2	Freight and cartage	78	35
"	3	Duty	89	25
Feb.	10	Storage	16	70
		Commission, 4%		

5.

Montreal, July 3, 1944.

Account Sales of Tea.
Sold for acct. of T. LIPTON.

By TODD & HEPBURN.

		SALES.			
June	20	20 half-chests, 1,210 lbs., @65c.,30 dys.			
"	30	44 " 2,640 " @72c., cash			
July	3	130 " 7,635 " @68c.,20 dys.			
		CHARGES.			
June	18	Freight	25		
"	19	Cartage	2	25	
June	25	Cash, advanced	250		
		Commission, 3%			

6.

Toronto, June 17, 1944.

Account Sales of Pork.
Sold for acct. of BROWN & SMITH.

By W. J. LONG & SONS.

		SALES.			
May	28	100 bbls. @ $17.60, 30 dys.			
June	1	130 " @ 16.35, 1 mo.			
"	10	240 " @ 17.50, 2 mos.			
"	17	30 " @ 15.25, cash			
		CHARGES.			
May	25	Freight	375		
"	25	Cartage	43	50	
June	17	Storage	12	50	
"	"	Cash, advanced	2,500		
		Commission, 3%			

COMMERCIAL ARITHMETIC

TEST PAPER—No. 14

PARTNERSHIP—PARTIAL PAYMENTS—CASH BALANCE

Your work for this paper is to be submitted to the instructor for correction and criticism.

1. Mills, Johnson and Brown are partners investing one-third, one-fifth, and seven-fifteenths of the capital respectively, and agreeing to share gains and losses equally. At the end of the year the assets are: Cash, \$1,785; Barley, \$2,500; Corn, \$1,752; Rye, \$350; Oats, \$1,650; Wheat, \$5,000. The liabilities are: Bills Payable, \$1,562; S. Myers, \$1,200; P. Wilson, \$1,875. If the net loss for the year was \$750, find each partner's capital now.

2. G and M entered into partnership and continued for 1 yr. At the commencement G had $\frac{2}{5}$ of the capital and M $\frac{3}{5}$. Gains and losses were to be shared equally. At the end of the year they had: Merchandise, \$2,760; Cash, \$3,240; Bills Receivable, \$1,320; Real Estate, \$1,850; Accounts Receivable, \$930. They owed: Bills Payable, \$1,875; Mortgage Payable, \$1,000; Accrued Interest, \$75; Accounts Payable, \$2,150. G withdrew \$832; M \$768. Total Gain \$2,100. Give each partner's investment and Present Worth.

3. W. E. Mathews and Harold Atkinson were partners in the grocery business. Mathews invested \$1,000 in cash, and Atkinson invested \$2,000 in merchandise. Each was to draw \$50 per month for services, and the net gain or net loss was to be divided in proportion to each partner's investment. At the end of the year the gain on merchandise was \$2,850. The office expenses were \$615. There was a private account against Mathews of \$275, and one against Atkinson of \$410, which amounts were deducted from their salary. Find the amount due to each partner on final settlement.

4. On a claim for $3,000, dated August 12, 1941, and bearing interest at 7%, payments were made as follows: December 15, 1941, $30; April 1, 1943, $550; January 20, 1944, $85; June 12, 1944, $1,651.50. How much was due May 30, 1945?

5. I held a bond against Ira Fox, dated May 1, 1941, for $4,000, interest at 6%. The following payments were endorsed on this bond: May, 21, 1942, $800; June 10, 1943, $1,200; August 10, 1944, $1,500. What was due May 1, 1945?

6. Find the cash balance of the following account, October 2, 1945—

A. H. Hamill

	$		$
May 12, To Goods, 30 days	700	June 18, By Cash . .	200
June 5, To Goods, 60 days	450	Aug. 20, By Cash . .	400
July 8, To Goods, 90 days	825	Sept. 3, Note, 30 days .	300
Aug. 6, To Goods Net .	340		

7. Inksater Bros.

1945.	$	1945	$
June 10, To Mdse., 4 mo.	850	Aug. 10, By Cash . .	560
July 23, To Mdse., 3 mo.	600	Sept. 29, By Cash . .	470
Sept. 15, To Mdse., 4 mo.	730	Oct. 7, Note, 10 days .	350

(*a*) What amount would settle the above account on January 10, 1946, interest at 5%?

(*b*) On what date should a 3 months' note for the balance of the account be dated so as to fall due on the equitable date of payment?

8. J. K. Anderson

1946	$	1946	$
Jan. 11, Mdse. for Cash .	750	Mar. 3, By Cash . .	700
Feb. 1, Mdse. at 3 mo. .	600	Apr. 14, By Cash . .	500
Mar. 15, Mdse. at 6 mo. .	1500	Apr. 15, By Cash . .	300
May 3, Mdse. at 4 mo. .	900		

Find the equitable date for the payment of the balance of the above account.

9. Put the following items into the form of an account sales, find the net proceeds and date of payment—

A. B. Harrison, Montreal, sold a consignment of goods from Chase & Co., Toronto, as follows: November 15, 1945, 135 chests Tea at $45, on 30 days; November 20, 75 sacks Coffee at $28, on 2 mo. December 1, 256 kegs Lard at $4.50, 30 days; December 1, 285 tubs Butter at $18.37 on 2 mo. Paid freight December 1, $23.75; Cartage, $5.40; Storage, December 10, $7.80; Commission, 2½%.

10. Average the following account sales—

	Sales		
July 11, 200 bbl. 30 days.	$1,100.00		
Aug. 5, 350 bbl. 10 days.	2,170.00		
Aug. 20, 250 bbl. 30 days.	1,500.00		
Sept. 2, 200 bbl. 60 days.	1,150.00		
			$5,920.00
	Charges		
July 1, Freight	$450.25		
July 1, Cartage	30.75		
July 3, Storage	150.00		
Commission	148.00		
			779.00
Balance Due			$5,141.00

It's Service that Measures Success

It isn't the cut of the clothes that you wear,
Nor the stuff out of which they are made,
Though chosen with taste and fastidious care,
And it isn't the price that you paid ;
It isn't the size of your pile in the bank,
Nor the number of acres you own,
It isn't a question of prestige or rank,
Nor of sinew and muscle and bone ;
It isn't the servants that come at your call,
It isn't the things you possess,
Whether many, or little—or nothing at all,
It's service that measures success.
It isn't a question of name, or of length
Of an ancestral pedigree,
Nor a question of mental vigor and strength
Nor a question of social degree ;
It isn't a question of city or town,
Nor a question of doctrine or creed,
It isn't a question of fame or renown,
Nor a question of valorous deed ;
But he who makes somebody happy each day,
And he who gives heed to distress,
Will find satisfaction the richest of pay,
For it's service that measures success.

—From the " Family Herald " and " Weekly Star."

TRUE DISCOUNT

411. The **Present Worth** of a debt, payable at some time in the future, is its value at the present time, and is such a sum that, if put out at interest, it will amount to the given debt when it becomes due.

412. The **True Discount** is the difference between the amount of the debt and its present worth.

413. **Illustration.**—Jones owes Smith $525, to be paid at the end of one year. If Smith can get 5% interest for his money, what sum should he be willing to accept from Jones to-day in

payment of the debt? What discount would Smith be allowing Jones by accepting this amount?

SOLUTION

If Jones were to pay Smith \$100 to-day, Smith would put the \$100 out at interest, and by the end of the year he would have \$105.

So \$100 paid to-day is equivalent to \$105 paid at the end of the year.

Or \$105 paid at the end of the year is equivalent to \$100 paid to-day.

And \$525 paid at the end of the year is equivalent to \$500 paid to-day.

Smith should be willing to accept \$500 to-day in payment of the debt, and he would by so doing allow Jones a discount of \$525 – \$500 = \$25.

The present worth of the debt would be, therefore, \$500, and the true discount on it would be \$25.

It will be well right here to note the difference between bank discount and true discount.

Suppose Jones gives Smith his note at one year for the amount of the debt of \$525, and Smith takes the note to a bank and gets it discounted. The bank would charge him a discount of 5% of the face of the note, which would be \$26.25, and Smith would receive \$525 – \$26.25 = \$498.75. The true discount is really the interest on the present worth of the debt; while the bank discount is really the interest on the amount of the debt.

The bank discount in this case amounts to \$26.25, and the true discount to \$25. The difference between the bank discount and the true discount is \$1.25, which is the interest on \$25 for the given time (one year) at the given rate (5%).

This may be stated generally thus—

The difference between the bank discount and the true discount is the interest on the true discount.

This may be proved generally as follows—

Bank Discount = Interest on the debt.

True Discount = Interest on the present worth of the debt.

True Discount = Interest (on the debt – true discount).

True Discount = Interest on the debt – interest on true discount.

True Discount = Bank discount – interest on true discount.

∴ Bank discount is greater than true discount by the interest on the true discount.

Or

The bank discount = the given percentage for the given time of the debt.

The true discount = the given percentage for the given time of the present worth of the debt.

But the debt = the present worth of the debt + the true discount.

So the bank discount exceeds the true discount by the given percentage for the given time of the true discount.

In our illustration the bank discount is $\frac{5}{100}$, or $\frac{1}{20}$, of the debt, and the true discount is $\frac{25}{525}$, or $\frac{1}{21}$, of the debt. By comparing these fractions it will be seen that the numerators are the same, but that the denominator of the fraction representing the true discount is made up of the sum of the two terms of the fraction representing the bank discount.

This relation is always true, and may be proved as follows—

Suppose, say, the bank discount is $\frac{3}{16}$ of the debt.

Bank discount is really the interest on the debt.

$\therefore$ \$16 in the given time at the given rate will gather \$3 interest.

$\therefore$ \$16 in the given time at the given rate will amount to \$19.

$\therefore$ \$19 will have a present worth of \$16.

Or \$19 will have a true discount of \$3.

And the true discount will be $\frac{3}{19}$ of the debt.

Comparing $\frac{3}{16}$ and $\frac{3}{19}$ it will be seen that the numerators are the same but that the denominator of the fraction $\frac{3}{19}$ is made up of the sum of the two terms of the fraction $\frac{3}{16}$.

Similarly, having given the fraction which the true discount is of the debt, the fraction representing the bank discount can be derived from it. Thus, if the true discount is $\frac{4}{19}$ of the sum, we know that the fraction representing the bank discount must be $\frac{4}{15}$ of it.

From the foregoing it will be gathered that the bank discount is always greater than the true discount.

Bank discount is theoretically incorrect, but practically it is the only discount used where the question of present worth of any negotiable paper is under consideration. Business men have also grown accustomed to using this style of discount, even in figuring the present worth of any amount due on account.

True discount is theoretically correct, but for ordinary business transactions is not practical or valuable. It is used, however, where comparisons are instituted, and, as we shall see later, the principle is also applied in all work in finding the present worth of bonds or debentures and annuities.

EXERCISE 124

Find the present worth and true discount of—

1. \$503.36 for 1 yr. at 4%
2. \$752.40 for 9 mos. at 6%
3. \$109.89 for 3 mos. at 7%
4. \$129.01 for 90 dys. at 4%
5. \$588.80 for 60 dys. at 5%
6. \$75.85 for 6 mos. at 5%
7. \$918.50 for 48 dys. at 5%
8. \$2,585.87 for 63 dys. at 7%
9. \$1,500 for 90 dys. at 6%

Find the true discount on—

10. \$1,317.24 for 18 dys. at 5%

11. $1,250 for 7 mos. at 4%

12. $800 for 5 mos. at 7%

13. Which is better, and how much, to buy flour at $7.25 per barrel on 9 mos.' time, or to pay $6.75 cash, money being worth 6% ?

14. If money is worth 5%, what cash offer will be equivalent to an offer of $1,385 for a bill of goods on 90 dys.' credit ?

15. A offers flour at $8 per barrel on 8 mos.' credit, and B offers the same grade of flour at $7.50 per barrel on 6 mos.' credit. Which offer is the better and how much, money being worth 6% ?

16. The terms on which a bill of goods amounting to $478.25 was bought are 2% 10 dys., net 60 dys. If money is worth 7%, which would be better, to pay the bill on the 10th day or on the 60th, and how much better ?

17. I bought a bill of goods amounting to $583.60 on 60 dys.' credit, and immediately sold them for $615.75 on 90 dys.' credit. Money being worth 6%, find my per cent. of profit or loss.

18. I carried a quantity of tweed in stock for 3 mos., and then sold it at an advance of 25% on first cost, extending to the purchaser a credit of 6 mos., without interest. If money is worth 5% per annum, what was my per cent. of profit or loss ?

19. Which is the better, and how much, to buy property worth $2,600, at 6% discount for cash, or on 8 mos.' time without interest, when money is worth 6% ?

20. How large a note legally due in 90 dys., with interest at 6%, will pay a debt of $685.24 due in 90 dys. without interest ?

21. How much will I gain if I borrow money at 8% to pay a debt of $4,500, due in 6 mos., if 6% discount is deducted from the face of the debt for cash ?

22. What is the difference between the interest and true discount on $1,345 for 9 mos. at 7% ?

23. Sold a bill of goods on 8 mos.' credit for $387.20. If the selling price was $24.60 less than the goods cost, and money is worth 5% per annum, how much was the loss and the loss per cent. ?

24. A merchant bought a bill of goods amounting to $937·60 on 5 mos.' credit, and the seller offered a discount of 5% for cash. If money is worth 7% per annum, how much would the merchant gain by accepting the seller's offer ?

25. If the interest is $\frac{5}{12}$ of the principal, what fraction of the principal is the true discount for the same time and at the same rate?

26. If the true discount is $\frac{3}{11}$ of the sum, what fraction of the sum is the bank discount for the same time and at the same rate?

27. The interest is \$9 and the true discount for the same time at the same rate is \$8. Find the sum.

28. The difference between the interest and the true discount for 9 mos. at 7% is \$.98. Find the sum.

29. The interest is $\frac{3}{7}$ of the principal and the difference between the interest and true discount is \$15.30. Find the principal.

30. The bank discount on a certain sum of money is \$30 and the true discount on the same sum for the same time is \$25. Find the sum.

31. The true discount on \$1,378 for 6 mos. is \$53. Find the true discount on the same sum for 9 mos.

32. The interest is \$3 and the difference between the interest and true discount for the same time and rate is 25c. Find the principal.

33. The true discount on a sum of money for 8 mos. at 9% is \$44.46. Find the sum.

34. The simple interest on a certain sum of money for 2 yrs. is \$50, and the true discount for the same time and rate is \$45. Find the sum and the rate per cent. per annum.

35. If \$20 is the true discount on \$420 for 6 mos., how much is the true discount on the same sum for 12 mos.?

36. If \$10 is the true discount on \$110 for 8 mos., on what sum would \$10 be the true discount for 4 mos.?

37. If \$20 is the interest on \$180 for a certain time, what is the true discount on \$180 for the same time at the same rate?

38. If \$105 is the true discount on \$665 due some time hence, what should be a proper discount off a bill of \$665, which has only half the time to run?

39. The bank discount on a certain sum for 8 mos. at 6% is \$5.98. What would be the true discount on the same sum for the same time and rate?

40. If \$11.10 is the true discount on \$196.10 for 9 mos., for how long would \$24.50 be the true discount on \$269.50 at the same rate?

41. I have two notes, both drawn for 8 mos., together amounting to \$112. I have them both discounted at 9%, one at bank discount and one at true discount. If the total discount is \$6.45, find the face of each note.

42. The difference between the true and bank discounts on a certain sum of money for 3 yrs. at 8% is \$30. Find the sum.

43. The true discount on a certain sum of money at 8% for 60 dys. is \$82.24. What is the bank discount on the same sum for the same time at the same rate?

44. A certain sum of money loaned at simple interest amounts to \$334.40 in 9 mos., and in 7 mos. more to \$345.60. Find the sum and rate.

45. The present worth of \$359.04, due a certain number of days hence, the rate of interest being 6%, is \$350.40. Find the number of days.

46. The present worth of \$405.60, due a certain number of months hence, is \$390. If the rate of interest is 6%, find the number of months.

47. Find the difference between the true and bank discounts on \$350 for 48 dys. at 5%.

48. What must be the amount of a note which matures in 93 dys. for the difference between the true and bank discounts on it to be 1c.? Discount rate 5%.

49. A tradesman marks his goods with two prices, one for cash and one for credit of 6 mos. What relation should the prices bear to each other, allowing interest at $7\frac{1}{2}$%? If the credit price of an article be \$33.20, what is the cash price?

50. Find the sum of money whose true discount for one year at 10% is greater by $\$3\frac{58}{189}$ than the sum of the true discounts of one-half of it at 8% and the other half at 12% for 1 yr.

51. Bought a farm for \$10,000, payable one-half cash, the remainder in 1 yr., with interest at 6%. I sell immediately for \$12,000, payable in 3 mos., with interest at 4%. What is my present gain, money being worth 5% per annum?

414. Comparison of Rates of Interest and Discount

Illustration 1

Toronto, Ont., May 30, 1944.

Seventy days after date I promise to pay to.....
F. D. White.....*or order at The*
Royal Bank of Canada, Church St. Branch, Toronto,.....
One Hundred.....$\frac{00}{100}$ *Dollars,*
value received. W. BENNER.

Suppose that F. D. White got E. Bowlby to discount the above note at 6% on May 30, what rate of interest would Mr. Bowlby make on the money he so invested?

SOLUTION

The note will be legally due 73 dys. after the date of discount.

The amount of discount for 73 dys. at 6% on \$100 = \$1.20.

Mr. Bowlby would give Mr. White \$98.80 for the note. He would then wait 73 dys. until the note matured, when he would receive \$100 from Mr. Benner. The investment of \$98.80 for 73 dys. produces \$100 – \$98.80 = \$1.20 interest.

Interest on \$98.80 for 73 dys. = \$1.20.

Interest on \$98.80 for 1 yr. would be $\$1.20 \times \frac{365}{73} = \$6.00$

Interest on \$100.00 for 1 yr. would be $\$6.00 \times \frac{10000}{9880} = \$6\frac{18}{247}$

Therefore, Mr. Bowlby would really make $6\frac{18}{247}\%$ interest on his investment.

415. *Illustration* 2

Suppose that Mr. Bowlby had wanted to make 6% interest on his money, what rate of discount would he have charged Mr. White?

SOLUTION

The interest on \$100 for 73 dys. at 6% = \$1.20.

Then the amount of \$100 for 73 dys. at 6% = \$101.20.

Therefore, to make 6% interest on his money, he would have to charge discount at the rate of \$1.20 on \$101.20 for 73 dys.

Discount on \$101.20 for 73 dys. = \$1.20.

Discount on \$101.20 for 1 yr. would be $\$1.20 \times \frac{365}{73} = \$6.$

Discount on \$100.00 for 1 yr. would be $\$6 \times \frac{100}{101.20} = \$5\frac{235}{253}.$

Therefore, to make 6% interest on the money he invested in discounting the note, Mr. Bowlby would have to charge discount at the rate of $5\frac{235}{253}\%$.

EXERCISE 125

1. What rate of interest is made when a note nominally due in 60 dys. is discounted at 5% ?

2. A bank discounted a note legally due in 90 dys. at 8% per annum. What was the actual rate of interest made by the bank on the transaction ?

3. At what rate should a note legally maturing in 45 dys. be discounted to produce 5% interest per annum on the investment ?

4. A moneylender wishes to make 7% on the money he invests. What rate of discount should he charge on notes nominally maturing in 30 dys. ?

5. A note of $700, dated Aug. 12, 1945, and payable 90 dys. after date, was discounted Sept. 26, 1945. What was the rate per cent. of discount if the proceeds were $694.40 ?

6. In question 5, what rate of interest did the purchaser make on his investment ?

7. What rate of interest was made when a note legally due in 57 dys. was discounted at 4% ?

8. The true discount on $922.10 for 48 dys. is $9.60. Find the rate. What rate of bank discount would give the same amount of discount ?

9. A note legally maturing in 85 dys. was discounted at 9%. What rate of interest did the purchaser make on his investment ?

10. A note dated January 1, 1944, at 90 dys., for $730, with interest at 5%, was discounted at a bank on January 21st at 7%. Find the rate of interest made by the bank on the amount advanced.

ANNUITIES

416. An **Annuity** is a specified sum of money paid annually or at equal periods, half-yearly, quarterly, and so on.

417. A **Certain Annuity** is one which begins and ends at a fixed time.

For instance, A is told on Jan. 1, 1942, that, starting with Jan. 1, 1943, he will be entitled to receive $100 each year for three years. This might be called a three-year annuity of $100, and might be presented to the mind thus—

Jan. 1, 1942 . {	$100	$100	$100
	Jan. 1, 1943.	Jan. 1, 1944.	Jan. 1, 1945

If this annuity is to be derived from an investment of money, loaned at 5%, it would require the use of $2,000 for three years to secure the income. The principal amount of $2,000 would need to be put out at interest on Jan. 1, 1942, to have the first $100 ready on Jan. 1, 1943.

This explains the law that an annuity begins, not at date of first payment, but one annuity interval before.

418. A **Perpetual Annuity**, or **Perpetuity**, is one which continues for ever. Thus, $100, if paid each year for ever, would require the perpetual use of $2,000 capital at 5%.

419. An **Annuity in Possession**, or an **Immediate Annuity**, is one that begins immediately.

420. A **Deferred Annuity**, or an **Annuity in Reversion**, is one that begins at some future time.

421. An **Annuity in Arrears**, or **Forborne**, is one on which the payments are not made when due.

422. The **Amount**, or **Final Value**, of an annuity is the sum to which all its payments, with interest on each, will amount at its termination.

423. QUESTIONS OF THE FIRST ASPECT

Given the annuity, the time, and the rate, to find its final value.

Suppose in the case mentioned above that A did not get his $100 each year as intended. The amount due him at the end of the three years might be expressed as follows—

$100 plus interest on it for 2 yrs.
$100 plus interest on it for 1 yr.
$100

In other words, he would be entitled to his $300 and also to the interest for the time for which he is kept out of the use of his money. Note that the first interest period is one year less than the number of years marking the duration of the annuity.

Annuities in actual practice are worked out on the lines of compound interest. It is absolutely necessary, therefore, that a good knowledge of compound interest should be in hand. Granting this, we have acquired, it may be insensibly, the introductory steps in calculating the final value of an annuity.

Let us explain our position. Suppose we have to find the amount of $1 for 3 yrs. at 3%, compound interest.

$1 .03 $1.03 end of 1st year	$1 .03 $ .03
.0309 $1.0609 end of 2nd year	$1.03 .03 $ .0309
.031827 $1.092727 end of 3rd year	$1.0609 .03 $ .031827

The amount of $1 is $1.092727, or the interest alone is $.092727. Now, of what does this interest of $.092727 consist? Simply the interest on $1 each year—3c.—deposited at the end of each year, and also allowed to gather interest. Suppose, for instance, we kept our $1 in one bank, and took the interest every year and placed it in another bank. The amount we would have in the second bank at the end of the time would be the amount of an annual deposit of 3c., with the interest at the rate of 3% compound, added to it.

Now, an annual deposit is called an annuity, and we see that in finding the amount of $1 for 3 yrs. at 3%, compound interest, we cannot help finding the amount of an annuity of 3c. for 3 yrs. at 3%, compound interest.

Knowing, then, the amount of an annuity of 3c. to be $.092727, we can easily find the amount of an annuity of $1. Thus—

If annuity of 3c. amounts to $.092727

$\therefore$ annuity of 1c. would amount to $\frac{\$.092727}{3}$

And annuity of 100c. would amount to $100 \times \frac{\$.092727}{3}$

That is, we multiply our figure by 100 and divide it by 3, or, which is the same thing, we divide our figure by .03.

From all of which we deduce this useful rule—

424. Rule.

To find the value at maturity of an annuity of $1, *we find the compound interest of* $1 *for the given time at the given rate, and divide by the given rate expressed decimally.*

When we know what an annuity of $1 amounts to, we can easily find by multiplication the amount of any annuity.

Now, even in finding the compound interest, we can take a shorter method than that already expressed. The amount of $1 for 3 yrs. at 3% can be expressed thus—

$$(1.03)^3$$

To get the interest alone we would subtract our principal of $1 from this, giving us

$$(1.03)^3 - 1$$

and when we divide this interest by the rate we have

$$\frac{(1.03)^3 - 1}{.03}$$

or, *putting it generally for all cases, the amount of* $1 *annuity*

$$= \frac{(1 + \text{rate})^{\text{time}} - 1}{\text{rate}}$$

425. Illustration 1.—What is the amount of an annuity of $400 for 23 yrs. at 5% compound interest?

SOLUTION BY RULE ESTABLISHED

$$\$400 \times \left\{\frac{(1.05^{23} - 1}{.05}\right\}$$

$$= \$400 \times \left\{\frac{(3.071524) - 1}{.05}\right\}$$

$$= \$400 \times \frac{(2.071524)}{.05}$$

$$= \frac{\$828.6096}{.05}$$

$$= \$16572.19$$

426. Illustration 2.—Find the amount accumulated at the end of 15 yrs. by a person who deposits in a bank at the beginning of each year the sum of $200, the bank paying 4% interest, compounded half-yearly.

SOLUTION

Since each deposit is made at the beginning of the year, the first deposit bears interest for 15 yrs., not for 14 yrs., as in the case of a 15-yr. annuity; and there will be no deposit at the end of the 15th year. We can, therefore, get the result by finding the amount of a 15-yr. annuity for the same time and rate, and adding to it the amount of one payment for 15 yrs. and subtracting one payment. Or, which is the same thing, to the amount of the 15-yr. annuity we can add the compound interest on one payment for 15 yrs.

The rate of 4%, when compounded semi-annually, is equal to an effective rate of 4.04% per annum.

$$(1.0404)^{15} = (1.02)^{30} = 1.81136$$

$$\text{Result} = \$200 \times \left(\frac{1.0404^{15} - 1}{.0404} + 1.0404^{15} - 1 \right) = \$4178.91$$

EXERCISE 126

1. What is the final value of an annuity of $350 for 10 yrs. at 6% per annum, compounded yearly?

2. What will an annuity of $74 amount to in 30 yrs. at 4%, compound interest?

3. What is the final value of an annual pension of $210 for 12 yrs. at 4%, compound interest?

4. A man pays $150 yearly for 15 yrs. for an endowment policy of $2,500. Find the accumulated value of payments, reckoning money at 6% per annum.

5. Find the amount of an annuity of $500 for 3 yrs., reckoning money at 10% per annum, compounded semi-annually.

6. If $500 is deposited at the beginning of each year for 8 yrs., what amount is due at the end of the time, money being worth 5% per annum, compound interest?

7. What amount will a man have to his credit who deposits in a savings bank $450 at the beginning of each year for 10 yrs., if the bank allows interest at the rate of 5% per annum, compounded half-yearly?

8. If a man deposits in a savings bank $100 at the beginning of each year for 3 yrs., what amount will there be to his credit at the end of the third year if the bank allows interest in the meanwhile at 3%, compounded half-yearly?

427. QUESTIONS OF THE SECOND ASPECT

Given the amount of the annuity, the time, and the rate, to find the annuity.

Illustration 1.—What sum of money deposited at the end of each year for the next ten years will amount to $800, money being worth 5%?

SOLUTION

A deposit of $1 at the end of each year for 10 yrs. will amount to

$$\frac{\$1 \times (1.05^{10} - 1)}{.05}$$

$800 will, then, be the amount of an annual deposit of

$$\$800 \div \frac{(1.05^{10} - 1)}{.05} = \frac{\$800 \times .05}{1.05^{10} - 1} = \frac{\$40}{.62889} = \$63.60$$

Illustration 2.—The town of Woodstock borrowed $20,000, and agreed to pay 5% compound interest. What sum must be set apart annually as a sinking fund to pay the debt in 12 yrs.?

SOLUTION.—This time the question within the main question is to find what is due at the end of the time, viz., the amount of $20,000 for 12 yrs. at 5% compound interest.

$$\text{Amount} = \$20{,}000 \times (1.05)^{12}$$
$$= \$35{,}917.12$$

$$\text{Then} \quad \$35{,}917.12 \div \frac{(1.05)^{12} - 1}{.05}$$
$$= \$2{,}256.58 \text{ Answer}$$

Illustration 3.—A farmer has a mortgage on his farm of $5,340, principal payable in 5 yrs. from Jan. 1, 1942, interest payable half-yearly at 5%. He has provided for the interest, but desires to place in a savings bank an annual deposit which, compounded at 4% every 6 mos., will give him enough to pay off the mortgage at the end of 5 yrs. What is the annual deposit to be made on

Jan. 1, 1942, and the four following years, to have $5,340 at end of 5 yrs. ?

SOLUTION.—A reference to Illustration 2, Questions of First Aspect, will show the effect of the deposit being made at the beginning of the year, instead of at the end—the value of $1 annuity must be increased by the compound interest on $1 for the full time. The solution then appears as follows—

$$\$5{,}340 \div \left\{ \frac{(1.0404)^5 - 1}{.0404} + (1.0404)^5 - 1 \right\}$$

$$= \$946.87 \text{ Answer}$$

EXERCISE 127

1. What annual deposit for 15 yrs. at 5% per annum will amount to $5,000 ?

2. A railroad company issued sinking fund bonds at 6% for $200,000, payable in 10 yrs. If at compound interest, what sum must be set apart annually to meet interest and principal when due ?

3. What sum must be set apart annually to rebuild a bridge costing $30,000, estimated to last 17 yrs., allowing 5% compound interest ?

4. A merchant is mortgagor on a mortgage of $5,000 due in 1946. He has provided for interest, but intends to pay principal out of proceeds of four mortgages of $1,250, each of which he holds, and which expire one in 1942, 1943, 1944, 1945. He proposes to invest sufficient of the principal of these at compound interest, 4% half-yearly, to provide for the $5,000 he owes in 1946. How much of each does he invest ?

5. A man pays $240 yearly for 15 yrs. for an endowment policy of $4,000. Reckoning money worth 6% per annum, payable yearly, how much is he paying each year for the life risk ?

6. A corporation obtains a loan of $100,000, to be paid during or at the end of 15 yrs. Show, by finding in each case the annual tax to be collected, which of the following ways is the best, and by how much—

(*a*) Pay the interest annually at 5% per annum, and create a sinking fund to meet the debt, to be invested at 4% per annum, convertible half-yearly.

(*b*) Repay loan and interest at 5½% per annum in 15 equal annual instalments.

(*c*) Invest annually an amount bearing interest at $6\frac{1}{2}\%$ per annum which, at the end of 15 yrs., will repay the loan and interest at 6% per annum. ($1{,}055^{15} = 2.232479$; $1.065^{15} = 2.57184$.)

428. QUESTIONS OF THE THIRD ASPECT

Given the necessary data, to find the present worth of an annuity.

Illustration 1.—What is the present value of an annuity of \$154 for 19 yrs. at 5% compound interest?

SOLUTION

$$\text{Final value of annuity} = \frac{\$154\,(1.05^{19} - 1)}{.05}$$

Amount of \$1 for 19 yrs. at 5% compound interest $= \$(1.05)^{19}$

$\$(1.05)^{19}$ has for its present value \$1.

$\frac{\$154\,(1.05^{19} - 1)}{.05}$ has for its present value $\frac{\$154\,(1.05^{19} - 1)}{.05} \div 1.05^{19}$

$$= \frac{\$154\,(1.05^{19} - 1)}{.05\,(1.05)^{19}} = \frac{\$3{,}080\,(1.52695)}{2.52695} = \$1{,}861.14.$$

Illustration 2.—What is the present value of an annuity of \$200, deferred 5 yrs. and to run 10 yrs., if money is worth 5% per annum, payable yearly?

SOLUTION

$$\text{Final value of annuity} = \frac{\$200\,(1.05^{10} - 1)}{.05}$$

Present worth for 15 yrs. (5 yrs. + 10 yrs.)

$$= \frac{\$200\,(1.05^{10} - 1)}{.05} \times \frac{1}{1.05^{15}} = \$1{,}210.04.$$

Illustration 3.—A town issues debentures for \$12,000, bearing interest at 6%, payable yearly, and to run 5 yrs. For what sum should they sell, money being worth 5% yearly?

SOLUTION.—In this case there are really two questions to solve, viz., to find the present worth of the debenture itself and the present worth of the interest on the debenture. The interest, 6% of \$12,000, gives us a figure of \$720 to be paid each year. The two results added give us the complete P.W.

$$\frac{\$12{,}000}{1.05^5} + \frac{\$720 \times (1.05^5 - 1)}{.05\,(1.05)^5} = \$12{,}519.54$$

Illustration 4.—The work of calculating the present worth of a bond or debenture may be considerably shortened by the application of the following formula:

Let C = redemption value
K = present worth of C
c = coupon rate (the rate of interest stated in the bond)
i = income rate
A = present value of bond

FORMULA

$$A = K + \frac{c}{i} \times (C - K)$$

Applying the above formula to the problem in Illustration 3, we get the following solution—

$$C = \$12{,}000.00$$
$$K = \frac{\$12{,}000.00}{1.05^5} = \frac{\$12{,}000.00}{1.2762816} = \$9{,}402.31$$
$$A = \$9{,}402.31 + \tfrac{6}{5} \times (\$12{,}000.00 - \$9{,}402.31)$$
$$= \$9{,}402.31 + \$3{,}117.23 = \$12{,}519.54.$$

Illustration 5.—If money is worth 5%, find the present value of a perpetuity of \$450.

SOLUTION.—To produce \$450 yearly for ever would require the perpetual use of $\$450 \times \frac{100}{5} = \$9{,}000$.

The present value of the perpetuity is, therefore, \$9,000.

EXERCISE 128

1. If money is worth 5% per annum, what should be paid for an annuity of \$400 to run 4 yrs.?

2. Find the present worth of an annuity of \$1,000 to run 15 yrs., the first payment to be made at the end of 1 yr., calculated at 4% per annum.

3. A man agrees to pay for a farm \$1,600 a year for 5 yrs. What sum paid now would be equivalent to this price, money being worth 4% per annum, payable yearly?

4. Compare the present values of two scholarships payable half-yearly, one of \$50 to continue $2\frac{1}{2}$ yrs., and the other for \$35 to continue $4\frac{1}{2}$ yrs., reckoning 5% compound interest, compounded half-yearly.

5. What is the present value of a perpetual annuity of $250, money being worth 4% per annum, payable yearly?

6. A farm yields a rental of $400 yearly. What is the value of this farm, money being worth 6% per annum?

7. Find how much should be paid for a perpetual annuity of $100, deferred 7 yrs., calculated at 5% yearly.

NOTE.—The expression, "deferred 7 years," means that the annuity begins in 7 years; this means that the first payment will be made at the end of the eighth year. The present worth in this case will be for 7 yrs.

8. What is the present value of a perpetual annuity of $250, the first payment to be made at the end of 6 yrs., calculated at 4% per annum?

9. Find the present value of an annuity deferred 1 yr. and to run 4 yrs., calculated at 5% yearly.

10. A mortgage of $2,500, bearing interest at 8% per annum, payable half-yearly, has 4 yrs. to run. Find its present value, calculated at 6% per annum, payable half-yearly.

11. A mortgage of $5,000, bearing interest at 6% per annum, payable yearly, has 10 yrs. to run. Find its present value, money being worth 5% per annum, payable half-yearly?

12. Find the present value of a mortgage for $10,000, bearing interest at 6% per annum, payable half-yearly, and having 3 yrs. to run, money being worth 4% per annum, payable half-yearly.

13. Bonds bearing 6%, payable annually, have 3 yrs. to run. What can I afford to pay for them so that I shall make 5% per annum on my money as long as it is outstanding?

14. What should be paid for a $1,000 coupon bond maturing 5 yrs. hence, and bearing 4% interest, payable annually, so that the investor will receive 5% per annum compound interest on his money?

15. What can I afford to pay for a $2,000 bond maturing 5 yrs. hence, and bearing interest at 7%, payable annually, so that I shall be making 6% compound interest on my money?

16. What can I afford to pay for a debenture for $100, having 3 yrs. to run and bearing interest at 5%, payable yearly, so that I may realize 4½% per annum on the investment?

17. What amount will be required to be raised annually at the

beginning of each year for 5 yrs. so that there may be sufficient to provide a sinking fund of $25,000 at the end of the 5th year, and to pay interest on $25,000 in the meanwhile at 5% per annum, money being worth 4%?

18. Find the present value of an annuity of $200, payable for 12 yrs., the first payment to be made at the end of 2 yrs., money at 3% per annum yearly.

19. What sum of money, deposited at the end of each year for the next 5 yrs., will then be sufficient to purchase a perpetual annuity of $50, deferred 2 yrs., money being worth 6% yearly?

20. What sum of money, deposited at the end of each year for the next 6 yrs., will then be sufficient to purchase an annuity of $500, deferred 2 yrs., to run 5 yrs., money being worth 4% per annum, payable yearly?

21. What sum of money deposited at the end of each year for the next 5 yrs., will then be sufficient to purchase a perpetual annuity of $100, deferred 2 yrs., money being worth 5%?

22. Money being worth 6% per annum, compound interest,

(*a*) What would be the cost of a 5-yr. annuity of $400?

(*b*) What annuity to run 5 yrs. could be bought for $2,000?

23. What is the present value of a mortgage of $5,000, bearing interest at 6%, payable yearly, having 5 yrs. and 6 mos. to run, the next payment of interest being due in 6 mos., money being worth 5% per annum, payable yearly?

24. A farm bears a mortgage of $3,000 at 8% interest, payable half-yearly; the mortgage has 5 yrs. to run. What sum paid now would be equivalent to reducing the interest on the mortgage to 5%, money being worth 4% per annum, payable half-yearly?

25. Find the present value of an annuity of $100, deferred 3 yrs. and to run 4 yrs., calculated at 5% yearly.

COMMERCIAL ARITHMETIC

TEST PAPER—No. 15

TRUE DISCOUNT—ANNUITIES

Your work for this paper is to be submitted to the instructor for correction and criticism.

1. After carrying a stock of silk for 4 months, I sold it at an advance of 30% on first cost, extending to the purchaser a credit of 1 yr. without interest. If money is worth 5% per annum, what was my per cent of profit or loss ?

2. The simple interest on a certain sum of money for 2 yr. is $50, and the true discount for the same time and rate is $45. Find the sum and the rate per cent per annum.

3. The bank discount on a sum of money is $90, and the true discount on the same sum for the same time and rate is $75. Find the sum.

4. What is the final value of an annual pension of $225 for 12 yr. at 5% compound interest ? $(1{\cdot}05^{12} = 1{\cdot}7958563.)$

5. A borrower obtains $1,500 at 5% to be repaid in five equal annual instalments of principal and interest. Give yearly payment (compound interest).

6. A town issues Debentures for $5,000.00, payable at the end of 5 yr., with interest yearly at 7%. What amount of levy is required each year to pay interest and provide a Sinking Fund earning 5% yearly to retire the Debentures at maturity ?

7. The bonus to be paid for a lease has been fixed at the present value of a rent of $1,000 per annum, payable at the end of each year

for the next 3 yr., and calculated on the basis of 4% per annum, compounded half-yearly.

Find the amount paid as bonus.

8. A town issues debentures to the amount of $20,000.00, bearing interest at 5% per annum. One of your clients asks you what amount he should pay for them that he may receive 4% on his investment, they having 4 yr. to run.

(*a*) Find the amount he should pay.

(*b*) Find how much of each of the four equal annual payments that he would receive will be for repayment of principal and how much should be considered by him to be interest.

9. Taking the same bond issue as that in Problem 8, calculate what your client should pay so that his investment will yield him a return of 6% per annum and show the application of the four equal annual payments.

SECTION 4

INVOLUTION AND EVOLUTION

INVOLUTION

429. **Involution** is the operation of finding any power of a given number.

430. A **Power** of a number is the product obtained by using it a number of times as a factor.

431. A **Square** of a number is the second power of that number or the product obtained by multiplying the number by itself. Thus the square of $4 = 4 \times 4 = 16$.

432. The **Cube** of a number is the third power of the number or the product obtained by multiplying together three factors, each of which is the number. Thus the cube of $4 = 4 \times 4 \times 4 = 64$.

433. The **Exponent** is a figure written to the right of the number and a little above it, indicating the number of times the factor is to be used. Thus in 4^2 the exponent 2 indicates that the factor 4 is to be used twice.

EXERCISE 129

1. What is the square, or second power, of 25 ?
2. What is the cube, or third power, of 5 ? 24 ? 168?
3. What is the fourth power of 4 ? 8 ? 16 ? 25 ?
4. What is the third power of $\frac{3}{4}$? $\frac{5}{6}$? $\frac{7}{8}$?
5. Expand the following: 6^3, 9^2, 8^4, 10^5, 23^3.
6. Expand the following: $(\frac{2}{9})^3$, $(\frac{4}{5})^4$, $(3\frac{1}{2})^3$, $(14\frac{2}{7})^2$, $(5.06\frac{1}{4})^3$.
7. Find the difference between 35^3 and 90^2.

EVOLUTION

434. **Evolution** is the operation of finding any root of a given number.

435. The **Square Root** of a given number is that number whose square is the given number.

EXAMPLES.—4 is the square of 2, $\therefore$ 2 is the square root of 4 ; 9 is the square of 3, $\therefore$ 3 is the square root of 9 ; 100 is the square of 10, $\therefore$ 10 is the square root of 100.

436. The **Cube Root** of a given number is that number whose cube is the given number.

EXAMPLES.—8 is the cube of 2, $\therefore$ 2 is the cube root of 8; 125 is the cube of 5, $\therefore$ 5 is the cube root of 125; 1,000 is the cube of 10, $\therefore$ 10 is the cube root of 1,000.

There are two ways of denoting Evolution. In the first or older notation, the square root of a given number is denoted by prefixing the symbol $\surd$ to the given number; the cube root is denoted by prefixing $\sqrt[3]{}$.

EXAMPLES.—$\surd 64$ denotes the square root of 64; $\sqrt[3]{64}$ denotes the cube root of 64.

The second method employs fractional exponents.

Thus, $64^{\frac{1}{2}}$ denotes the square root of 64; $64^{\frac{1}{3}}$ denotes the cube root of 64.

437. To extract the square root of a number.

RULE

1. *Point off the given number into periods of two figures each, beginning at the decimal point.*

2. *Find the highest square contained in the left-hand period and place its root to the right of the number, in the place occupied by the quotient in division.*

3. *Subtract the square of the digit put in the root, from the left-hand period, and to the remainder bring down the next period to the right, for a new dividend.*

4. *Double the part of the root already found for a* TRIAL DIVISOR.

5. *Find how many times the trial divisor is contained in the dividend, exclusive of the right-hand digit, and place the figure thus obtained both in the root and also to the right of the trial divisor.*

6. *Multiply the divisor thus completed by the digit last put in the root; subtract the product from the dividend, and to the remainder bring down the next period for a new dividend.*

7. *Again, double the part of the root already found for a new* TRIAL DIVISOR; *proceed as in* 5 *and* 6, *and continue the process until all the periods are brought down.*

438. Illustration.—What is the square root of 22420225?

	22|42|02|25(4735
	16
87	642
	609
943	3302
	2829
9465	47325
	47325

SOLUTION.—Here 22 is the left-hand period and the highest square in 22 is 16, of which the square root is 4. We place 4 in the root and subtract 16 from 22. This leaves a remainder 6, to which we bring down the next period, 42, and thus obtain 642 for the new dividend. Our next step is to find the trial divisor, which we obtain by doubling the part of the root already found. This gives us 8 (= 4 doubled), and we ask how many times 8 will go into 64 (the dividend exclusive of the right-hand digit). Bearing in mind that we are to put the digit thus obtained both in the root and in the divisor, and that the complete divisor will be over 80, we find that the required digit is 7, which we accordingly place both in the root and in the divisor. The complete divisor is 87, which multiplied by 7, gives 609, and this subtracted from 642 gives a remainder 33, to which we bring down the next period, 02, and thus get 3302 for the next dividend.

Again, doubling the part of the root already found, we obtain 94 (= 47 doubled) for a trial divisor, and as this will go into 330 (the dividend exclusive of the right-hand digit) 3 times, we place 3 both in the root and in the divisor.

Multiplying the 943 thus obtained by 3, subtracting and bringing down the next period, we get 47325 for the next dividend. The next trial divisor is 946 (= 473 doubled), which will go into 4732 (the dividend exclusive of the right-hand figure) 5 times; and we therefore place 5 both in the root and in the divisor. Multiplying and subtracting, we find no remainder. 4735 is therefore the square root of 22420225.

PROOF.—$4735 \times 4735 = 22420225$.

NOTE 1.—If there is a *remainder* after the root of the last period is found, annex *periods of ciphers*, and proceed as before. The figures of the root thus obtained will be *decimals*.

NOTE 2.—If the trial divisor is not contained in the dividend, *annex a cipher* both to the root and to the divisor, and bring down the next period.

NOTE 3.—It sometimes happens that the remainder is *larger* than the divisor; but it does not necessarily follow that the *figure* in the root is too *small*.

439. To extract the square root of a decimal.

RULE

Begin at the units' place, and proceed towards the left and right, to separate into periods of two figures each, then extract the root as in whole numbers.

NOTE 1.—The *left-hand* period in *whole numbers* may have but *one* figure; but in *decimals*, each period must have *two* figures. Hence, if the number of decimals is odd, a *cipher* must be annexed to complete the period.

NOTE 2.—It must be kept in mind that no period should contain an integer and decimal, and that, if there is an odd number of decimal places in the given number, the last period must be completed by annexing a cipher.

440. To extract the square root of a fraction.

RULE

Reduce the fraction to its simplest form and find the square root of each term separately.

NOTE 1.—If the denominator of the given fraction, when reduced, is an imperfect square, reduce the fraction to a *decimal*, and proceed as above.

NOTE 2.—Mixed numbers should be reduced to improper fractions, or the fractional part to a decimal.

EXERCISE 130

Find the square root of—

1. 576	9. 39601	17. 11.9025
2. 1225	10. 88209	18. 41249.61
3. 2025	11. 751689	19. .697225
4. 4356	12. 106929	20. 54.6121
5. 11025	13. $\frac{225}{289}$	21. 3
6. 18225	14. $\frac{169}{625}$	22. 376.4
7. 15129	15. $\frac{256}{361}$	23. 12.823
8. 103041	16. $\frac{196}{841}$	24. 4367.2

441. To extract the cube root of a number.

Question.—What is the cube root of 184,608,795,384?

(*a*)	(*b*)	(*c*)	
	5		184,608,795,384
			125
15	6	7500	59608
		936	
		8436	50616
			8992795
168	9	940800	
		15201	8604009
		956001	388786384
1707	4	97128300	388786384
		68296	
		97196596	

Divide the number into periods of three figures each, beginning at the decimal point.

(1) The first period is 184, and the nearest perfect cube to 184 is 125, which is the cube of 5. Then 5 is the first digit in our answer.

Put 5 in column (*b*), the answer column.

Subtract 125 from 184, and bring down second period.

(2) Multiply 5 by 3, and put the result, 15, in column (*a*).

Now multiply 5 squared by 300 = 7500. Put this in column (*c*) opposite 15.

Now divide 7500 into 59608. We find it goes 7 times. Now if we put 7 in column (*b*), and multiply 157 by 7, and add result to 7500, we get 8599. But if we multiply 8599 by 7, our result is greater than 59608. Therefore 7 is too great for our second digit of answer.

So dividing 59608 by 7500, take 6 for quotient instead of 7.

Put 6 down in column (*b*) as second figure of answer.

Multiply 156, that is, the 15 in column (*a*) and 6 in column (*b*), by 6, and add result to 7500. Thus we get 8436.

Now multiply 8436 by 6, and we get 50616.

Subtract this from 59608, and our remainder is 8992, to which we attach the third period.

(3) Now to find third figure of answer.

Multiply 15 by 10, and to it add 6×3, or 18. 6 is the second figure of column (*b*), and 15 the first in column (*a*).

Thus we get in column (*a*) 168.

Square 56, and multiply result by 300 = 940800.

This divided into 8992795 gives 9.

Put down 9 in column (*b*) as third figure of answer.

Multiply 1689 (168 in first column, 9 in second) by 9 = 15201, and add result to 940800 = 956001.

Now multiply 956001 by 9 = 8604009, and subtract this from 8992795 = 388786.

(4) To 388786 attach fourth period.

Multiply 168 by 10, and add 9 × 3, or 27.

Thus 1680 + 27 = 1707. Put this in column (*a*). Square 569, and multiply by 300 = 97128300.

Divide this into 388786384 = 4.

Put 4 in column (*b*) as fourth figure in answer, and proceed as in (2) and (3).

Thus our answer is 5694 exactly.

NOTE 1.—If there is a *remainder* after the root of the last period is found, annex periods of ciphers, and proceed as before. The root figures thus obtained will be *decimals*.

NOTE 2.—If a trial divisor is *not contained* in the dividend, put a *cipher* in the root, *two ciphers* on the right of the divisor, and bring down the next period.

NOTE 3.—If the product of the divisor completed into the figure last placed in the root *exceeds* the dividend, the root figure is too large. Sometimes the remainder is *larger* than the divisor completed; but it does not necessarily follow that the root figure is *too small*.

442. To extract the cube root of a decimal.

RULE

Begin at the units' place, and proceed both toward the left and right to separate into periods of three figures each, then extract the root as in whole numbers.

NOTE.—The *left-hand* period in *whole numbers* may have but *one* or *two* figures, but in *decimals* each period must have *three* figures. Hence, ciphers must be annexed to the right of the decimal to complete the periods, when necessary.

443. To extract the cube root of a fraction.

RULE

Reduce the fraction to its lowest terms, then extract the root of its numerator and denominator.

NOTE 1.—When the denominator is not a *perfect cube*, the fraction should be reduced to a decimal, and the root of the decimal be found as above.

NOTE 2.—A *mixed* number should be reduced to an improper fraction.

EXERCISE 131

Find the cube root of—

1. 4096
2. 32768
3. 74088
4. 493039
5. 250047
6. 614125
7. 14706125
8. 84027672
9. 354894912
10. 673373097125
11. 122615327232
12. 389.017
13. .194104539
14. 48228.544
15. 2
16. 14.6
17. $\frac{1831}{1728}$
18. $\frac{250}{686}$
19. $\frac{3}{4}$
20. $405\frac{28}{125}$

Find the fourth roots of—

21. 531441
22. 4100625
23. 1575.2961

24. Find the sixth root of 4826809.

25. What are the dimensions of a cubical block whose volume is 35,937 cu. in.?

26. A bin, whose capacity is 6,750 cu. in., is twice as long as it is wide or high. What are its dimensions?

27. A pile of wood has the form of a rectangular solid, 168 ft. long, 126 ft. wide, and 28 ft. high. What would be the length of one of its edges if it were in the form of a cube?

28. How large a cube could be made by using the total contents of a rectangular mound of earth that is 216 ft. long, 48 ft. wide, and 36 ft. high?

29. The dimensions of a rectangular box are as 1 : 2 : 3. If the box has the same capacity as one 6 ft. long, 4 ft. 2 in. wide, and 3 ft. 9 in. deep, find its measurements.

30. The cubical base of a monument contains 274,625 cu. ft. How many square feet in one of its faces?

31. There are 231 cu. in. in a gallon. Find the inside dimensions correct to tenths of an inch of a cubical tank that will hold 1,000 gals.

32. If 160,103,007 cubical blocks of stone, each 1 foot square, were placed in a cubical pile, what would be the length of each edge?

PRACTICAL MENSURATION

444. Mensuration treats of the measurement of lines, angles, surfaces, and solids.

SURFACES

445. A **Line** is that which has length, but not breadth or thickness.

446. A **Straight Line** is one that does not change direction. It is the shortest distance between two points.

447. A **Curved Line** is one that changes its direction at every point. It is one of which no three consecutive points are in the same direction.

448. Parallel Lines are lines that have the same direction and are equally distant at all points.

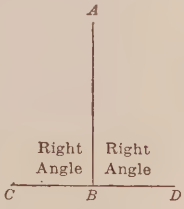
449. A **Horizontal Line** is one that is parallel to the horizon or water level.

450. A **Vertical Line** is one that is perpendicular to a horizontal plane.

451. A **Surface** is that which has length and breadth only.

If a straight edge laid anywhere upon a surface touches at every point, the surface is a plane surface or a plane.

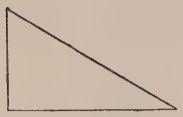
452. A **Right Angle** is an angle formed when one straight line meets another so as to make the adjacent angles equal. The lines forming the angles are said to be *perpendicular to each other*.



In the accompanying diagram ABC and ABD are right angles, and the lines AB and CD are perpendicular to each other.

453. A **Triangle** is a plane figure with three plane sides and three plane angles. The side on which the triangle stands is the *base*, the opposite corner the *vertex*, and the shortest distance from the vertex to the base, or the base extended, is the *height* or *altitude* of the triangle.

Triangle

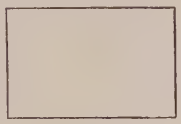


Right-angled Triangle

454. A **Right-angled Triangle** is a triangle having a right angle.

455. The **Hypotenuse** of a right-angled triangle is the side opposite the right angle.

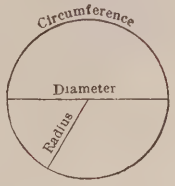
456. The **Perpendicular** is the side that forms a right angle with the base.



Rectangle

457. A **Rectangle** is a plane figure having four straight sides and four square corners. When the four sides are equal, the figure is usually called a *square*.

458. The **Perimeter** of any plane figure is the length of the line or lines inclosing it.

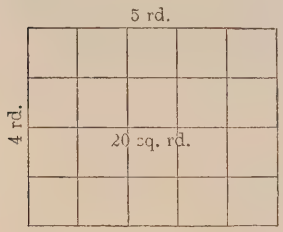


When a plane figure is bounded by a curved line, every point of which is equally distant from the centre, it is called a *circle*. The perimeter of a circle is called its *circumference;* a line passing through the centre and terminating in the circumference, the *diameter;* one-half of the diameter, the *radius*.

459. A **Polygon** is a plane figure having more than four straight sides.

460. To find the area of a rectangle or square.

Illustration.—What is the area of a garden 5 rd. long by 4 rd. wide?



SOLUTION

1 sq. rd. × 4 × 5 = 20 sq. rds.

RULE

Multiply the length by the breadth and the result will be the area.

EXERCISE 132

1. How many acres are there in a field 220 yds. long and 40 rd. wide?

2. How many bricks 8 in. long and 4 in. wide will pave a yard that is 116 ft. long and 46 ft. wide?

3. Which would be the more economical and by how much: to pave a walk 4 ft. wide and 200 ft. long with stone at 22c. per square yard or with brick at $1.02 per square yard?

4. Find the cost at 16c. per square yard of making a walk 4 ft. wide around the outside of a lot 10 rd. long and 130 ft. wide.

5. If the walk in the preceding question had been around the inside of the lot, how much would it have cost?

6. A farm cost $8,250 at $75 an acre. If it is 968 yds. long, how many rods wide is it?

7. Two fields contain 40 ac. each. One is in the form of a square and the other is 4 times as long as it is wide. Find the difference in the cost of fencing them at 45c. per rod.

8. A field containing 8 ac. is 5 times as long as it is wide. Find its perimeter in yards.

9. A rectangular piece of ground is 60 yds. long and contains $\frac{1}{3}$ of an acre. It contains a grass plot bordered by a walk 6 ft. wide. Find the area of the plot.

10. Find the difference between the perimeter of a square field containing $22\frac{1}{2}$ ac. and the perimeter of a rectangular field of equal area, the length of the latter being $1\frac{1}{3}$ times its width.

461. To find the area of a triangle when the base and altitude are given.

Illustration.—Find the area of a triangle whose base and altitude are 6 ft. and 8 ft. respectively.

SOLUTION.—In the accompanying diagram assume that the base ($C\ B$) is 6 ft. and the altitude ($A\ D$ is 8 ft. It will be seen that the altitude divides the triangle into two right-angled triangles, each of which is one-half of a rectangle whose sides are 8 ft. and 3 ft. Two triangles, each one-half of a rectangle 8 ft. by 3 ft., are equal to one rectangle 8 ft. by 3 ft. The area of the triangle given is, then, the product of these two dimensions, or 24 sq. ft.

RULE

Multiply one-half the base by the altitude.

462. To find the area of a triangle when the sides are given.

Illustration.—What is the area of a triangle whose sides are 12 ft., 16 ft., and 18 ft.?

SOLUTION

$(12 + 16 + 18) \div 2 = 23$

$23 - 18 = 5$ $\qquad 23 \times 5 \times 7 \times 11 = 8855$

$23 - 16 = 7$

$23 - 12 = 11$ $\qquad \sqrt{8855} = 94.1$ sq. ft. Ans.

RULE

From half the sum of the three sides, subtract each side separately; then multiply the half sum and the three remainders together, and extract the square root of the product.

463. To find the hypotenuse of a right-angled triangle.

Illustration.—The base of a right-angled triangle is 12 ft. and the perpendicular is 9 ft. What is the hypotenuse?

SOLUTION

$12^2 = 144$

$9^2 = 81$

$144 + 81 = 225$

$\sqrt{225} = 15$

RULE

To the square of the base add the square of the perpendicular; the square root of the sum will equal the hypotenuse.

464. To find the base or perpendicular.

Illustration.—The hypotenuse of a right-angled triangle is 25 in. and the perpendicular is 15 in. What is the base?

SOLUTION

$25^2 = 625$

$15^2 = 225$

$625 - 225 = 400$

$\sqrt{400} = 20$

RULE

From the square of hypotenuse, subtract the square of the given side; the square root of the remainder will be the required side.

EXERCISE 133

1. Find the area of a triangle whose base is 12 ft. 6 in. and altitude 6 ft. 9 in.

2. At 40c. a square yard, find the cost of paving a triangular court, its base being 105 ft. and its altitude 21 yds.

3. The base of a triangular field, containing 1 ac. is $90\frac{1}{3}$ yds., in length. What is the altitude?

4. At 10c. a square foot, what will it cost to lay sod on a triangular yard whose sides are 8, 15, and 17 ft. respectively?

5. The sides of an iron plate are 20, 21, and 29 ft. respectively. What is it worth at $2.70 a square yard?

6. The hypotenuse of a right-angled triangle is 50 ft. and the base is 40 ft. What is (*a*) the perpendicular, (*b*) the area of the triangle?

7. A ladder 17 ft. long was placed so as to just reach the top of a building 15 ft. high. How far from the base of the building was the ladder placed?

8. A triangular sail whose edges are respectively 33, 56, and 65 ft., was made of canvas that cost 80c. a square yard. What did the sail cost?

9. What is the distance diagonally across a floor that is 40 ft. long and 30 ft. wide?

10. Two upright poles are respectively 57 ft. and 82 ft. high. If the poles are 60 ft. apart, what must be the length of a line that reaches from the top of one to the top of the other?

THE CIRCLE

465. Principles.

1. The circumference = the diameter $\times$ 3.1416 nearly.

2. Therefore the diameter = the circumference $\div$ 3.1416 nearly.

3. The area of a circle = the square of the radius $\times$ 3.1416 nearly.

4. The area of a circle = the circumference $\times$ half the radius = 3.1416 times the square of the radius.

5. Therefore the radius of a circle = square root of (the area $\div$ 3.1416) nearly.

NOTE.—The fraction of $3\frac{1}{7}$ is commonly used in place of the decimal 3.1416, and is near enough for common practical operations.

466. To find the circumference of a circle.

Illustration 1.—What is the circumference of a circle whose radius is 14 ft. ?

SOLUTION

14 ft. × 2 = 28 ft. diameter,
28 ft. × $3\frac{1}{7}$ = 88 ft. Ans. (Prin. 1.)

RULE

Multiply the diameter by 3.1416, *or* $3\frac{1}{7}$.

467. To find the diameter of a circle.

Illustration 2.—The circumference of a circle is 352 ft. What is the diameter ?

SOLUTION

352 ÷ $3\frac{1}{7}$ = 112 ft. Ans. (Prin. 2.)

RULE

Divide the circumference by 3.1416, *or* $3\frac{1}{7}$.

468. To find the area of a circle when the diameter is given.

Illustration 3.—What is the area of a circle whose diameter is 28 ft. ?

SOLUTION 1

28 ft. ÷ 2 = 14 ft. radius
14 × 14 × $3\frac{1}{7}$ = 616 sq. ft. Ans. (Prin. 3.)

SOLUTION 2

28 ft. × $3\frac{1}{7}$ = 88 ft. the circumference. (Prin. 1.)
28 ft. ÷ 2 = 14 ft. the radius
88 × $\frac{1}{2}$ = 616 sq. ft. Ans. (Prin. 4.)

RULE

Multiply the square of the radius by 3.1416 *or* $3\frac{1}{7}$. *Or,*
Multiply the circumference by $\frac{1}{4}$ *of the diameter, or* $\frac{1}{2}$ *the radius.*

469. To find the radius, diameter, and circumference when the area is given.

Illustration 4.—The area of a circle is 616 sq. ft. Find the radius, diameter, and circumference.

SOLUTION

Radius $= \sqrt{616 \div 3\frac{1}{7}} = 14$ ft. (Prin. 5.)
14 ft. $\times$ 2 $= 28$ ft. the diameter
28 ft. $\times$ $3\frac{1}{7} = 88$ ft. the circumference. (Prin. 1.)

RULE

Divide the area by 3.1416 *or* $3\frac{1}{7}$ *and extract the square root of the quotient. Multiply the radius by* 2. *Multiply the diameter by* 3.1416 *or* $3\frac{1}{7}$.

EXERCISE 134

1. If it is 12 ft. 10 in. around the base of a tree, what is its diameter?

2. The radius of a circle is 2 ft. Find the whole perimeter of its semicircle.

3. What is the diameter of a wheel which turns around 1,000 times in going a mile?

4. The difference between the diameter and the circumference of a circle is 12 ft. Find its area.

5. A cow is tethered to a post driven in the centre of a lot 100 ft. square; the tether is just long enough for her to reach the fence. How much of the surface of the lot is she unable to crop?

6. The diameter of a carriage wheel is $24\frac{1}{2}$ in. Find how many turns the wheel makes in going 1 mi.

7. Find the cost of paving a $10\frac{1}{2}$ ft. drive around the outside of a circular plot 28 yds. in diameter, at 65c. a square yard.

8. Find the width of a circular path containing 120 sq. yds. which surrounds a circular pond whose circumference is 220 yds.

9. A circular pond, which is 4 mi. in diameter, has a driveway around it. A man wishing to reach a point directly across the pond from where he stands can drive at the rate of 10 mi. an hour and row at the rate of 6 mi. an hour. How many minutes can he save by going the quicker way?

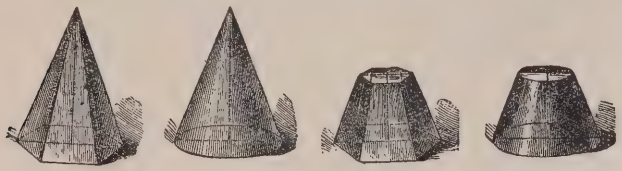
10. The ice on a pond, whose area is $\frac{1}{2}$ an acre, is 10 in. thick How many tons of ice may be taken from the pond supposing a cubic foot of ice to weigh 56 lbs.?

SOLIDS

470. A **Solid** is that which has length, breadth, and thickness.

471. A **Rectangular Solid** is a solid bounded by six rectangular sides or faces. When these sides are squares, the figure is called a *cube*.

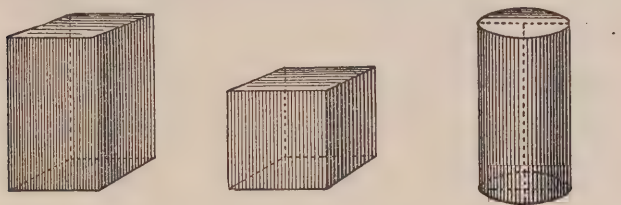
472. A **Cylinder** is a circular body of uniform diameter whose ends are equal and parallel circles.



Pyramid. Cone. Frustrum of a pyramid. Frustrum of a cone.

473. A **Pyramid** is a solid whose base is a polygon and whose sides terminate in a point called the *vertex*.

474. A **Cone** is a solid which has a circle for its base, and terminates in a point called the *vertex*.



Rectangular Solid. Cube. Cylinder.

475. The **Altitude** of a pyramid or cone is the perpendicular distance from the base to the vertex.

476. The **Slant Height** of a pyramid is the distance from the vertex to the middle point of any side of the base.

477. A **Frustrum** of a pyramid or cone is the part which is left after the top is cut off by a plane parallel to the base.

478. To find the solid contents of a cylinder.

Illustration.—Find the cost of digging a round well 25 ft. deep and 8 ft. in diameter, at 35c. per cubic yard.

SOLUTION

Area of base in square feet $= (4 \times 4 \times 3\frac{1}{7})$

Solid contents of well in cubic feet $= (4 \times 4 \times 3\frac{1}{7}) \times 25$

Solid contents of well in cubic yards $= \dfrac{4 \times 4 \times 3\frac{1}{7} \times 25}{27}$

Cost of digging $= \dfrac{35\text{c.} \times 4 \times 4 \times 3\frac{1}{7} \times 25}{27} = \$16.30$

RULE

Multiply the area of the base by the height of the cylinder.

479. The **Lateral Surface** of a cylinder is the surface of its curved sides.

The lateral surface of a cylinder is equal to the surface of a rectangular body, the length and height of which are equal to the circumference and height of the cylinder.

Thus, the lateral surface of the cylinder in the accompanying diagram is the area of the rectangle described by *A B C* and *D* back of the cylinder. Hence,

480. To find the area of the lateral surface of a cylinder.

RULE

Multiply the circumference of the base by the height of the cylinder.

MISCELLANEOUS

481. To find the convex surface of a pyramid or cone.

RULE

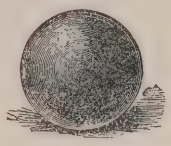
1. *Multiply the perimeter by one-half the slant height.*

2. *To find the entire surface, add the area of the base to the area of the convex surface.*

482. The **Altitude** of a frustrum is the perpendicular distance between its ends.

483. The **Slant Height** of a frustrum of a pyramid is the distance between the middle points of two parallel sides of one of its faces.

484. A **Sphere** or **Globe** is a solid terminated by a curve surface, every part of which is equally distant from a point within, called the *centre*.



Sphere or Globe.

485. The **Diameter** of a sphere is a straight line drawn through its centre and terminated both ends by the surface.

486. A **Hemisphere** is one-half a sphere.

487. The **Radius** of a sphere is a straight line drawn from its centre to any point in its surface.

488. To find the solid contents of a rectangular solid.

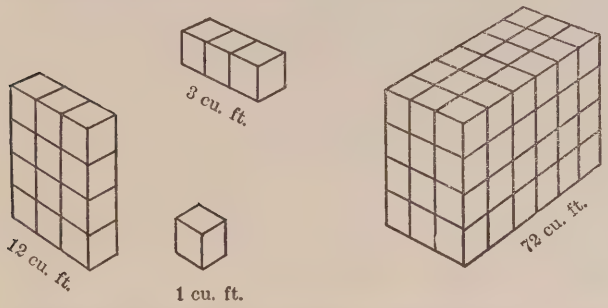


Illustration.—What is the volume of a solid 6 ft. long, 4 ft. high, and 3 ft. wide?

SOLUTION

1 cu. ft. × 3 × 4 × 6 = 72 cu. ft.

RULE

Multiply the length by the breadth by the thickness and the result will be the solid contents.

489. To find convex surface of a frustrum of a cone or pyramid.

RULE

1. *Multiply one-half the sum of the perimeters of the ends by the slant height.*

2. *To find the entire surface, add the areas of the ends to the area of the convex surface.*

490. To find the volume of a cone or pyramid.

RULE

Multiply area of the base by one-third the altitude.

491. To find the volume of the frustrum of a cone or pyramid.

RULE

$(A + a + \sqrt{A \times a}) \times h \times \frac{1}{3}$, *where "A" stands for the area of the larger end, "a" for the area of the smaller end, and "h" for the perpendicular height.*

492. To find the surface of a sphere.

Multiply the square of the diameter by $3\frac{1}{7}$.

493. To find the volume of a sphere.

Multiply the cube of the diameter by $3\frac{1}{7}$, *and divide the result by* 6.

494. To find the number of gallons in a cistern.

Find the volume in cubic inches and divide the result by 277.118.

NOTE.—There are 277.118 cu. in. in 1 gal.

495. To find the number of bushels of wheat in a bin or pile.

Find the volume in cubic inches and divide the result by 2150.42.

NOTE.—There are 2150.42 cu. in. in 1 bus.

496. To find the mean diameter of a cask (nearly).

Add to the head diameter $\frac{2}{3}$, *or, if the staves are but little curved,* $\frac{3}{8}$ *of the difference between the head and bung diameters.*

497. To find the volume of the cask in gallons.

Multiply the square of the mean diameter by the length (both in inches), and this product by .0034.

EXERCISE 135

1. How many cubic yards will a box hold if it is 10 ft. 6 in. long, 4 ft. wide, and 2 ft. 4 in. deep?

2. A room contains 1,536 cu. ft. of space. If it is 16 ft. long and 12 ft. wide, how high is it?

3. Rain falling uniformly for 5 hrs. on a roof, whose horizontal dimensions are 10 yds. by 15 ft., fills a tank 6 ft. 3 in. by 3 ft. by 2 ft. 6 in. Find the depth of the rainfall per hour.

4. If a cubic foot of water weighs 1,000 ozs., and a gallon of water weighs 10 lbs., how many gallons will be required to fill a rectangular tank 12 ft. long, 4 ft. wide, and 4 ft. deep?

5. To drain a swamp in Dereham, the Township Council had a ditch dug 1 mi. long, 3 ft. deep, 6 ft. wide at the surface, and 4 ft. wide at the bottom. Find the total cost at 9c. per cubic yard.

6. A gravel-bed whose surface has an area of 4 ac. contains gravel to an average depth of 6 ft. How many miles of road 11 ft. wide can be covered from the gravel in the bed if it be spread on to a uniform depth of 8 in.?

7. How many gallons will a can hold if it is 22 in. in diameter and 3 ft. high?

8. I have a cylindrical tank which contains 160 gal. It is 6 ft. 5 in. in diameter. How deep is it?

9. How many square inches of surface are there in a stove pipe 22 in. in circumference and 12 ft. long?

10. Find the number of cubic feet in a log 30 ft. long and 2 ft. in diameter at the larger and 1 ft. 10 in. at the smaller end.

EXERCISE 136

1. How many granite blocks 12 in. by 18 in. will be required to pave a mile of roadway 42 ft. in width?

2. How much would it cost to fence a field 40 rd. long and 30 rd. wide at 9c. a yard?

3. How many square feet in the upper surface of a board 16 ft. long and 6 in. wide at one end and 14 in. wide at the other?

4. Find the cost of paving a road of the uniform breadth of 4 yds. around the inside of a rectangular 2-ac. lot, 20 rd. wide, at 24c. a square yard.

5. A square space containing 256 sq. yds. is to be lengthened

by 4 ft. 3 in. in one of its dimensions and shortened by 3 ft. 4 in. in the other. What will then be its area?

6. A rectangular field is 40 rd. in length and 30 yds. in width. Find in feet the side of a square of equal area.

7. Find, in inches, the side of the greatest square stick of timber which can be cut from a tree whose circumference is 9 ft. 2 in.?

8. The whole perimeter of a semicircle is 90 in. Find its radius.

9. Find the difference in feet between the perimeters of a circular and a square field, if each contains 2 ac.

10. Find the area of a uniform walk 2 yds. wide around a circular pond which contains $15\frac{2}{5}$ ac.

EXERCISE 137

1. How many bricks 8 in. long, 4 in. wide, and 2 in. thick can be packed in a box 4 ft. 6 in. long, 3 ft. wide, and 2 ft. deep?

2. The side of a square field is 48 rd. Find the length of the side of a square field containing $2\frac{1}{4}$ times as much land.

3. Find the number of acres in a triangular field whose sides are 1,056 yds., 1,980 yds., and 2,244 yds.

4. One blackboard in a school is 4 ft. wide and 16 ft. long, the other is $3\frac{1}{2}$ ft. wide and 24 ft. long. How many feet must I cut off the length of the larger blackboard so that the remainder will have the same area as the smaller one?

5. Find the cost of gilding the entire outside surface of a covered box, 3 ft. long, 2 ft. 6 in. wide, and 1 ft. 9 in. deep, at $1.20 per square foot.

6. How much will it cost to paint the outside and both floors of a two-storey cottage, 36 ft. long, 33 ft. wide, and 18 ft. high, at 10c. per square yard, the walls to be 18 in. thick and no allowance to be made for cornices, openings, or partitions?

7. Find the expense of sodding a plot of ground, which is 40 yds. long and 100 ft. wide, with sods each a yard in length and a foot in width, the sods, when laid, costing 75c. per hundred.

8. A box is made of plank 2 in. thick and without a lid. Its outside measurements are: depth, 16 in.; width, 18 in.; and

length, 24 in. How much will it cost to have it painted, inside and outside, at 9c. per square foot?

9. A garden roller is 3 ft. $7\frac{1}{2}$ in. wide and 5 ft. $10\frac{2}{3}$ in. in circumference. How much ground does it pass over in making three complete revolutions?

10. A locomotive, running at the rate of 35 mi. per hour, has a driving wheel which makes 4 revs. in 1 sec. Find the diameter of the wheel.

COMMERCIAL ARITHMETIC

TEST PAPER—No. 16

INVOLUTION AND EVOLUTION—MENSURATION

Your work for this paper is to be submitted to the instructor for correction and criticism.

1. Find the square root of—
   (*a*) 189,475,225.
   (*b*) 281·9041.
   (*c*) ·000729.

2. Find the cube root of—
   (*a*) 65,939,264.
   (*b*) 119,204 to two places of decimals.

3. A field containing 40 ac. is 4 times as long as it is wide. Find the cost of fencing it at $1 per rod.

4. How large a cube can be made by using the total contents of a regular mound of earth that is 1,331 ft. long, 405 ft. wide, and 25 ft. high ?

5. The adjacent sides of a parallelogram measure 132 ft. and 84 ft. respectively, and the area of the parallelogram is two-thirds of that of a square of equal perimeter. Find the perpendicular distance between each pair of parallel sides.

6. Find the area of a quadrilateral, the longest of whose sides measure 23·29 chains, and the perpendiculars on this side from the opposite angles of the quadrilateral 17·75 chains and 13·45 chains respectively, the distance of the feet of these perpendiculars from the adjacent angles being 3·64 chains and 2·40 chains respectively, and from each other 17·25 chains.

7. Find the volume of a cube, the area of whose surfaces is 100·86 sq. in.

8. The sides of a triangle are 21, 43, 35 ft. respectively. Find its area.

9. How many square yards of canvas will be required to make a conical tent 9 ft. high, and having a base of 4 ft. radius, no allowance being made for seams.

10. A circular path is 400 yd. in length on its inner edge. What will be its length 5 ft. out from that edge all around ?

PRACTICAL MEASUREMENTS

LUMBER

498. The unit of measurement for lumber is the board foot. It is 1 ft. long, 1 ft. wide, and 1 in. thick.

Lumber less than 1 in. in thickness is considered inch lumber in measuring.

In measuring the width of a board, a fraction greater than a half runs the width up to the next even number; a fraction less than a half is rejected. Thus, a board $4\frac{5}{8}$ in. wide is considered 5 in. wide; while a board $4\frac{3}{8}$ in. wide is considered 4 in. wide.

Lumber is usually sold at so much a thousand feet, board measure.

To find the number of board feet or feet of lumber in a board, plank, joist, etc., multiply together the three numbers representing the number of times the unit (1 board feet) is repeated in each direction.

499. Illustration.—Find the number of board feet in a plank 10 ft. long, 15 in. wide, and 3 in. thick.

SOLUTION 1

The plank is 10 ft. long, and a board foot is 1 ft. long, so the unit is repeated 10 times in length.

The plank is 15 in. wide, and a board foot is 1 ft. or 12 in. wide, so the unit is repeated $1\frac{1}{4}$ times in width.

The plank is 3 in. thick, and a board foot is 1 in. thick, so the unit is repeated 3 times in thickness.

So the number of board feet in the plank $= 10 \times 1\frac{1}{4} \times 3 = 37\frac{1}{2}$ board feet.

SOLUTION 2

$$\frac{\text{Contents of plank}}{\text{Contents of board foot}} = \frac{10\text{ (ft.)} \times 15\text{ (in.)} \times 3\text{ (in.)}}{1\text{ (ft.)} \times 12\text{ (in.)} \times 1\text{ (in.)}} = 37\tfrac{1}{2}\text{ bd. ft.}$$

Short Method

Multiply the length in feet by the width and thickness in inches, and divide the product by 12, *and the result will be the number of board feet of lumber.*

In charging, or billing lumber, the number of pieces are entered first, then the thickness and width in inches, then the feet in length. For example, in recording 6 pcs., 4 in. thick by 6 in. wide and 20 ft. long, the form would be thus: 6 pcs. 4 in. × 6 in. – 20 ft., and would be called off by the salesman, " 6 *four-by-sixes* - 20 ft.," *four-by-sixes* being the name by which he selects and sells stock.

Instead of writing " inches " and " feet," lumber billing clerks use (″) for inches, and (′) for feet; thus, 3 in. by 4 in. – 17 ft. long, is written: 3″ × 4″– 17′.

When the width of a board tapers uniformly, the average width is found by finding one-half the sum of the two ends.

EXERCISE 138

1. How many feet of lumber in a floor 15 ft. long, 12 ft. wide, and 1 in. thick?

2. A bridge 84 ft. long and 20 ft. wide is covered with oak plank 2½ in. thick. What is it worth at $16 per thousand?

3. How much lumber 1 in. thick will be required to cover a walk 5 ft. wide around the outside of a lawn 300 yds. long and 200 yds. wide?

4. What is the number of board feet in a stick of timber 30 ft. long and 18 in. square?

5. A 2-in. plank, 9 in. wide at one end and 15 in. wide at the other, tapers gradually. How much lumber is there in it, the length being 18 ft.?

6. At $20 per thousand, find the whole cost of 5 scantling 20 ft. long, 4 in. wide, 3 in. thick; 9 scantling 18 ft. long, 5 in. wide, 4 in. thick; 6 scantling 14 ft. long, 6 in. wide, 5 in. thick.

7. At $32.50 per thousand, what will be the cost of—
8 scantlings 3″ × 4″– 18′; 12 scantlings 4″ × 5″– 16′;
8 scantlings 5″ × 6″– 14′.

8. At $19.50 per thousand, what will be the total cost of—
9 boards 1″ × 2″– 14′; 6 boards 1½″ × 18″– 16′;
15 boards 2″ × 14″– 20′; 8 boards 1¼″ × 12″– 18′.

9. At \$24 per thousand, what will be the cost of the lumber required to enclose a field 40 rd. square with a board fence if the boards are 15 ft. long, 5 in. wide, and 1 in. thick, and the fence 5 boards high?

10. At \$21.50 per thousand, what will be the cost of the lumber in a line fence 160 rd. long, if the boards are 11 ft. long, 7 in. wide, and 1 in. thick, and the fence 4 boards high?

11. What will it cost to fence 10 mi. of railway, both sides, with six rounds of 6 in. boards at \$15 per thousand feet?

12. What will it cost, at \$16 per thousand, to fence a field 40 rd. by 60 rd. with one round of 12 in. boards and five of 6 in. boards?

13. What will be the cost per mile to fence a railway with six strands of barbed wire, which weighs 1 lb. per rod, at 8c. a pound?

14. Find the cost of a quarter-mile of fence, with the posts 8 ft. apart, a 12-in. base, a $2'' \times 4''$ rail at top, and five rows of 6 in. boards. The posts cost 10c. each, and the lumber \$12 per thousand.

15. If lumber is \$20 per thousand, find the cost of the boards and scantling required for a sidewalk 54 ft. long and 4 ft. wide. The boards are 1 in. thick, and are laid on two rows of scantling $2'' \times 4''$.

16. A contractor undertakes to lay a sidewalk 8 ft. wide, on both sides of a street one-eighth of a mile in length. The plank used is to be 2 in. thick, and the walk is to be supported by three continuous lines of scantling 4 in. square. Determine the cost of the lumber at \$15 per thousand feet board measure.

17. A lot, 60 ft. wide and 120 ft. long, is to be enclosed on the two sides and the back by a tight board fence 6 ft. high. The posts are to be placed 6 ft. apart, and to cost 15c. each. There are to be two string pieces of scantling, 2 in. thick and 4 in. wide, from post to post, on which to nail the boards. Lumber is \$15 per 1,000 ft.

(*a*) Find the cost of the posts.

(*b*) Find the cost of the boards.

(*c*) Find the cost of the scantlings.

18. A certain sidewalk is 250 yds. long, 10 ft. wide, and made of plank 2 in. thick. The planks rest on three continuous lines of scantling $3'' \times 6''$. Find the cost of the material at $18 per thousand.

19. At $16 per thousand, find the cost of the 2-in. plank for a side walk half a mile long and 6 ft. wide.

20. Find the cost of 720 boards, 14 ft. long, 8 in. wide, and $1\frac{1}{2}$ in. thick, at $12 per thousand feet.

21. At $15 per thousand, board measure, what will be the cost of 2-in. plank for a 4-ft. sidewalk, half a mile long?

22. At $12 per thousand, what will be the cost of 2-in. plank for a 3-ft. sidewalk on the street sides of a rectangular corner lot 55 ft. wide and 108 ft. 8 in. long?

23. What length of 2-in. plank, 18 in. wide, will contain 48 board feet?

24. What length of 3-in. plank, 10 in. wide, will contain 40 board feet?

25. What is the width of a 2-in. plank, 16 ft. long, which contains 40 board feet?

26. What is the thickness of a piece of timber, 40 ft. long and 15 in. wide, that contains 400 ft. of lumber?

27. What will be one-half of the cost of a line fence 40 rd. long and five boards high, of boards 16 ft. long, 1 ft. wide, and 1 in. in thickness, the posts to be placed 8 ft. apart at a cost of $16 per thousand for lumber and $25 per hundred for posts?

28. At $18 per thousand for lumber and $22 per hundred for posts, what will be the cost of lumber and posts for enclosing a lot 40×160 ft. with a picket fence, the pickets being 4 ft. long, 3 in. wide, and 1 in. thick, allowing 3 in. space between pickets, the posts being placed 8 ft. apart, two 2×4's being used as stringers, and a baseboard 10 in. wide extending below the pickets?

ROOFING

500. The unit in measuring for flooring or roofing is the square. A square contains 100 sq. ft.

501. Shingles are 16 in. long, and average 4 in. in width. They are generally laid 4 in. to the weather. Hence each shingle will

cover 16 sq. in., and 9 will be required for each square foot, or 900 for each square of roofing.

Allowing for waste, 1,000 shingles, laid 4 in. to the weather, are estimated to cover a square.

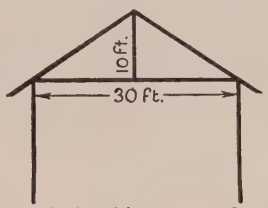
Shingles are sold by the bunch, each bunch containing 250 shingles; hence four bunches are required for a square of roofing.

Dealers will not sell part of a bunch.

502. Pitch.—In placing the rafters on a building the term "pitch of the roof" is often mentioned. Such expressions as "one-third pitch," "one-half pitch" are quite common.

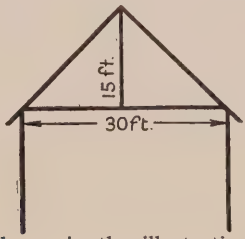
Suppose a building were 30 feet wide and the designated pitch were one-third; this means that the peak of the roof would be $\frac{1}{3}$ of 30 feet, or 10 feet above the level of the walls.

Illustration.



If it were one-half pitch, this means that the peak would be $\frac{1}{2}$ of 30 feet, or 15 feet above the level of the walls.

Illustration.

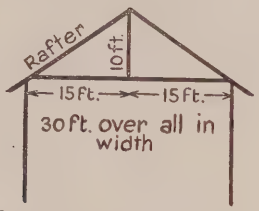


The rafters, as shown in the illustration, hang over various lengths to form the eaves, depending upon the size and type of building.

To find the length of the rafters, in order to figure the number

of square feet of sheeting lumber required and the number of shingles needed, becomes an easy matter.

SOLUTION



The distance from the peak to where the rafter comes to the outside of the wall is found by the following formula—

$$\sqrt{15^2 + 10^2} = \sqrt{225 + 100} = \sqrt{325} = 18.03\text{ft.}$$

To this length, 18.03 ft. must be added the overhang of 2 ft., 27 in., 30 in., or whatever it may be, which gives the entire length of the rafter.

Sometimes a roof is made 1 ft. or 2 ft. over or less than one-third pitch, or 1 ft. or 2 ft. less than one-half pitch. Suppose a building were 45 ft. wide and the pitch designated 1 ft. over one-third pitch. Then the distance from the level of the walls to the peak would be $45 \div 3 = 15$ ft. plus 1 ft., which would be 16 ft.

EXERCISE 139

1. Making no allowance for waste, how many shingles, laid $4\frac{1}{2}$ in. to the weather, will be required for a square of roofing?

2. Making no allowance for waste, how many shingles, laid 5 in. to the weather, will be required for a square of roofing?

3. How many squares of roofing are there in a double roof, 45 ft. long, with 20 ft. rafters?

4. At $10 per square, what will be the cost of the slate for a double roof 45 ft. long and 28 ft. wide?

5. How many bunches of shingles are required for a shed roof 40 ft. long and 3 ft. wide?

6. How many bunches of shingles are required for a double roof, 70 ft. long, with 30 ft. rafters?

7. Find the cost, at $4.50 per thousand, of the shingles necessary for a double roof, 56 ft. long, with rafters 25 ft. long.

8. Find the cost, at $4.20 per thousand, of the shingles for the roof of a building, 52 ft. 8 in. long and 32 ft. wide, having a gable 12 ft. high and the rafters having an 18 in. heel.

9. A building, 64 ft. long and 36 ft. wide, has a gable 14 ft. high. In roofing the building, the rafters were given an 18 in. heel. Find—

(*a*) The number of bunches of shingles required for the roof.

(*b*) The value of the sheeting, at $16 per thousand.

(*c*) The value, at $23 per thousand, of the rafters, if they are 2 in. × 4 in. and are 2 ft. from centre to centre.

10. How many shingles, 18 in. long and 4 in. wide, lying one-third to the weather, are required to shingle a double roof, 54 ft. long, with rafters 22 ft. long, the first row of shingles being double and no allowance being made for waste?

LATHING AND PLASTERING

503. Standard laths are 4 ft. long, $1\frac{1}{2}$ in. wide, and are laid $\frac{1}{2}$ in. apart. 1,000 laths of standard size will, therefore, cover about 74 sq. yds. Allowing for waste, however, 1,000 laths are estimated to cover 70 sq. yds.

Laths are put up in bundles of 100 and 50. So a bundle of 100 is estimated to cover 7 sq. yds., and a bundle of 50 to cover $3\frac{1}{2}$ sq. yds.

But all laths are not of standard size. Some are only 32 in. long and some are only $1\frac{1}{4}$ in. wide.

In reckoning the amount of laths required, calculate the superficial area to be lathed, and deduct the area of all openings.

504. In reckoning the cost of labor for lathing and plastering, it is customary to deduct only one-half the area of the openings; but there is no set rule to this effect, and it is well to have a distinct understanding with the contractor on this point before the job is commenced.

EXERCISE 140

1. How many bundles of 100 standard laths are required for the walls and ceiling of a room 27 ft. long, 18 ft. wide, and 10 ft. high, containing two doors 7 ft. by 4 ft., and four windows 6 ft. by 4 ft.?

2. In the preceding question, had the lath been 32 in. long,

and put up in bundles of 50 each, how many bundles would have been required?

3. Making no allowance for waste, how many square yards would 1,000 laths, 32 in. long and 1½ in. wide, cover if laid three-eighths of an inch apart?

4. Find the cost of plastering a room 30 ft. long, 27 ft. wide, and 12 ft. high, at 20c. a square yard, if there are two doors each 3 ft. 6 in. by 7 ft., and three windows each 3 ft. 4 in. by 6 ft., and the plasterer be allowed pay for one-half the area of the openings for his trouble in plastering around them.

5. A room, 16 ft. long, 14 ft. 6 in. wide, and 10 ft. high, has a skirting board 1 ft. high, two doors 7 ft. by 3 ft. 10 in., and two windows 6 ft. by 4 ft. Find the number of bundles of 50 standard laths required to lath it.

6. Find the cost, at 22c. a square yard, of plastering the walls of a room 23 ft. 6 in. long, 15 ft. wide, and 8 ft. 4 in. high, having one door 7 ft. 6 in. by 3 ft. 8 in., three windows 6 ft. 9 in. by 4 ft. 2 in., and a skirting board 11 in. high, allowing one-half the area of the openings.

7. Find the cost, at 22c. a square yard, of plastering a room 18 ft. long, 15 ft. 6 in. wide, and 10 ft. 4 in. high, containing two doors 7 ft. 4 in. by 4 ft., two windows 6 ft. by 3 ft. 10 in., one mantel-piece 5 ft. by 3 ft. 6 in., and a 10 in. baseboard (deduct one-half the area of the openings).

BRICK AND STONE WORK

505. It may be stated at the outset that no set rule can be given for the calculation of either the amount of material required or the amount to be expended in labor in either of these cases. Bricks are of varying dimensions. They are laid according to specifications in varying thicknesses of mortar, and the judgment of the contractor, in facing the particular conditions, so often modifies even his own general plan of figuring, that it would be absurd to say that if certain methods of calculation are followed, we will get results as contractors in Canada or any other place would estimate them.

506. What is true of brick work is just as true of stone work.

There are many kinds of stone work and many places where the judgment of the contractor is brought into play. The best that we can hope to do in a work of this character is to lay down certain general rules, which at least have the sanction of good authority, in the hope that the student who learns to apply his ability under these rules will have no difficulty in making calculations, no matter what changes he may be told to make, according to the part of the country in which he is working or the peculiar methods of the contractor for whom he is working.

Stone Work

507. The perch is a common unit of measurement. In stone work it is $16\frac{1}{2}$ ft. long, $1\frac{1}{2}$ ft. wide, and 1 ft. high, and contains exactly $24\frac{3}{4}$ cu. ft. In estimating 25 cu. ft. are figured as a perch. Stone masonry is measured by two systems: quarryman's measurement and mason's measurement. The quarryman's measurement is concerned with the figuring of the quantity of material to be used. The mason's measurement is more concerned with figuring the amount of work to be done, and the consequent price to be paid for it.

In measuring for material, the actual contents of the wall are measured. That is, all openings are taken out and all corners are measured single. In buying the stone, it may be bought by the cord, which is of the same dimensions as a cord of wood; namely, 8 ft. by 4 ft. by 4 ft., or 128 cu. ft. It is estimated that 4 perches or 100 cu. ft. of wall will take 1 cord of stone, 1 bbl. of lime, or, say, $2\frac{1}{2}$ bus., and 5 bbl. of sand.

In estimating the amount of work to be done, corners are always doubled. That is, the girt measurement of the walls is taken. No allowance is made for openings less than 3 by 5 ft., and only half the amount of openings larger than 3 by 5 ft.

Brick Work

508. Brick walls are of various degrees of thickness. For instance, there is the wall which is $4\frac{1}{2}$ in. thick, or, as it is sometimes called, the half-brick wall. There is the 9-in. wall, which is sometimes called the single brick wall. There is the 13-in. wall or the brick and a half wall, the 18-in. wall or the double brick, the 22-in. wall or the $2\frac{1}{2}$ brick, and so on according to the thickness of the wall. In consequence of variations in size of bricks, no exact rule for the volume of laid brick can be given. The following scale is, therefore, a fair average—

7 bricks to the superficial foot in 4-in. wall
14 bricks to the superficial foot in 9-in. wall
21 bricks to the superficial foot in 13-in. wall
28 bricks to the superficial foot in 18-in. wall
35 bricks to the superficial foot in 22-in. wall

If the wall is thicker than 22 in., for each additional $4\frac{1}{2}$ in. in thickness add 7 bricks per square foot.

The labor and material of brick work are estimated by the 1000 brick. In measuring up brick walls, it is not customary to deduct for openings less than 2 ft. square. To allow for corners, in getting the superficial area of the walls, from the girt measurement subtract four times the thickness of the wall and then multiply by the height.

EXERCISE 141

1. How many cords of stone will be required for the foundations of a house 39 ft. by 27 ft., the stonework to be 6 ft. high and 18 in. thick?

2. In the preceding question, for how many cubic yards would the masons be paid?

3. How many cords of stone are required for the walls of a house 40 ft. long, 27 ft. wide, and 20 ft. high, the wall to be 1 ft. thick, if it is to have four doors 7 ft. 6 in. by 4 ft., and eight windows 5 ft. by 4 ft.?

4. In the preceding question, for how many cubic yards will the men who do the work be paid?

5. A house is to be built 35 ft. long, 28 ft. wide, and 22 ft. high, and is to have ten windows $4\frac{1}{2}$ ft. by 6 ft., and five doors 8 ft. by 4 ft. 6 in., the wall to be 9 in. thick. How many bricks will be required?

6. If the wall in the preceding question were made 13 in. thick, how many bricks would be required?

APPENDIX

THE METRIC SYSTEM OF MEASUREMENT

509. The Metric System of Measurement gets its name from the metre, which is the unit of length of the system.

All the units of measurement are derived in a simple manner from the metre.

This system of measurement is used in all countries for scientific purposes on account of its exactness, and in many countries it is used for ordinary purposes.

Measures of Length

510. The Standard Metre is a bar of platinum, which is supposed to be the length of one ten-millionth ($\frac{1}{10000000}$ or .0000001) of a quarter of the earth's circumference, measured by a line passing through Paris, France, from the equator to the pole.

511. TABLE

10 millimetres,	marked	mm.,	are 1 centimetre,	marked	cm.	= $\frac{1}{100}$ or .01	metre
10 centimetres,	"	cm.,	" 1 decimetre,	"	dm.	= $\frac{1}{10}$ or .01	"
10 decimetres,	"	dm.,	" 1 metre,	"	m.		
10 metres,	"	m.,	" 1 dekametre,	"	Dm.	= 10 metres	
10 dekametres,	"	Dm.,	" 1 hektometre,	"	Hm.	= 100 "	
10 hektometres,	"	Hm.,	" 1 Kilometre,	"	Km.	= 1000 "	
10 Kilometres,	"	Km.,	" 1 Myriametre,	"	Mm.	= 10000 "	

512. Comparative Lengths are as follows—

	INCHES.	FEET.	YARDS.
1 Metre =	39.37079	3.2808992	1.0936331
1 Decimetre =	3.93708	.3280899	.1093633
1 Centimetre =	.39371	.032809	.0109363
1 Millimetre =	.03937	.0032809	.0010936

513. The Metre is used in measuring cloth and short distances; the Kilometre is used in measuring long distances.

It will be noted that the system is a decimal one, since each unit is 10 times as large, or $\frac{1}{10}$ as small, as some other unit, so that each unit is 10, 100, 1,000, etc., times the standard, or $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$, etc., of the standard. This relation among the units exists in every table of the metric system. So that, if the standard is known, all the other units can be formed by prefixing the same syllable to the standards.

The prefix milli is derived from the Latin *mille*, meaning thousand
" centi " " " *centum*, " hundred
" deci " " " *decem*, " ten
" deca or deka " " Greek *deka*, " ten
" hecto or hekto " " " *hekaton*, " hundred
" kilo " " " *khiloi*, " thousand
" myria " " " *myria*, " ten thousand

In abbreviating the names of the units, those in each table that are larger than the standard are capitalized, while the others are not.

Since, in the Metric System, 10, 100, 1000, etc., units of a lower denomination make a unit of a higher denomination, it follows that any one of the metric measures may be expressed in terms of another measure by simply moving the decimal point to the right or left.

Land or Surface Measure

514. The Are is the unit of land measure (or area). It is a square whose side is 10 metres.

TABLE

100 centiares, marked ca., are 1 Are, marked a.
100 ares " a., " 1 hektar, " Ha.

For measuring other surfaces, squares of the metre and its subdivisions are used.

An Are is 100 sq. m., marked m^2. It is about 119.6 sq. yds. The Hektar is nearly $2\frac{1}{2}$ ac. (2.47).

The Are is the principal unit of surface of small plots of land. The area of a farm is expressed in hektars; of a country, in square kilometres.

515. Measures of Capacity

The Litre is the unit of capacity. It is a cubic decimetre, that is, a cube whose edge is a decimetre long.

TABLE

10 centilitres, marked cl., are 1 decilitre, marked dl.
10 decilitres " dl., " 1 litre, " l.
10 litres, " l., " 1 dekalitre " Dl.
10 dekalitres " Dl., " 1 hektolitre, " Hl.

The measures commonly used are the litre and the hektolitre. The litre is very nearly a quart; it is used in measuring milk, wine, etc., in moderate quantities. The hektolitre is about 2 bus. $3\frac{1}{3}$ pk.; it is used in measuring grain, fruit, roots, etc., in large quantities.

For measuring wood the Stere is used; it is a cubic metre (= 35,316 cu. ft.).

Measures of Weight

516. The Gram is the unit of Weight. It is the weight of a cubic centimetre of water.

TABLE

10 milligrams, marked mg., are 1 centigram, marked cg.
10 centigrams, " cg., " 1 decigram, " dg.
10 decigrams, " dg., " 1 gram, " g.
10 grams, " g., " 1 dekagram, " Dg.
10 dekagrams, " Dg., " 1 hektogram, " Hg.
10 hektograms, " Hg., " 1 kilogram, " Kg.
10 kilograms, " Kg., " 1 myriagram, " Mg.
10 myriagrams, " Mg., " 1 quintal, " Q.
10 quintals or 1000 kilograms are 1 Metric ton, marked M.T.

The weights commonly used are the Gram, Kilogram, and Metric Ton. The Gram is used in mixing medicines, in weighing the precious metals,

and in all cases where great exactness is required. The kilogram (commonly called the " Kilo ") is the usual weight for groceries and coarse articles generally; it is very nearly $2\frac{1}{5}$ lbs. Avoir. The metric ton is used for weighing hay and other heavy articles; it is about 204 lbs. more than our ton.

517. Legal and Approximate Values are as follows—

Denomination.	Legal Value.	Approximate Value.
Metre	39.37 in.	3 ft. $3\frac{3}{8}$ in.
Centimetre	.39371 in.	$\frac{2}{5}$ in.
Kilometre	.62137 mi.	$\frac{5}{8}$ mi.
Square Metre	1.196 sq. yds.	$10\frac{3}{4}$ sq. ft.
Are	119.6 sq. yds.	4 sq. rds.
Hektar	2.471 ac.	$2\frac{1}{2}$ ac.
Cubic Metre	1.308 cu. yds.	$35\frac{1}{3}$ cu. ft.
Litre	1.76077 pts.	$1\frac{3}{4}$ pts.
Hektolitre	2.8375 bus.	2 bus. $3\frac{1}{3}$ pk.
Gram	15.432 gr. Troy	$15\frac{1}{2}$ gr.
Kilogram	2.2046 lb. Avoir.	$2\frac{1}{5}$ lbs.
Metric Ton (or tonneau)	2204.6 lbs. "	1 T. 204 lbs.
Stere	0.27590 cord	$\frac{1}{4}$ cord

COMMERCIAL ARITHMETIC

TEST PAPER—No. 17

PRACTICAL MEASUREMENTS—METRIC SYSTEM

Your work for this paper is to be submitted to the instructor for correction and criticism.

1. If the paperhanger charges $3, and the wall-paper costs 80c. a double roll, and the border 4c. a yd., find the cost of papering a room 18 ft. 9 in. long, 16 ft. 8 in. wide, and 13 ft. 6 in. high, allowing for two doors, each 3 ft. 9 in. wide, and 3 windows, each 4 ft. 2 in. wide ; also for a baseboard 18 in. deep.

2. Find the cost of carpeting a room 22 ft. 8 in. by 13 ft. 4 in. if the carpeting be 27 in. wide, and cost $1.80 per yard, there being a waste of 8 in. per strip in matching, the strips running lengthwise.

3. At 15c. a square yard, find the cost of plastering the walls and ceiling of a room 21 ft. long, 14 ft. wide, and 12 ft. high, with 4 openings, each 8 ft. by 4 ft.

4. At $3.60 per thousand, find the cost of the shingles for a roof of a building 60 ft. long, 40 ft. wide, having a gable 12 ft. high, and the rafters having an 18 in. heel.

5. Find the cost of lumber required for a tight board fence 43 ft. long and 5 ft. high, with two rows of scantlings 2 in. by 4 in. Lumber is to cost $25.00 per M. and scantling $22.00 per M. feet.

6. A square field containing 10 ac. is to be fenced in with a post and board fence 6 boards high. The posts are placed 11 ft. apart and cost 25c. each. The boards are 5 in. wide and lumber is worth $16.00 per M. Find cost of the fence.

7. Find the cost of gravelling 4 miles of road 9 ft. wide with gravel 9 in. deep at 60c. per load. (1 load = 1 cub. yd.)

8. Bought 25 hhd. of wine, of 225 litres each, at the rate of $·156 a litre. How much did it cost ?

9. Using the approximation, 1 metre = 39·37 in., find the number of litres in 1 cub. ft.

10. A race track 14 metres wide surrounds a circular field containing 61·1226 hektares. Find the total cost of the track, including (*a*) the value of the land at $250 per hektare ; (*b*) the cost of fencing both sides at $2.50 per dekametre, and (*c*) the cost of gravelling its surface to a depth of 2 decimetres at 60c. a stere.

ANSWERS

ANSWERS

SECTION 1

Page 2—(*a*) 1, 2, 4, 5, 6, 7. (*b*) 1, 2, 5, 8. (*c*) 1, 2, 4, 6. (*d*) 2, 5, 8. (*e*) 1, 2, 6. (*f*) 1, 5. (*g*) 2, 5.

Page 3—**1,** 2, 2, 2, 2, 2, 2. **2,** 2, 2, 2, 2, 3, 3. **3,** 2, 2, 2, 2, 2, 2, 2, 2, 2. **4,** 2, 3, 5, 5, 7. **5,** 3, 509. **6,** 3, 3, 5, 5, 7. **7,** 2, 2, 2, 3, 3, 3, 11. **8,** 2, 2, 2, 7, 7, 7. **9,** 11,397. **10,** 3, 3, 5, 11, 13. **11,** 5, 5, 5, 5, 5. **12,** 2, 2, 2, 3, 3, 11, 23. **13,** 11, 11, 179. **14,** 3, 3, 3, 5, 191. **15,** 3, 7, 11, 167.

Page 5—**1,** 6. **2,** 5. **3,** 6. **4,** 4. **5,** 24. **6,** 12. **7,** 30. **8,** 25. **9,** 12. **10,** 24. **11,** 4. **12,** 243. **13,** 171. **14,** 375. **15,** 462. **16,** 211. **17,** 107. **18,** 257. **19,** 79. **20,** 89. **21,** 4.

Page 5—**1,** 225 sq ft. **2,** 19 ft. **3,** 24. **4,** 18. **5,** 16 ft. **6,** 4 ft., 95 ft.

Page 7—**1,** 352. **2,** 1,020. **3,** 285. **4,** 1,053. **5,** 900. **6,** 660. **7,** 2,200. **8,** 360. **9,** 60. **10,** 1,200. **11,** 8,640. **12,** 3,584. **13,** 330. **14,** 630. **15,** 1,078.

Page 8—*Proper Fractions*—$\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{8}$, $\frac{12}{16}$, $\frac{2}{3}$, $\frac{3}{6}$, $\frac{4}{8}$, $\frac{5}{6}$, $\frac{8}{9}$, $\frac{7}{10}$, $\frac{2}{11}$, $\frac{15}{16}$, $\frac{15}{32}$, $\frac{47}{125}$, $\frac{17}{864}$, $\frac{85}{115}$. *Improper Fractions*—$\frac{3}{2} = 1\frac{1}{2}$, $\frac{5}{4} = 3\frac{1}{4}$, $\frac{12}{3} = 4$, $\frac{17}{5} = 3\frac{2}{5}$, $\frac{16}{6} = 2\frac{4}{6}$, $\frac{8}{7} = 1\frac{1}{7}$, $\frac{16}{9} = 2\frac{7}{9}$, $\frac{14}{9} = 1\frac{5}{9}$, $\frac{21}{9} = 2\frac{3}{9}$, $\frac{18}{10} = 1\frac{8}{10}$, $\frac{23}{10} = 2\frac{3}{10}$, $\frac{15}{11} = 1\frac{4}{11}$, $\frac{17}{15} = 1\frac{2}{15}$, $\frac{74}{11} = 6\frac{8}{11}$. *Mixed Numbers*—$4\frac{1}{2} = \frac{9}{2}$, $2\frac{3}{4} = \frac{11}{4}$, $6\frac{1}{3} = \frac{19}{3}$, $4\frac{1}{3} = \frac{13}{3}$, $6\frac{1}{5} = \frac{31}{5}$, $5\frac{1}{6} = \frac{31}{6}$, $2\frac{1}{6} = \frac{13}{6}$, $14\frac{3}{7} = \frac{101}{7}$, $11\frac{1}{9} = \frac{100}{9}$, $5\frac{1}{10} = \frac{51}{10}$, $7\frac{1}{11} = \frac{78}{11}$, $8\frac{1}{15} = \frac{121}{15}$, $23\frac{1}{3} = \frac{70}{3}$, $7\frac{5}{6} = \frac{65}{6}$, $4\frac{8}{11} = \frac{52}{11}$, $9\frac{7}{30} = \frac{277}{30}$, $2\frac{8}{17} = \frac{42}{17}$, $9\frac{23}{25} = \frac{233}{25}$, $29\frac{7}{250} = \frac{7257}{250}$, $10\frac{18}{225} = \frac{2262}{225}$, $29\frac{3}{8} = \frac{235}{8}$.

Page 10—**1,** $\frac{10}{15}$. **2,** $\frac{8}{15}$. **3,** $\frac{6}{21}$. **4,** $\frac{14}{21}$. **5,** $\frac{15}{18}$. **6,** $\frac{18}{36}$. **7,** $\frac{28}{36}$. **8,** $\frac{9}{27}$. **9,** $\frac{9}{120}$. **10,** $\frac{55}{88}$. **11,** $\frac{28}{52}$. **12,** $\frac{10}{115}$. **13,** $\frac{24}{128}$. **14,** $\frac{40}{192}$. **15,** $\frac{77}{147}$.

Page 11—**1,** $\frac{2}{6}$. **2,** $\frac{5}{12}$. **3,** $\frac{13}{32}$. **4,** $\frac{2}{25}$. **5,** $\frac{2}{5}$. **6,** $\frac{3}{11}$. **7,** $\frac{7}{11}$. **8,** $\frac{11}{12}$. **9,** $\frac{5}{7}$. **10,** $\frac{3}{5}$. **11,** $\frac{9}{11}$. **12,** $\frac{13}{16}$. **13,** $\frac{5}{7}$. **14,** $\frac{7}{8}$. **15,** $\frac{3}{4}$. **16,** $\frac{3}{7}$. **17,** $\frac{1}{3}$. **18,** $\frac{8}{11}$. **19,** $\frac{14}{25}$. **20,** $\frac{121}{128}$.

Page 11—**1,** $\frac{3}{6}$, $\frac{4}{6}$. **2,** $\frac{2}{4}$, $\frac{1}{4}$. **3,** $\frac{6}{10}$, $\frac{7}{10}$. **4,** $\frac{30}{36}$, $\frac{31}{36}$, $\frac{32}{36}$. **5,** $\frac{12}{24}$, $\frac{8}{24}$, $\frac{19}{24}$. **6,** $\frac{125}{150}$, $\frac{64}{150}$, $\frac{25}{150}$. **7,** $\frac{240}{720}$, $\frac{225}{720}$, $\frac{600}{720}$, $\frac{528}{720}$. **8,** $\frac{468}{1260}$, $\frac{868}{1260}$, $\frac{560}{1260}$, $\frac{35}{1260}$. **9,** $\frac{270}{300}$, $\frac{220}{300}$, $\frac{255}{300}$, $\frac{252}{300}$. **10,** $\frac{20}{60}$, $\frac{32}{60}$, $\frac{15}{60}$, $\frac{23}{60}$. **11,** $\frac{105}{180}$, $\frac{60}{180}$, $\frac{120}{180}$, $\frac{28}{180}$. **12,** $\frac{42}{72}$, $\frac{44}{72}$, $\frac{39}{72}$, $\frac{46}{72}$.

Page 13—**1,** $2\frac{13}{60}$. **2,** $2\frac{37}{60}$. **3,** $19\frac{11}{12}$. **4,** $3\frac{17}{20}$. **5,** $2\frac{13}{180}$. **6,** $3\frac{11}{60}$. **7,** $12\frac{23}{60}$. **8,** $10\frac{5}{18}$. **9,** $39\frac{1}{14}$. **10,** $34\frac{23}{112}$. **11,** $2\frac{107}{112}$. **12,** $20\frac{5}{56}$. **13,** $17\frac{1018}{1848}$. **14,** $30\frac{433}{3432}$. **15,** $1098\frac{9}{16}$. **16,** $1588\frac{1}{2}$. **17,** $1212\frac{17}{40}$. **18,** $1250\frac{3}{9}$.

Page 14—**1,** $\frac{1}{12}$. **2,** $\frac{4}{8}\frac{1}{8}$. **3,** $\frac{1}{8}$. **4,** $\frac{5}{48}$. **5,** $\frac{11}{24}$. **6,** $\frac{1}{4}$. **7,** $\frac{1}{15}$. **8,** $\frac{1}{18}$. **9,** $\frac{19}{56}$. **10,** $\frac{11}{36}$. **11,** $\frac{3}{4}$. **12,** $3\frac{13}{15}$. **13,** $\frac{11}{12}$. **14,** $2\frac{17}{40}$. **15,** $1\frac{8}{36}$. **16,** $9\frac{1}{4}$. **17,** $63\frac{1}{8}$. **18,** $136\frac{3}{4}$. **19,** $93\frac{3}{8}$. **20,** $10\frac{31}{60}$. **21,** $32\frac{7}{12}$. **22,** $63\frac{3}{5}$. **23,** $54\frac{28}{48}$. **24,** $55\frac{6}{11}$. **25,** $93\frac{13}{17}$.

Page 16—**1,** 15. **2,** $15\frac{1}{3}$. **3,** $93\frac{3}{4}$. **4,** 135. **5,** $10\frac{2}{25}$. **6,** $21\frac{7}{8}$. **7,** $79\frac{1}{2}$. **8,** $348\frac{3}{4}$. **9,** $583\frac{1}{8}$. **10,** $2312\frac{1}{2}$. **11,** $4866\frac{3}{8}$. **12,** $906\frac{3}{4}$. **13,** $18\frac{3}{8}$. **14,** $23\frac{7}{16}$. **15,** $34\frac{2}{15}$. **16,** $10\frac{15}{16}$. **17,** $7\frac{7}{8}$. **18,** $24\frac{15}{16}$. **19,** $129\frac{1}{5}$. **20,** $117\frac{1}{2}$. **21,** $213\frac{1}{8}$. **22,** $\frac{35}{128}$. **23,** $50\frac{1}{8}$. **24,** $46,406\frac{1}{4}$. **25,** $5,937\frac{1}{2}$. **26,** $189\frac{1}{16}$. **27,** $326\frac{3}{8}\frac{1}{2}$. **28,** $6,471\frac{9}{16}$. **29,** $997\frac{1}{12}$.

Page 17—**1,** $\frac{2}{7}$. **2,** $\frac{2}{17}$. **3,** $\frac{4}{35}$. **4,** $\frac{7}{54}$. **5,** $9\frac{3}{5}$. **6,** $1\frac{1}{5}$. **7,** $2\frac{1}{4}$. **8,** $1\frac{1}{4}$. **9,** $23\frac{1}{3}$. **10,** $3\frac{11}{14}$. **11,** $1\frac{1}{14}$. **12,** $3\frac{1}{4}$.

Page 18—**1,** $7\frac{1}{14}$. **2,** $663\frac{17}{81}$. **3,** $52\frac{7}{14}$. **4,** $\frac{3}{7}$. **5,** $1\frac{59}{117}$. **6,** $1\frac{1225}{1458}$. **7,** $17\frac{1}{8}$.

Page 18—1, $80\frac{9}{16}$ lbs. **2,** $26\frac{57}{160}$ gals. **3,** $6\frac{3}{5}$ lbs. **4,** 75 bus. **5,** $\$46.02\frac{13}{16}$. **6,** $93.75. **7,** $22\frac{1}{2}$ dys. **8,** $\frac{7}{10}$.

Page 18—1, $20\frac{13}{48}$ mi. **2,** $14\frac{29}{32}$ yds. **3,** $5,040. **4,** $40,000. **5,** $\frac{5}{6}$. **6,** $26,160. **7,** $10\frac{1}{8}$. **8,** $202,500.00.

Page 19—1, $230.40. **2,** $190\frac{39}{250}$ lbs. **3,** $5\frac{1}{7}$. **4,** $16,700. **5,** 44 bbls. **6,** $50. **7,** $30. **8,** B, $18.75; C, $26.25.

Page 20—1, $25\frac{209}{880}$ mi. **2,** 270 lbs. **3,** $34\frac{5}{17}$. **4,** $12,705. **5,** $13\frac{37}{191}$ dys. **6,** A, $5\frac{3}{11}$ dys.; B, $6\frac{18}{25}$ dys.; C, $9\frac{15}{17}$ dys.

Page 20—1, 18 dys. **2,** $\frac{2}{7}$ dys.; 60c. **3,** $6\frac{22}{45}$ lbs. **4,** $\$148.57\frac{31}{176}$. **5,** 60 lemons. **6,** $3\frac{3}{4}$ hrs. **7,** $13\frac{1}{3}$ min. **8,** 70 yds.

Page 21—1, $42. **2,** $10,000. **3,** $387\frac{109}{704}$ lbs. **4,** $12,054. **5,** $\frac{13}{144}$. **6,** 1,520 tons. **7,** $\frac{37}{56}$ water, $\frac{19}{56}$ spirit. **8,** 2,806 bus.

Page 25—1, $5,750; $6,500. **2,** $1,639.50; $2,125.50. **3,** 247 ac.; 290 ac. **4,** A, $1,137; B, $615; C, $615. **5,** S., $1,309; B., $1,309; J., $911. **6,** 1st, $4,695; 2nd, $4,375; 3rd, $5,285. **7,** 1st, 987 mi.; 2nd, 1,135 mi.; 3rd, 1,429 mi. **8,** A, $9,510; B, $3,625; C, $6,370.

Page 26—1, A, $496; B, $372. **2,** 1,191; 3,573. **3,** $1,030; $3,605. **4,** Ship, $12,500; cargo, $11,250. **5,** 4,290; 3,003. **6,** 567; 1,512. **7,** 1st, 570 lbs.; 2nd, 760 lbs.; 3rd, 1,140 lbs. **8,** 1st, 376 lbs.; 2nd, 564 lbs.; 3rd, 705 lbs.

Page 26—1, 66 boys; 88 women; 110 men. **2,** Boy, 75c.; man, $1.50. **3,** Sheep, $5.00; lamb, $3.50. **4,** Man, $895; boy, $358. **5,** 1st, $134.20; 2nd, $33.55; 3rd, $67.10. **6,** A, $3.60; B, $4.00; C, $5.20. **7,** Tweed, 82 yds.; silk, 63 yds. **8,** 56 bus.; 69 bus.

Page 27—1, A, $6,000; B, $10,000. **2,** A, $2,000; B, $1,500; C, $1,200. **3,** A, 60c.; B, 80c. **4,** A, $49.50; B, $48.00. **5,** A, $\$6.29\frac{17}{27}$; B, $\$6.20\frac{10}{127}$. **6,** $37\frac{1}{2}$c. **7,** Sugar, $\$41.29\frac{5}{7}$; coffee, $\$55.06\frac{2}{7}$. **8,** $\$2,526\frac{6}{19}$; $\$2,105\frac{5}{19}$; $\$3,368\frac{8}{19}$.

Page 28—1, A, $\$3,529\frac{49}{119}$; B, $\$2,420\frac{20}{119}$; C, $\$6,050\frac{50}{119}$. **2,** J., $1,500; H., $2,400. **3,** A, $2,250; B, $3,600; C, $3,000. **4,** 72. **5,** A, $18; B, $27; C, $60. **6,** A, 50; B, 60; C, 70. **7,** 630 ac. **8,** $14,784; $16,362; $22,674.

Page 29—1, Man, $33.60; woman, $24; child, $14.40. **2,** Man, $1.40; woman, 84c.; boy, 60c. **3,** Man, $11.34; woman, $7.56; $5.04. **4,** $35; $52.50; $84. **5,** $2,160. **6,** 100 ac. **7,** $200. **8,** $35.

Page 30—1, 48 ft.; 72 ft. **2,** 300 persons. **3,** Gunpowder, $133\frac{1}{3}$ cwt.; nitre, 100 cwt.; sulphur, $13\frac{1}{3}$ cwt. **4,** A, $508; B, $438; C, $254. **5,** House, $1,200; lot, $900. **6,** A, $2,500; B, $3,000; C, $4,000. **7,** $25\frac{5}{7}$ lbs.

Page 33—1, Seventy-five hundredths. **2,** Seventy-five thousandths. **3,** Three-hundred-twenty-five thousandths. **4,** Three-hundred-twenty-five ten-thousandths. **5,** Two-hundred-fifteen thousandths. **6,** Two-hundred-fifteen ten-thousandths. **7,** Six-hundred-twenty-five thousandths. **8,** Six-hundred-twenty-five ten-thousandths. **9,** Six-hundred-twenty-five hundred-thousandths. **10,** Nine-hundred-thirty-seven thousandths. **11,** Nine-hundred thirty-seven ten-thousandths. **12,** Nine-hundred-thirty-seven hundred-thousandths. **13,** Eleven-thousand-six-hundred-twenty-five hundred-thousandths. **14,** Eighty-three-thousand-seven-hundred-fifteen hundred-thousandths. **15,** Five-hundred-seventeen-thousand-three-hundred-eighty-two

millionths. **16,** Eight-million-two-hundred-sixty-seven-thousand-five-hundred-thirteen ten-millionths. **17,** Seventeen-thousand-eight-hundred-thirty-four ten-millionths. **18,** Five and eighty-three-thousand-two-hundred-ninety-one millionths. **19,** Seven and fifty-six-thousand-eight-hundred-thirty-two ten-millionths. **20,** Eight and ten-thousand-five ten-millionths. **21,** Six and ten-million-seventy-thousand-eighty-two hundred-millionths. **22,** One and fifteen-thousand-eight-hundred-millionths. **23,** Six and five-thousand-eight-hundred-thirty-two-three-quarter millionths. **24,** Four and ten-million-seventy-five-thousand-eight-two-third hundred-millionths.

Page 33—1, .7. **2,** .07. **3,** .027. **4,** .3225. **5,** .0075. **6,** .80008. **7,** .00909. **8,** .00008. **9,** .00324. **10,** .0034. **11,** 8.24. **12,** 25.3125. **13,** .000226. **14,** .06045. **15,** .00005. **16,** 2.5. **17,** .5. **18,** .000204. **19,** .0024. **20,** .0524. **21,** .0025. **22,** 4.035. **23,** .000087. **24,** 1,200.21.

Page 34—1, $\frac{1}{2}$. **2,** $\frac{4}{5}$. **3,** $\frac{1}{4}$. **4,** $\frac{17}{20}$. **5,** $\frac{3}{50}$. **6,** $\frac{3}{8}$. **7,** $\frac{5}{8}$. **8,** $\frac{3}{10}$. **9,** $\frac{11}{125}$. **10,** $\frac{388}{625}$. **11,** $16\frac{3}{4}$. **12,** $5\frac{8}{125}$. **13,** $\frac{1}{160}$. **14,** $5\frac{1}{800}$. **15,** $18\frac{1}{32}$. **16,** $7\frac{258}{800}$. **17,** $3\frac{4}{3125}$. **18,** $5\frac{1}{32}$. **19,** $50\frac{1}{2500}$. **20,** $\frac{5}{8}$. **21,** $\frac{5}{8}$. **22,** $\frac{53}{160}$. **23,** $\frac{1}{125}$. **24,** $25\frac{1}{6}$.

Page 36—1, .5. **2,** .75. **3,** .8. **4,** .625. **5,** .875. **6,** .5555 +. **7,** .3125. **8,** .3333 +. **9,** .6666 +. **10,** .0625. **11,** .4166 +. **12,** .226. **13,** .76. **14,** .01562 +. **15,** .00024. **16,** 19.66 +. **17,** 415.62 +. **18,** 75.583 +. **19,** 91.281 +. **20,** 278.0468 +. **21,** 67.5555 +. **22,** 8.05468 +. **23,** 15.54545 +. **24,** 46.34287 +. **25,** 75.19531 +. **26,** 812.931281 +. **27,** 73.418333 +. **28,** 2.9117187 +. **29,** 18.2736666 +.

Page 37—1, 120.0379. **2,** 340.912962. **3,** 8741.2062. **4,** 970.17047. **5,** 228.845495. **6,** 1737.92777625. **7,** 69.21875. **8,** 4.71627. **9,** $3\frac{853}{15000}$. **10,** 627.176875. **11,** 55.09375. **12,** 488.699066.

Page 38—1, 16.1524. **2,** 2.3806. **3,** .43876. **4,** .23296. **5,** 1.8316. **6,** .00521. **7,** 3.9249. **8,** 1.405. **9,** 168.098. **10,** .01. **11,** .6322. **12,** 8.3416.

Page 39—1, .6. **2,** .153. **3,** .0744. **4,** 2.418. **5,** .02315. **6,** .318971. **7,** .1762. **8,** 765.357. **9,** .0000322. **10,** 12.2136. **11,** .00041588. **12,** .212577. **13,** 2.208015. **14,** 5.7431. **15,** .0105105. **16,** .9375.

Page 41—1, .0225. **2,** .0004928. **3,** .0027847 +. **4,** .0110792. **5,** 3.375. **6,** .4928. **7,** 6.104. **8,** .37466 +. **9,** 1,500. **10,** 112.66 +. **11,** 235,750. **12,** 127,000. **13,** 483.3333 +. **14,** 1.168 +. **15,** .0553 +. **16,** 2.80246 +. **17,** 1.1075 +. **18,** .020 +. **19,** .0277 +. **20,** 1120 +. **21,** 1.71 +. **22,** 1.6666 +. **23,** 4.051927 +. **24,** 2.548 +.

Page 42—1, 158.43. **2,** 19,303.821. **3,** 1.4019. **4,** .316. **5,** .0004. **6,** 19.162. **7,** .26005.

Page 46—1, 6.328. **2,** 485.46. **3,** .11233. **4,** 11.221. **5,** 332.941. **6,** 11.8629. **7,** .992425. **8,** .021.

Page 47—1, $350.31. **2,** $228.33. **3,** $278.04. **4,** $924.57. **5,** $1,156.62. **6,** $2,316.25. **7,** $2,162.31. **8,** $1,276.16. **9,** $2,682.79. **10,** $3,276.83.

Page 49—1, .3. **2,** .6. **3,** .16. **4,** .83. **5,** .428571. **6,** .857142. **7,** .416. **8,** .81. **9,** .3571428. **10,** .46428571. **11,** .43. **12,** .523809. **13,** .27. **14,** .316. **15,** .538461. **16,** $\frac{5}{9}$. **17,** $\frac{8}{11}$. **18,** $\frac{52}{111}$. **19,** $\frac{17}{111}$. **20,** $\frac{41}{111}$. **21,** $\frac{2045}{9999}$. **22,** $\frac{347}{1111}$. **23,** $\frac{121}{550}$. **24,** $4\frac{19}{180}$. **25,** $\frac{787}{1665}$. **26,** $7\frac{1}{999}$. **27,** $2\frac{1751}{3300}$. **28,** $\frac{151}{495}$. **29,** $\frac{52}{1650}$. **30,** $\frac{27}{11000}$.

Page 50—1, 50c. **2,** 15.82\frac{1}{2}$. **3,** 27 prs. **4,** 2.08. **5,** $\frac{37}{40}$. **6,** .01$\frac{1}{2}$. **7,** .11. **8,** 168,000. **9,** $5\frac{17}{75}$. **10,** 30.98.

Page 50—1, 4.60$\frac{5}{9}$. **2,** 319.37\frac{1}{2}$. **3,** 2,000 sheep. **4,** W., $7,678.50; d., $5,119. **5,** $6,120. **6,** $11.13. **7,** $877.50. **8,** 488. **9,** 63.295 ac. **10,** .042.

Page 51—1, 304.212 gals. **2,** .0013. **3,** 40 gals. **4,** 102.983 tons. **5,** 9.045 bus. **6,** 263.2702. **7,** $42,880. **8,** 150.08 ac. **9,** 5.0420168 +. **10,** £472 12s. $4\frac{8}{9}$d.

Page 52—1, $76.4953125. **2,** .03493. **3,** $6\frac{114}{2069}$. **4,** $816.66\frac{2}{3}$. **5,** $232.66\frac{2}{3}$. **6,** $14.90. **7,** .13; $3,000.

Page 62—1, 256,915 in. **2,** 12,604d. **3,** 203 qts. **4,** 155,243 in. **5,** 505 gi. **6,** 333,515 cu. in. **7,** 7,448,775 sq. in. **8,** 17,043 lk. **9** 136,049 ozs. **10,** 3,965 in. **11,** 4,131$\frac{3}{4}$ sq. ft. **12,** 444,088$\frac{3}{8}$ sq. ft. **13,** 4,989,931 sec. **14,** 218 cu. ft. **15,** 269 pts **16,** 129,832 in.

Page 63—1, 25 tons 8 cwt. 46 lbs. 15 oz. **2,** 120 rd. 9 in. **3,** 7 tons 13 cwts. 1 st. **4,** 534 gals. 2 qts. **5,** 2 dys. 3 hrs. 36 min. **6,** 5 cu. yds., 26 cu. ft., 436 cu. in. **7,** £33 14s. 1d. 3 far. **8,** 3,734 cords, 8 cu. ft. **9,** 444 reams, 17 quires, 18 sheets. **10,** 78 half-crowns, 2s. 4d. **11,** $45.63. **12,** 40 bus. 1 qt.

Page 64—1, 18 hrs. **2,** 186 rds. 3 yds. 2 ft. **3,** 17 dys. 3 hrs. 25 min. 42$\frac{8}{7}$ sec. **4,** 101 sq. rds. 24 sq. yds. 6 sq. ft. 108 sq. in. **5,** 11 cu. ft. 432 cu. in. **6,** 3 qts. 1$\frac{1}{5}$ pts. **7,** $\frac{1}{5}$ pt. **8,** $1\frac{18}{89}$ in. **9,** 200 rds. **10,** 14 cwt. 54 lbs. **11,** 100 sq. rds. **12,** 3 cd. ft. 6.08 cu. ft. **13,** 2.15 qts. **14,** 5 ozs. 1 dr. 8$\frac{4}{7}$ grs.

Page 65—1, $\frac{39}{512}$. **2,** .012803. **3,** $\frac{5}{176}$. **4,** $\frac{137}{2850}$. **5,** $\frac{889}{9801}$. **6,** $231.78. **7,** $1,220.50. **8,** $367.66. **9,** $606.57. **10,** $858.90. **11.** $145.43. **12,** $1,034.03.

Page 66—1, £279 6s. 0d. **2,** 189 mi. 307 rds. 4 yds. 1 ft. 4 in. **3,** £719 14s. **4.** 21.9416 s.

Page 67—1, 3 mi. 76 rds. 4 yds. 2 ft. 6 in. **2,** 1 bus. 2 pks. 1 qt. **3,** 20 gals. 3 qts. **4,** 5 tons 18 cwts. 41 lbs. **5,** £2,955 17s. 3 far.

Page 68—1, 142 bus. 2 pks. 3 qts. **2,** 119 cd. 53 cu. ft. 432 cu. in. **3,** 4,338 acs. 23 sq. rds. 5 sq. yds. 7 sq. ft. 36 sq. in. **4,** 1373 bus. 2 pks. **5,** 1,010 mi. 35 rds. 2 ft. 6 in.

Page 69—1, 1 bus. 3 pks., 1 gal. 3$\frac{1}{9}$ qts. **2,** 1 ton 1 cwt. 82 lbs. 8 ozs. **3,** £4 15s. 10d. 3$\frac{3}{8}$ far. **4,** $19.97. **6,** (*a*) £38 3$\frac{21}{73}$d., (*b*) £513 13s. 11$\frac{49}{73}$d.

SECTION 2

Page 77—1, 22,525. **2,** 31,225. **3,** 69,330. **4,** 65,800. **5,** 65,975. **6,** 57,870. **7,** 74,200. **8,** 80,810. **9,** 299,500. **10,** 148,800.

Page 78—1, 4,507,425. **2,** 9,063,404. **3,** 8,739,450. **4,** 2,218,064. **5,** 7,409,059. **6,** 3,786,282. **7,** 10,189,206. **8,** 14,116,544. **9,** 21,454. **10,** 20,589. **11,** 21,902. **12,** 28,915.

Page 79—1, 28,224. **2,** 64,152. **3,** 61,138. **4,** 48,015. **5,** 15,904. **6,** 11,968. **7,** 103,460. **8,** 95,876. **9,** 513,432. **10,** 720,896. **11,** 1,526,688. **12,** 417,600. **13,** 2,088,384. **14,** 7,494,900. **15,** 11,367,504. **16,** 5,824,140. **17,** 4,818,170. **18,** 60,713,750.

Page 81—1, $1, $5, $9. **2,** $1, $9, $19. **3,** $1, $3, $5. **4,** $8, $12. $20. **5,** $1, $6, $12. **6,** $1, $6, $5. **7,** $30, $12, $18. **8,** $6, $18, $50. **9,** $2, $4, $16. **10,** $15, $25, $45. **11,** $14, $63, $94.50. **12,** $6, $10. $14. **13,** $6, $10, $16. **14,** 8\frac{3}{4}$, $13, $26. **15,** $18, $24, $21. **16,** $63. $99, $36. **17,** $26.88, $148.44, $116.56. **18,** $163.63, $99.75, $409.94, **19,** $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{5}$. **20,** 9,375, 468.75.

Page 83—1, $35.88. **2,** $18.23. **3,** $52.65. **4,** $69.26. **5,** $47.09.

6, $143.52. **7,** $17.14. **8,** $34.22. **9,** $14.62. **10,** $104.08. **11,** $956.87. **12,** $43.13. **13,** $650. **14,** $2,250. **15,** $9.36. **16,** $1,094.98.

Page 83—2, 90 yds. **3,** $602.68. **4,** $26.250. **5,** 253 girls. **6,** 544 sheep. **7,** $623.90. **8,** $624.12. **9,** R.E., $4,320; B.S., $6,480; C.L., $6,480. **10,** $225.

Page 84—1, $3,562.50. **2,** $11,760. **3,** $3,784. **4,** $70. **5,** $1,890. **6,** $626.40. **7,** $27,324. **8,** $76.50. **9,** $22,500.

Page 85—1, 3,640. **2,** $648. **3,** $975. **4,** $9,780. **5,** $11,578. **6,** $160. **7,** 255 students. **8,** 5,850.

Page 86—1, 1,800. **2,** 500 sheep. **3,** $2,158.50. **4,** 500 ac. **5,** $3,400. **6,** $153. **7,** $563.50. **8,** 90 lbs.

Page 91—1, $39.59. **2,** $17.56. **3,** $26.08. **4,** $54.80. **5,** $453.49. **6,** $2,055.54.

Page 93—1, $18.26. **2,** 10 dys, $199.96; cash $11.20. **3,** 10 dys., $135.59; net 17.85; total, $153.84. **4,** $16.57. **5,** $54.47.

Page 96—1, 37%. **2,** $37\frac{1}{2}$%. **3,** $21\frac{1}{4}$%. **4,** $56\frac{1}{5}$%. **5,** $34\frac{8}{20}$%. **6,** $322.87. **7,** $4,863.75; $4,620.56.

Page 97—1, $450. **2,** $216. **3,** $143.43. **4,** $14.20. **5,** $128.30. **6,** $243.71. **7,** $136.80. **8,** $155.52. **9,** $219.90. **10,** $192.24. **11,** $188.03. **12,** $36.19. **13,** $428.40. **14,** $234.44. **15,** Latter; $2.10. **16,** $12.96. **17,** 160 hats.

Page 98—1, 28%. **2,** 40%. **3,** 40%. **4,** $49\frac{3}{5}$%. **5,** 62%. **6,** 46%.

Page 98—1, $3.20. **2,** $14.81. **3,** $6.66\frac{2}{3}$. **4,** $41.15. **5,** $30.

Page 108—1, $24. **2,** $21.60. **3,** $12.60. **4,** $18. **5,** $64. **6,** $65. **7,** $57.60. **8,** $120. **9,** $24. **10,** $1.60. **11,** $7.80. **12,** $4.50.

Page 109—1, $7.25. **2,** $11.84. **3,** $5.89. **4,** $33.97. **5,** $51.98. **6,** $18.40. **7,** $41.96. **8,** $108.89.

Page 109—1, $27.18. **2,** $141.14; $235.23. **3,** $72.68; $93.43. **4,** $65.23. $70.66. **5,** $158.02; $182.33.

Page 110—1, $1,300. **2,** $750. **3,** $1,133\frac{1}{3}$. **4,** $192. **5,** $300. **6,** $350. **7,** $85. **8,** $630. **9,** $975. **10,** $840. **11,** $500. **12,** $1,000.

Page 110—1, $500. **2,** $2,000. **3,** $2,400. **4,** $80. **5,** $201.17\frac{11}{16}$. **6,** $440. **7,** $4,380. **8,** $2,189.47. **9,** $2,555. **10,** $3,285. **11,** $730. **12,** $511. **13,** $584. **14,** $438.

Page 113—1, $2.44. **2,** $7.26. **3,** $6.01. **4,** $11.92. **5,** $14.58. **6,** $11.20. **7,** $71.45. **8,** $6.21. **9,** $12.12. **10,** $3.34. **11,** $40.12. **12,** $7.36. **13,** $4.11. **14,** $21.90. **15,** $2.70. **16,** $14.84. **17,** $11.27. **18,** $3.27. **19,** $23.21. **20,** $7.20. **21,** $8.06. **22,** $12.01. **23,** $.39. **24,** $.44. **25,** $1.54. **26,** $69.80. **27,** $6.75. **28,** $13.68. **29,** $128.17. **30,** $9.46. **31,** $16.51. **32,** $47.06. **33,** $68.14. **34,** $188.66. **35,** $100.62. **36,** $17.36.

Page 117—1, Feb. 6, 1943; 33 dys. **2,** Feb. 7, 1943; 34 dys. **3,** Mar. 14, 1943; 33 dys. **4,** Mar. 12, 1943; 31 dys. **5,** Mar. 13, 1944; 33 dys. **6,** Mar. 12, 1944; 32 dys. **7,** Mar. 1, 1943; 63 dys. **8,** Mar. 3, 1943; 65 dys. **9,** Mar. 3, 1943; 64 dys. **10,** Mar. 3, 1943; 63 dys. **11,** Mar. 3, 1943; 62 dys. **12,** Sept. 3, 1943; 34 dys. **13,** Oct. 3, 1943; 64 dys. **14,** Jan. 4, 1944; 34 dys. **15,** Jan. 2, 1944; 34 dys. **16,** $500.00; $501.51; $531.10. **17,** $506.14; $507.65; $537.24. **18,** $503.51; $505.02; $534.61. **19,**

$506.14; $508.25; $549.67. **20,** $504.38; $505.89; $535.48. **21,** Apr. 16, 1924; $425.00; $428.67. **22,** Apr. 19, 1924; $425.00; $428.49.

Page 118—1, 6%. **2,** 4%. **3,** 4%. **4,** 3%. **5,** 8%. **6,** 3%. **7,** 3½% **8,** 5%. **9,** 6%.

Page 120—1, Apr. 8, 1944; 94 dys.; $5.68; $361.82. **2,** Mar. 3, 1944; 31 dys.; $1.80; $421.90. **3,** Mar. 2, 1944; 52 dys.; $.99; $125.46. **4,** Sept. 1, 1944; 185 dys.; $16.84; $536.75. **5,** Apr. 16, 1944; 60 dys.; $1.36; $164.33. **6,** Dec. 18, 1943; 33 dys.; $.41; $75.16. **7,** Mar. 3, 1944; 89 dys.; $3.00; $242.71. **8,** Dec. 21, 1944; 219 dys.; $13.11; $351.12. **9,** June 17, 1943; 47 dys.; $1.30; $167.45. **10,** Sept. 3, 1943; 33 dys.; $1.32; $241.93. **11,** $573.99. **12,** $1,506.61. **13,** $129.36. **14,** $417.65 in their favor. **15,** $161.17. **16,** $59.34.

Page 123—1, $913.97. **2,** $1,340.23. **3,** $430.96. **4,** $3,528.25. **5,** $622.52. **6,** $356.12. **7,** $7,236. **8,** $8,563.75.

Page 125—1, Secord, $180; Caldwell, $220; Secord pay Caldwell $20. **2,** Leeson, $703.33; Martyn, $513.33; Baker, $566.67; Martyn and Baker pay Leeson $81.11 and $27.78 respectively (gains and losses divided equally). **3,** White, $478.33; Johnson, $350; White pay Johnson $73.89. **4,** Brown, $481.27; James, $328.90; Brown pay James $4.83. **5,** Kent, $774.98; Todd, $500.30; Tufford, $385.95; Todd debited $53.44, Kent credited $36.65, Tufford credited $16.79.

Page 129—1, $11,125.00. **2,** $1,700.00. **3,** $2,428.93. **4,** $53.33. **5,** 4%; 10%. **6,** 20%. **7,** $5.77. **8,** S.P., $42.00; gain, $2.00; $4\frac{16}{21}$%. **9,** $4,357.14$\frac{2}{7}$.

Page 130—1, Loss, $32,994.35; 9.948 % loss. **2,** Loss, $1,597.12; .44 + %. **3,** Gain, $24,271.28; gain, 8.792 + %; gain, 9.64%.

Page 131—1, 12½%. **2,** 29¾%. **3,** 10%. **4,** 8%. **5,** 90%. **6,** 77⅓%. **7,** 27%. **8,** 84⅓%. **9,** 16⅔%. **10,** 15%. **11,** 2%. **12,** 20%. **13,** 75%. **14,** ½%. **15,** ½%. **16,** ½%. **17,** $42\frac{8}{7}$%. **18,** 25%. **19,** 20%. **20,** 13%. **21,** 100%. **22,** 16⅔%. **23,** 9%. **24,** 20%. **25,** 25%.

Page 134—1, $4,500. **2,** 1,000 bus. **3,** 63 gals.; 57.96 gals. **4,** 584 bus. 1 pk. 1 gal. **5,** $251.67. **6,** $260. **7,** $1,010.60. **8,** $3,600. **9,** $360. **10,** 10c. **11,** $30. **12,** $4.80. **13,** $2.25. **14,** $14. **15,** $8.33⅓. **16,** $19.20. **17,** $14.40. **18,** $48.

Page 134—1, $101.04. **2,** $63. **3,** $45. **4,** $240 loss. **5,** $5.70. **6,** $416. **7,** 6⅔% loss. **8,** $4,297.28.

Page 135—1, G., $24.75; S.P., $173.25. **2,** $6,406.25. **3,** Gain, $74. **4,** Gain, $105. **5,** Gain, $37.50. **6,** 18¾%. **7,** $3,200.00. **8,** $8,000.00. **9,** $30.00; $50.00. **10,** 22% loss. **11,** 33⅓%. **12,** $58.60. **13,** 12½% gain on sales. **14,** 70.47 + %.

Page 137—1, Com., $225; Pro., $11,025. **2,** $93.75. **3,** $1,110. **4,** $3,937.50. **5,** $2,004.20. **6,** $5,162.80.

Page 139—1, 1⅛%. **2,** $\frac{3}{32}$%. **3,** $\frac{3}{16}$%. **4,** 5%. **5,** 4%. **6,** 2%. **7,** ¼%. **8,** 3½%. **9,** 4½%. **10,** 1⅓%; 2½%. **11,** $2\frac{2}{5}$%. **12,** 6%; 4½%. **13,** 3½%. **14,** 2¾%.

Page 141—1, 9,000 bus. **2,** $16,910.40. **3,** $90,000. **4,** 50 bales. **5,** $536. **6,** 7,360 bus.

Page 141—1, $6,000. **2,** 12,000 yds. **3,** $1,760. **4,** 5½c. **5,** 10,666⅔ yds. **6,** 225 bbls.

Page 142—1, $700. **2,** 50%. **3,** 2,000 lbs.; $16.20. **4,** $2,832. **5,** $7,375.

Page 143—1, 17⅜%. **2,** $4,608. **3,** $1.50. **4,** S.P. Wheat, $15,468.75, C.P. Cotton, $14,437.50. **5,** (*a*) 41 carloads, (*b*) $965 commission, (*c*) $285 unexpended. **6,** 72 c. **7,** $594. **8,** 5½%.

Page 153—1, Dividend, $50; % on Invt., 4.405%. **2,** Dividend, $375; % on Invt., 13.514%. **3,** Total Invt., $2,950; % on Invt., 4.407%. **4,** (*a*) 144 shares, (*b*) $14 unexpended, (*c*) Increased $52. **5,** Outlay, $10,955; Income $450; % on Invt., 4.107+%.

Page 154—1, 5½%. **2,** 5%. **3,** 4⅛%. **4,** 6%. **5,** 5%. **6,** 3%. **7,** 6%. **8,** 3½%. **9,** 4%. **10,** 8%.

Page 154—1, $3,100. **2,** $18,600 **3,** $25,920. **4,** $13,125. **5,** $60,100.

Page 155—1, 58 shares and $90.50 unexpended. **2,** 80 shares. **3,** 40 shares. **4,** 160 shares. **5,** 24 shares. **6,** 40 shares. **7,** 50 shares. **8,** 48 shares. **9,** 70 shares. **10,** 120 shares. **11,** 48 shares. **12,** 50 shares. **13,** 90 shares. **14,** 25 shares. **15,** 64 shares. **16,** 96 shares.

Page 156—1, $133⅓. **2,** $85⅝. **3,** $87½ **4,** $119.80. **5,** $87.25.

Page 156—1, 5%. **2,** 6.726+%. **3,** 6.65+%. **4,** 7.936+%. **5,** 7.604+%. **6,** $7,256.25. **7,** $2,452.25. **8,** $4,122.25. **9,** 710 shares and $3.10 unexpended. **10,** Latter by .303%. **11,** $209.25. **12,** $6,997.50. **13,** $50. **14,** $1,288 and $7.15 unexpended. **15,** Increase $55.

Page 160—1, $294.45. **2,** $87.95. **3,** $172.53. **4,** $624.47 (using Oct. 23 for date of second sale). **5,** $67.49. **6,** $4,496.70; $4,602.95.

Page 162—1, $135. **2,** $22.40. **3,** $412.50. **4,** $110.25. **5,** $228.92 **6,** $210.

Page 163—1, 3 mills; $12. **2,** 12 mills. **3,** 9 mills. **4,** 11 mills. **5,** 138c. **6,** 98c. **7,** 6 mills. **8,** $56,000. **9,** $2,500,000.

Page 167—1, $8,685.66. **2,** $227.50. **3,** Insured receives $20,000 under ordinary policy; $15,000 under 80% co-insurance policy. **4,** $16.15. **5,** Saves $228 annually. **6,** $8,888.89. **7,** $2,843.75. **8,** $244,788.

Page 168—1, 66⅔c. **2,** 65c. **3,** 96c. **4,** 7%; 87½c. **5,** $1.50. **6,** $15,600. **7,** $300,000. **8,** $12,873.56. **9,** $47,733⅓.

Page 171—1, $1,958.75. **2,** $6,000. **3,** $900. **4,** $75. **5,** $975. **6,** $192. **7,** $399. **8,** $1,119.20. **9,** $4,278.42. **10,** $905.05. **11,** $1,970.90.

Page 171—1, Duty $6.30. **2,** Duty $6.45. **3,** Duty $265.80. **4,** Duty $154.35. **5,** Duty $42. **6,** Duty $26.40. **7,** Duty $46.48.

Page 175—1, 173 lbs. **2,** 49c. **3,** 30$\frac{11}{16}$c. **4,** (*a*) 465; (*b*) $497.55; (*c*) $1.07. **5,** $6,000. **6,** W. 4246.30; T, 3,643.84. **7,** L, $2,283.13; T, $1,765. **8,** $23.24. **9,** $85.44. **10,** $483.96.

Page 179—1, $2.74. **2,** $7.96. **3,** $72.15. **4,** A, $540; B, $66.0. **5,** Hess, $2,080; Dunn, $1,540; Norris, $1,700. **6,** A, $2696.38; B, $2876.13; C, $2927.49.

Page 181—1, 11 : 4. **2,** 4 : 18 : 3. **3,** $16\frac{55}{1337}$ qts., $24\frac{1068}{1337}$ qts., $20\frac{1319}{1337}$ qts., $18\frac{244}{1337}$ qts. **4,** 24 lbs., 36 lbs. **5,** 40c.; 31c. **6,** 45c.; 65c. **7,** 12,359 ft. **8,** A, $6,000; B, $10,000.

Page 182—1, 30.98. **2,** 51 gals. **3,** $3,200; 15 mos. **4,** $11,200. **5,** 80 at 55c.; 120 at 75c. **6,** 15 gals. **7,** 12 gals. **8,** 14 bus. rye; 14 bus. barley.

Page 183—1, 20 lbs. at 33c.; 60 lbs. at 37c.; 40 lbs. at 45c. **2,** 14 lbs. at 14c.; $23\frac{1}{3}$ lbs. at 18c.; $46\frac{2}{3}$ lbs. at 30c. **3,** 2 lb. at 10c.; 4 lb. at 13c.; 12 lb. at 15c.; 18 lbs. at 17c.; 9 lbs. at 18c. **4,** 9 : 2. **5,** 35c. **6,** 50%. **7,** (*a*) 14 gals.; (*b*) 30 gals. **8,** 2 qts.

Page 188—1, $1,182.19. **2,** $1,149.95. **3,** $630.67. **4,** $2,083.40. **5,** $1,939.95. **6,** $247.48. **7,** $567.84. **8,** $25.95. **9,** 1.1698585. **10,** 2.012197. **11,** 2.563304. **12,** $638.09. **13,** $407.19.

Page 189—1, $150. **2,** $875.25. **3,** $1,016.79. **4,** $4,448.99. **5,** $316.68. **6,** $1,051.56. **7,** $402.49. **8,** $560.

Page 190—1, 6%. **2,** 7%. **3,** 4%. **4,** 3%.

Page 190—1, 20 yrs. **2,** 13 yrs. **3,** 22 yrs. **4,** 7 yrs.

Page 190—1, $8,567.76. **2,** $4,559.90. **3,** $3,302.73. **4,** $7,246.46. **5,** $9,038.14. **6,** $902.01. **7,** $1,200. **8,** $1,845. **9,** 6%. **10,** 7%.

Page 191—1, 11 yrs. **2,** $9\frac{1}{2}$ yrs. **3,** 6.09%. **4,** 5.912%. **5,** $4,448.99.

SECTION 3

Page 199—1, $1,797.00. **2,** $938.74. **3,** $4,091.39. **4,** $2,550.79. **5,** $7,993.36. **6,** $4,611.59. **7,** $1,865.53. **8,** $2,890.40. **9,** $1,300.80. **10,** $119.41. **11,** $3,492.42. **12,** $1,859.73. **13,** $423.30. **14,** $955.68. **15,** $703.57.

Page 200—1, $109\frac{1}{4}$. **2,** $109\frac{1}{2}$. **3,** $108\frac{3}{4}$. **4,** 23.9c. **5,** $23\frac{1}{2}$c. **6,** 40.37 +. **7,** 3.95c. **8,** 3c. **9,** 3.915 c.

Page 201—1, £244 17s. 4d. **2,** £560. **3,** 25,000 mks. **4,** 55,035.10 fr. **5,** £1,195 17s. 6.30 + d. **6,** 1872.59 reichsmarks. **7,** 24,048.61fr. **8,** 3,600 glds.

Page 202—1, 3075.6 fr. **2,** $838.19. **3,** 2.13 fr. **4,** $166.05. **5,** $4.86. **6,** $2,602\frac{1}{2}$ fl.

Page 205—1, $79.40. **2,** $52.20. **3,** $215.50.

Page 212—1, A's P.W., $16,987\frac{1}{2}$; B's P.W., $13,635\frac{1}{2}$; C's P.W., $12,677\frac{1}{2}$. **2,** H. Mann's P.W., $2,029.75; J. Ross' P.W., $3,970.25. **3,** A's P.W., $69,833\frac{1}{2}$; B's P.W., $21,600; C's P.W., $8,766\frac{2}{3}$. **4,** A's P.W., $19,078.03; B's P.W., $13,342.92; C's P.W., $11,879.05. **5,** $300. **6,** $420. **7,** A, $833\frac{1}{3}$; B, $1,033\frac{1}{3}$; C, $1,133\frac{1}{3}$. **8,** Thompson's net credit, $15,972.50; Wilson's net debit, $672.50. **9,** G's net capital, $18,396.28; H owes remaining partners $1,136.43 interest; H owes N $620.88. **10,** H's insolvency at commencement, $331.69; G's insolvency at commencement, $331.70; H's net capital at closing, $476.67; G's net capital at closing, $1,111.66. **11,** S's net capital at closing, $5,560; G's net capital at closing, $5,220; H's net capital at closing, $575. **12,** Wright's account shows deficit of $60, and Lang's a surplus of $465. There is only $405 cash on hand. Lang gets the cash, and must lose the $60 which Wright is unable to contribute. **13,** C, $118.48; G, $338.52. **14,** H, $9,335.67; J, $20,664.33. **15,** The proceeds of assets are $18,000. This is applied

as follows: To creditors, $10,500; to A (loan), $3,000; to B (loan), $750; to A (capital), $3,750.

Page 220—1, $543.09. **2,** $798.36. **3,** $773.76. **4,** $1,242.12. **5,** $526.83. **6,** $814.13. **7,** $481.60. **8,** $221.92. **9,** $192.60. **10,** $817.19. **11,** $319.18. **12,** $713.77. **13,** $3,690.75. **14,** $4,998.935. **15,** $900. **16,** $100.

Page 227—1, (*a*) $552.12; (*b*) Feb. 21, 1944. **2,** (*a*) $378.91; (*b*) Nov. 15, 1943. **3,** (*a*) $73.39; (*b*) Oct. 14, 1944. **4,** (*a*) $80.17; (*b*) Dec. 9, 1944. **5,** (*a*) $122.33.; (*b*) Feb. 22, 1945. **6,** (*a*) $333.66; (*b*) Aug. 8, 1943. **7,** (*a*) $198.04; (*b*) June 13, 1944. **8,** (*a*) $168.27; (*b*) Mar. 25, 1944. **9,** (*a*) $2.29; (*b*) Sept. 8, 1944. **10,** (*a*) $794.41; (*b*) Nov. 27, 1944.

Page 234—1, Mar. 14, 1944. **2,** June 7. **3,** June 7, 1944. **4,** Mar. 20, 1945. **5,** 21 dys. **6,** 157 dys.; Aug. 14. **7,** Sept. 21. **8,** 72 mos., or 6 yrs. **9,** 64 dys. **10,** July 29, next year. **11,** Feb. 22, 1944. **12,** May 6, 1945. **13,** Oct. 12, 1944. **14,** Apr. 18, 1945. **15,** May 20, 1943. **16,** Sept. 25, 1943. **17,** Jan. 7, 1944. **18,** Aug. 27, 1943. **19,** July 21, 1943.

Page 239—1, $1,907.25; Mar. 18, 1944. **2,** $786.39; Jan. 3, 1944. **3,** $188.60; Apr. 11, 1944. **4,** $1,660.15; Feb. 15, 1944. **5,** $7,365.48; July 18, 1944. **6,** $5,355.71; Aug. 7, 1944.

Page 243—1, $484; $19.36. **2,** $720; $32.40. **3,** $108; $1.89; **4,** $127.75; $1.26. **5,** $584; $4.80. **6,** $74; $1.85. **7,** $912.50; $6.00. **8,** $2,555; $30.87. **9,** $1,478.13; $21.87. **10,** $3.24. **11,** $28.50. **12,** $22.67. **13,** $6.75 cash, by 19c. per barrel. **14,** $1,368.13. **15,** Latter by 41c. per barrel. **16,** By paying on 10th day $5.03 would be saved. **17,** 4.9956 + % gain. **18,** 20.4456 + % gain. **19,** Former by $56. **20,** $675.25. **21,** $100.80. **22,** $3.52. **23,** $37.09; 9.0068% nearly. **24,** $20.31. **25,** $\frac{5}{17}$. **26,** $\frac{3}{8}$. **27,** $72. **28,** 374.22\frac{5}{9}$. **29,** $119. **30,** $150. **31,** $78. **32,** $33. **33,** $785.46. **34,** $450; $5\frac{5}{9}$%. **35,** 38\frac{2}{17}$. **36,** $210. **37,** $18. **38,** $57. **39,** $5.75. **40,** 15 mos. **41,** True, $79.50; Bank, $32.50. **42,** 645.83\frac{1}{3}$. **43,** 83.32\frac{276}{1785}$. **44,** $320; 6%. **45,** 150 dys. **46,** 8 mos. **47,** 1.5c. nearly. **48,** 62.39\frac{1789}{8640}$. **49,** 80.83, $32. **50,** $11,000. **51,** $1,922.75.

Page 248—1, $5\frac{315}{7237}$%. **2,** $8\frac{288}{1786}$%. **3,** $4\frac{1424}{1469}$%. **4,** $6\frac{3514}{38731}$%. **5,** $6\frac{1}{12}$%. **6,** $6\frac{197}{1488}$%. **7,** $4\frac{574}{2267}$%. **8,** 8%; $7\frac{5483}{9321}$%. **9,** $9\frac{1317}{7147}$%. **10,** $7\frac{1447}{14570}$%.

Page 252—1, $4,613.28. **2,** $4,150.29. **3,** $3,155.42. **4,** $3,700.88. **5,** $1,659. **6,** $5,013.28. **7,** $5,963.97. **8,** $318.50.

Page 254—1, $231.71. **2,** $27,173.59. **3,** $1,660.97. **4,** $1,131.06. **5,** $77.88. **6,** (*a*) $9,979.28; (*b*) $9,962.55; (*c*) $9,910.44.

Page 256—1, $1,418.38. **2,** $11,118.39. **3,** $7,122.92. **4,** $232.29; $278.98. **5,** $6,250. **6,** $6,666$\frac{2}{3}$. **7,** $1,421.36. **8,** $5,137.04. **9,** 3.377 times the annuity. **10,** $2,675.49. **11,** $5,360.85. **12,** $10,560.14. **13,** $102.72. **14,** $956.71. **15,** $2,084.25. **16,** $101.37. **17,** $5,640.07. **18,** $1,932.82. **19,** $131.57. **20,** $310.26. **21,** $328.30. **22,** (*a*) $1,684.95; (*b*) $474.79. **23,** $5,385.12. **24,** $404.22. **25,** $306.32.

Page 259—1, 625. **2,** 125; 13,824; 4,741,632. **3,** 256; 4,096; 65,536; 390.625. **4,** $\frac{27}{64}$; $\frac{125}{216}$; $\frac{343}{512}$. **5,** 216; 81; 4,096; 100,000; 12,167. **6,** $\frac{8}{729}$; $\frac{256}{625}$; $42\frac{7}{8}$; $204\frac{4}{49}$; $129\frac{10657}{10976}$, or 129.746337890625. **7,** 34775.

Page 262—1, 24. **2,** 35. **3,** 45. **4,** 66. **5,** 105. **6,** 135. **7,** 123. **8,** 321. **9,** 199. **10,** 297. **11,** 867. **12,** 327. **13,** $\frac{15}{17}$. **14,** $\frac{13}{25}$. **15,** $\frac{16}{19}$.

16, $\frac{4}{15}$. **17,** 3.45. **18,** 203.1. **19,** .835. **20,** 7.39. **21,** 1.73205. **22,** 19.40103. **23,** 3.58092. **24,** 66.08479.

Page 265—1, 16. **2,** 32. **3,** 42. **4,** 79. **5,** 63. **6,** 85. **7,** 245. **8,** 438. **9,** 708. **10,** 8,765. **11,** 4,968. **12,** 7.3. **13,** .579. **14,** 36.4. **15,** 1.26. **16,** 2.444. **17,** $\frac{11}{12}$. **18,** $\frac{5}{7}$. **19,** .908. **20,** $7\frac{3}{5}$. **21,** 27. **22,** 45. **23,** 6.3. **24,** 13. **25,** 33 in. **26,** 15 in.; 15 in.; 30 in. **27,** 84 ft. **28,** 72 ft. **29,** 2 ft. 6 in.; 5 ft.; 7 ft. 6 in. **30,** 4,225 sq. ft. **31,** 61.3 in. **32,** 543 ft.

Page 268—1, 10 ac. **2,** 24,012 bricks. **3,** \$71.11$\frac{1}{9}$. **4,** \$43.09$\frac{1}{4}$. **5,** \$40.81$\frac{7}{9}$. **6,** 100 rods. **7,** \$36. **8,** 1,056 rds. **9,** 11,536 sq. ft. **10,** 2.487 rods.

Page 270—1, $42\frac{5}{16}$ sq. ft. **2,** \$147. **3,** $107\frac{47}{27}$ yds. **4,** \$6. **5,** \$63. **6,** (*a*) 30 ft., (*b*) 600 sq. ft. **7,** 8 ft. **8,** \$82.13$\frac{1}{8}$. **9,** 50 ft. **10,** 65 ft.

Page 272—1, 4 ft. 1 in. **2,** $10\frac{2}{7}$ ft. **3,** 20.16 in. **4,** 24.64 sq. ft. **5,** $2,142\frac{6}{7}$ sq. ft. **6,** $822\frac{6}{7}$. **7,** \$225.22$\frac{1}{2}$. **8,** .54 yds. **9,** Drive, $2\frac{2}{7}$ min. **10,** $508\frac{1}{5}$ tons.

Page 277—1, $3\frac{17}{27}$ cu. yd. **2,** 8 ft. **3,** $\frac{1}{4}$ in. **4,** 1,200 gals. **5,** \$264. **6,** 27 mi. **7,** 49,402 gals **8,** 9.518 in. nearly. **9,** 3,168 sq. in. **10,** 86.6468 cu. ft.

Page 277—1, 147.840 blocks. **2,** \$69.30. **3,** $13\frac{1}{8}$ sq. ft. **4,** \$364.80. **5,** $259\frac{17}{54}$ sq. yds. **6,** 243.72 ft. **7,** 24.748 + in. **8,** $17\frac{1}{2}$ in. **9,** 134.117 ft. **10,** $1,948\frac{4}{7}$ sq. yds.

Page 278—1, 729 bricks. **2,** 72 rds. **3,** 216 ac. **4,** $5\frac{5}{7}$ ft. **5,** \$41.10. **6,** \$49.60. **7,** \$30. **8,** \$1.70$\frac{1}{2}$ (without bottom). **9,** $64\frac{3}{4}$ sq. ft. **10,** 49 in.

Page 280—1, 180 ft. **2,** \$67.20. **3,** 15,100 ft. **4,** 810 ft. **5,** 36 ft. **6,** \$11.60. **7,** \$24.18. **8,** \$21.78. **9,** \$132. **10,** \$132.44. **11,** \$4,752. **12,** \$184.80. **13,** \$153.60. **14,** \$82.60. **15,** \$5.76. **16,** \$396. **17,** (*a*) \$7.65; (*b*) \$27; (*c*) \$6. **18,** \$330.75. **19,** \$506.88. **20,** \$120.96. **21,** \$316.80. **22,** \$12. **23,** 16 ft. **24,** 16 ft. **25,** 15 in. **26,** 8 in. **27,** \$36.90. **28,** \$41.00.

Page 284—1, 800 shingles. **2,** 720 shingles. **3,** 18 squares. **4,** \$252. **5,** 5 bunches. **6,** 168 bunches. **7,** \$126. **8,** \$95.55. **9,** (*a*) 125 bunches; (*b*) \$49·77; (*c*) \$24.59. **10,** 14,580 shingles.

Page 285—1, 19 bundles. **2,** 54 bundles. **3,** $40\frac{10}{81}$ sq. yds. **4,** \$47.19. **5,** 22 bundles. **6,** \$12.59. **7,** \$20.89$\frac{1}{8}$.

Page 288—1, $8\frac{55}{64}$ cds. **2,** 44 cu. yds. **3,** 18$\frac{1}{8}$ cds. **4,** $94\frac{23}{27}$ cu. yds **5,** 31,584 bricks. **6,** 46,760 bricks.

ANSWERS TO EXAMINATIONS

EXAMINATION 1

Page 22—1, $2\frac{23}{70}$. **2,** \$14,000. **3,** \$80, \$35. **4,** \$67,837.50. **5,** \$248.71. **6,** \$6, \$15, \$16. **7,** 6$\frac{1}{4}$ bu. **8,** 405 sheep.

EXAMINATION 2

Page 23—1, Sulphur, $323\frac{31}{220}$; Saltpetre, $215\frac{94}{220}$; Charcoal, $161\frac{155}{220}$. **2,** 13 fields. **3,** $40\frac{1369}{4077}$c. **4,** \$5.00. **5,** 336 trees. **6,** 22$\frac{1}{2}$ days. **7,** they require $16\frac{5}{7}$ days; A receives \$44.60$\frac{220}{343}$; B receives \$30.39$\frac{123}{343}$. **8,** loss, 52c.

EXAMINATION 3

Page 52—1, .0751383957246471. **2,** 6.875. **3,** 24,933 thousand feet.

4, (*a*) 500 in book-keeping dept.; (*b*) 375 in shorthand and typing dept.; (*c*) 125 in English dept.; (*d*) 1,000 in entire school. **5,** 2.92125. **6,** $5,950 total gain for year; $5,000 net gain for year. **7,** $7,500. **8,** $5,513.80 gain.

EXAMINATION 4

Page 70—1, 10,560. **2,** 369. **3,** 1,628. **4,** 154. **5,** 15. **6,** 225. **7,** 231. **8,** 9.

EXAMINATION 5

Page 191—1, $33\frac{1}{3}\%$. **2,** 6%. **3,** 20%. **4,** $4\frac{1}{2}\%$. **5,** 72c. **6,** $57.38. **7,** $3.60. **8,** $2.50; 4.50. **9,** $7,500,000. **10,** Percentage: 41.89; 23.84; 71.44; 37.37.

EXAMINATION 6

Page 192—1, $992.00. **2,** $946.17. **3,** $43,316.00. **4.** $240.00; $200.00. **5,** $10.15. **6,** $4\frac{1}{5}\%$. **7,** $22.00. **8,** $65.01. **9,** $3,125.00. **10,** $12\frac{1}{2}\%$.

EXAMINATION 7

Page 193—1, $51,834.92. **2,** $25\frac{12\frac{1}{2}}{47\frac{1}{2}}\%$. **3,** $4,800.00. **4,** $157.50. **5,** $200.00. **6,** $2.21. **7,** $17\frac{3}{5}\%$. **8,** $175.00. **9,** $50,203.13. **10,** 8%.